Korean J. Math. **29** (2021), No. 3, pp. 519–526
http://dx.doi.org/10.11568/kjm.2021.29.3.519

COEFFICIENT ESTIMATES FOR A NEW GENERAL SUBCLASS OF ANALYTIC BI-UNIVALENT FUNCTIONS

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Abstract. In a very recent paper, Yousef *et al.* [Anal. Math. Phys. 11: 58 (2021)] introduced two new subclasses of analytic and bi-univalent functions and obtained the estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions belonging to these classes. In this study, we introduce a general subclass $\mathcal{B}_{\Sigma}^{h,p}(\lambda,\mu,\delta)$ of analytic and bi-univalent functions in the unit disk $\mathbb{U}$, and investigate the coefficient bounds for functions belonging to this general function class. Our results improve the results of the above mentioned paper of Yousef *et al.*

1. Introduction

Let $\mathcal{A}$ denote the class of all functions of the form

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots \tag{1}$$

which are analytic in the unit disk

$$\mathbb{U} = \{z : |z| < 1\}.$$

We also denote by $\mathcal{S}$ the class of all functions in the normalized analytic function class $\mathcal{A}$ which are univalent in $\mathbb{U}$.

Since univalent functions are one-to-one, they are invertible and the inverse functions need not to be defined on the entire unit disk $\mathbb{U}$. In fact, the Koebe one-quarter theorem [5] ensures that the image of $\mathbb{U}$ under every univalent function $f \in \mathcal{S}$ contains a disk of radius $1/4$. Thus every function $f \in \mathcal{A}$ has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U})$$

and

$$f\left(f^{-1}(w)\right) = w \qquad \left(|w| < r_0(f);\ r_0(f) \geq \frac{1}{4}\right).$$

In fact, the inverse function f^{-1} is given by

$$f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots. \tag{2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1). For a brief

Received June 21, 2021. Revised August 6, 2021. Accepted August 11, 2021.
2010 Mathematics Subject Classification: 30C45.
Key words and phrases: Analytic function, univalent function, bi-univalent function, coefficient bounds.

history and interesting examples of functions in the class Σ, see [8] (see also [1]). In fact, the aforecited work of Srivastava *et al.* [8] essentially revived the investigation of various subclasses of the bi-univalent function class Σ in recent years (see, for example, [3,4,6,7,9]).

Recently, Yousef *et al.* [11] introduced the following two subclasses of the bi-univalent function class Σ and obtained non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ of functions in each of these subclasses.

DEFINITION 1.1. (see [11]) For $\lambda \geq 1,\ \mu \geq 0,\ \delta \geq 0$ and $0 < \alpha \leq 1$, a function $f \in \Sigma$ given by (1) is said to be in the class $\mathcal{B}_{\Sigma}^{\mu}[\alpha, \lambda, \delta]$ if the following conditions hold for all $z, w \in \mathbb{U}$:

$$\left|\arg\left\{(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} + \xi\delta z f''(z)\right\}\right| < \frac{\alpha\pi}{2}$$

and

$$\left|\arg\left\{(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu} + \lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1} + \xi\delta w g''(w)\right\}\right| < \frac{\alpha\pi}{2},$$

where the function $g = f^{-1}$ is defined by (2) and $\xi = \frac{2\lambda+\mu}{2\lambda+1}$.

THEOREM 1.2. *(see [11]) Let the function f given by (1) be in the class $\mathcal{B}_{\Sigma}^{\mu}[\alpha, \lambda, \delta]$. Then*

$$|a_2| \leq \frac{2\alpha}{\sqrt{(\lambda+\mu+2\xi\delta)^2 + \alpha\left[2\lambda+\mu-(\lambda+2\xi\delta)^2 + (12-4\mu)\xi\delta\right]}}$$

and

$$|a_3| \leq \frac{4\alpha^2}{(\lambda+\mu+2\xi\delta)^2} + \frac{2\alpha}{2\lambda+\mu+6\xi\delta}.$$

DEFINITION 1.3. (see [11]) For $\lambda \geq 1,\ \mu \geq 0,\ \delta \geq 0$ and $0 \leq \beta < 1$, a function $f \in \Sigma$ given by (1) is said to be in the class $\mathcal{B}_{\Sigma}^{\mu}(\beta, \lambda, \delta)$ if the following conditions hold for all $z, w \in \mathbb{U}$:

$$\Re\left\{(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} + \xi\delta z f''(z)\right\} > \beta$$

and

$$\Re\left\{(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu} + \lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1} + \xi\delta w g''(w)\right\} > \beta,$$

where the function $g = f^{-1}$ is defined by (2) and $\xi = \frac{2\lambda+\mu}{2\lambda+1}$.

THEOREM 1.4. *(see [11]) Let the function f given by (1) be in the class $\mathcal{B}_{\Sigma}^{\mu}(\beta, \lambda, \delta)$. Then*

$$|a_2| \leq \min\left\{\frac{2(1-\beta)}{\lambda+\mu+2\xi\delta}\ ,\ \sqrt{\frac{4(1-\beta)}{(\mu+1)(2\lambda+\mu)+12\xi\delta}}\right\}$$

and

$$|a_3| \leq \begin{cases} \min\left\{\frac{4(1-\beta)^2}{(\lambda+\mu+2\xi\delta)^2} + \frac{2(1-\beta)}{2\lambda+\mu+6\xi\delta}\ ,\ \frac{4(1-\beta)}{(\mu+1)(2\lambda+\mu)+12\xi\delta}\right\} & ,\quad 0 \leq \mu < 1 \\ \frac{2(1-\beta)}{2\lambda+\mu+6\xi\delta} & ,\quad \mu \geq 1 \end{cases}.$$

Here, in our present sequel to some of the aforecited works (especially [11]), we introduce the following subclass of the analytic function class $\mathcal{A}$, analogously to the definition given by Xu *et al.* [9].

Definition 1.5. Let the functions $h, p : \mathbb{U} \to \mathbb{C}$ be so constrained that

$$\min\left\{\Re\left(h\left(z\right)\right), \Re\left(p\left(z\right)\right)\right\} > 0 \quad \left(z \in \mathbb{U}\right) \qquad \text{and} \qquad h\left(0\right) = p\left(0\right) = 1.$$

Also let the function $f \in \Sigma$ defined by (1) be in the analytic function class $\mathcal{A}$. We say that

$$f \in \mathcal{B}_{\Sigma}^{h,p}\left(\lambda, \mu, \delta\right) \qquad \left(\lambda \geq 1,\ \mu \geq 0,\ \delta \geq 0\right)$$

if the following conditions are satisfied:

$$(3) \qquad \left(1-\lambda\right)\left(\frac{f\left(z\right)}{z}\right)^{\mu} + \lambda f'\left(z\right)\left(\frac{f\left(z\right)}{z}\right)^{\mu-1} + \xi\delta z f''\left(z\right) \in h\left(\mathbb{U}\right) \quad \left(z \in \mathbb{U}\right)$$

and

$$(4) \qquad \left(1-\lambda\right)\left(\frac{g\left(w\right)}{w}\right)^{\mu} + \lambda g'\left(w\right)\left(\frac{g\left(w\right)}{w}\right)^{\mu-1} + \xi\delta w g''\left(w\right) \in p\left(\mathbb{U}\right) \quad \left(w \in \mathbb{U}\right),$$

where the function $g = f^{-1}$ is defined by (2) and

$$\xi = \frac{2\lambda+\mu}{2\lambda+1}.$$

Remark 1.6. We note that the class $\mathcal{B}_{\Sigma}^{h,p}\left(\lambda, \mu, \delta\right)$ reduces to the classes $\mathcal{N}_{\Sigma}^{h,p}\left(\lambda, \mu\right)$, $\mathcal{B}_{\Sigma}^{h,p}\left(\lambda\right), \mathcal{B}_{\Sigma}^{h,p}$ and $\mathcal{H}_{\Sigma}^{h,p}$ given by

$$\mathcal{N}_{\Sigma}^{h,p}\left(\lambda, \mu\right) = \mathcal{B}_{\Sigma}^{h,p}\left(\lambda, \mu, 0\right),$$
$$\mathcal{B}_{\Sigma}^{h,p}\left(\lambda\right) = \mathcal{B}_{\Sigma}^{h,p}\left(\lambda, 1, 0\right),$$
$$\mathcal{B}_{\Sigma}^{h,p} = \mathcal{B}_{\Sigma}^{h,p}\left(1, 0, 0\right),$$
$$\mathcal{H}_{\Sigma}^{h,p} = \mathcal{B}_{\Sigma}^{h,p}\left(1, 1, 0\right),$$

respectively, each of which was introduced and studied by Srivastava *et al.* [7], Xu *et al.* [10], Bulut [2] and Xu *et al.* [9], respectively.

Remark 1.7. There are many choices of the functions h and p which would provide interesting subclasses of the analytic function class $\mathcal{A}$. For example, if we let

$$h\left(z\right) = \left(\frac{1+z}{1-z}\right)^{\alpha} \qquad \text{and} \qquad p\left(z\right) = \left(\frac{1-z}{1+z}\right)^{\alpha} \qquad \left(0 < \alpha \leq 1\right)$$

or

$$h\left(z\right) = \frac{1+\left(1-2\beta\right)z}{1-z} \qquad \text{and} \qquad p\left(z\right) = \frac{1-\left(1-2\beta\right)z}{1+z} \qquad \left(0 \leq \beta < 1\right),$$

it is easy to verify that the functions h and p satisfy the hypotheses of Definition 1.5. If $f \in \mathcal{B}_{\Sigma}^{h,p}\left(\lambda, \mu, \delta\right)$, then $f \in \Sigma$,

$$\left|\arg\left\{\left(1-\lambda\right)\left(\frac{f\left(z\right)}{z}\right)^{\mu} + \lambda f'\left(z\right)\left(\frac{f\left(z\right)}{z}\right)^{\mu-1} + \xi\delta z f''\left(z\right)\right\}\right| < \frac{\alpha\pi}{2}$$

and

$$\left|\arg\left\{\left(1-\lambda\right)\left(\frac{g\left(w\right)}{w}\right)^{\mu} + \lambda g'\left(w\right)\left(\frac{g\left(w\right)}{w}\right)^{\mu-1} + \xi\delta w g''\left(w\right)\right\}\right| < \frac{\alpha\pi}{2},$$

or

$$\Re\left\{(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu}+\lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1}+\xi\delta zf''(z)\right\}>\beta$$

and

$$\Re\left\{(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu}+\lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1}+\xi\delta wg''(w)\right\}>\beta,$$

where the function g is defined by (2). This means that

$$f\in\mathcal{B}_{\Sigma}^{\mu}[\alpha,\lambda,\delta]\qquad(\lambda\geq 1,\ \mu\geq 0,\ \delta\geq 0,\ 0<\alpha\leq 1)$$

or

$$f\in\mathcal{B}_{\Sigma}^{\mu}(\beta,\lambda,\delta)\qquad(\lambda\geq 1,\ \mu\geq 0,\ \delta\geq 0,\ 0\leq\beta<1).$$

Our paper is motivated and stimulated especially by the works of Yousef *et al.* [11], we propose to investigate the bi-univalent function class $\mathcal{B}_{\Sigma}^{h,p}(\lambda,\mu,\delta)$ introduced in Definition 1.5 here and derive coefficient estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for a function $f\in\mathcal{B}_{\Sigma}^{h,p}(\lambda,\mu,\delta)$ given by (1). Our results for the bi-univalent function class $\mathcal{B}_{\Sigma}^{h,p}(\lambda,\mu,\delta)$ would generalize and improve the related works of Yousef *et al.* [11], Çağlar *et al.* [4], Srivastava *et al.* [7], Bulut [2] and Xu *et al.* [9, 10].

2. A set of general coefficient estimates

Throughout this paper, we assume that

$$\lambda\geq 1,\ \mu\geq 0,\ \delta\geq 0,\qquad\text{and}\qquad\xi=\frac{2\lambda+\mu}{2\lambda+1}.$$

In this section, we state and prove our general results involving the bi-univalent function class $\mathcal{B}_{\Sigma}^{h,p}(\lambda,\mu,\delta)$ given by Definition 1.5.

Theorem 2.1. *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion* (1) *be in the function class $\mathcal{B}_{\Sigma}^{h,p}(\lambda,\mu,\delta)$. Then*

$$|a_2|\leq\min\left\{\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{2(\lambda+\mu+2\xi\delta)^2}},\ \sqrt{\frac{|h''(0)|+|p''(0)|}{2[(\mu+1)(2\lambda+\mu)+12\xi\delta]}}\right\}\tag{5}$$

and

$$|a_3|\leq\min\left\{\frac{|h'(0)|^2+|p'(0)|^2}{2(\lambda+\mu+2\xi\delta)^2}+\frac{|h''(0)|+|p''(0)|}{4(2\lambda+\mu+6\xi\delta)},\right.$$
$$\left.\frac{[(3+\mu)(2\lambda+\mu)+24\xi\delta]\,|h''(0)|+|1-\mu|\,(2\lambda+\mu)\,|p''(0)|}{4(2\lambda+\mu+6\xi\delta)[(\mu+1)(2\lambda+\mu)+12\xi\delta]}\right\}.\tag{6}$$

Proof. First of all, we write the argument inequalities in (3) and (4) in their equivalent forms as follows:

$$(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu}+\lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1}+\xi\delta zf''(z)=h(z)\quad(z\in\mathbb{U})$$

and

$$(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu}+\lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1}+\xi\delta w g''(w)=p(w)\quad (w\in\mathbb{U}),$$

respectively, where $h(z)$ and $p(w)$ satisfy the conditions of Definition 1.5. Furthermore, the functions $h(z)$ and $p(w)$ have the following Taylor-Maclaurin series expansions:

$$h(z)=1+h_1z+h_2z^2+\cdots$$

and

$$p(w)=1+p_1w+p_2w^2+\cdots,$$

respectively. Now, upon equating the coefficients of

$$(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu}+\lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1}+\xi\delta z f''(z)$$

with those of $h(z)$ and the coefficients of

$$(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu}+\lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1}+\xi\delta w g''(w)$$

with those of $p(w)$, we get

$$(\lambda+\mu+2\xi\delta)\,a_2=h_1, \tag{7}$$

$$(2\lambda+\mu+6\xi\delta)\,a_3+(\mu-1)\left(\lambda+\frac{\mu}{2}\right)a_2^2=h_2, \tag{8}$$

$$-(\lambda+\mu+2\xi\delta)\,a_2=p_1 \tag{9}$$

and

$$-(2\lambda+\mu+6\xi\delta)\,a_3+\left[(\mu+3)\left(\lambda+\frac{\mu}{2}\right)+12\xi\delta\right]a_2^2=p_2. \tag{10}$$

From (7) and (9), we obtain

$$h_1=-p_1 \tag{11}$$

and

$$2(\lambda+\mu+2\xi\delta)^2\,a_2^2=h_1^2+p_1^2. \tag{12}$$

Also, from (8) and (10), we find that

$$[(\mu+1)(2\lambda+\mu)+12\xi\delta]\,a_2^2=h_2+p_2. \tag{13}$$

Therefore, from the equalities (12) and (13) we obtain

$$|a_2|^2\le\frac{|h'(0)|^2+|p'(0)|^2}{2(\lambda+\mu+2\xi\delta)^2}$$

and

$$|a_2|^2\le\frac{|h''(0)|+|p''(0)|}{2[(\mu+1)(2\lambda+\mu)+12\xi\delta]},$$

respectively. So we get the desired estimate on the coefficient $|a_2|$ as asserted in (5).

Next, in order to find the bound on the coefficient $|a_3|$, we subtract (10) from (8). We thus get

$$2(2\lambda+\mu+6\xi\delta)\,a_3-2(2\lambda+\mu+6\xi\delta)\,a_2^2=h_2-p_2. \tag{14}$$

Upon substituting the value of a_2^2 from (12) into (14), it follows that

$$a_3 = \frac{h_1^2 + p_1^2}{2(\lambda + \mu + 2\xi\delta)^2} + \frac{h_2 - p_2}{2(2\lambda + \mu + 6\xi\delta)}.$$

So we get

$$|a_3| \leq \frac{|h'(0)|^2 + |p'(0)|^2}{2(\lambda + \mu + 2\xi\delta)^2} + \frac{|h''(0)| + |p''(0)|}{4(2\lambda + \mu + 6\xi\delta)}.$$

On the other hand, upon substituting the value of a_2^2 from (13) into (14), it follows that

$$a_3 = \frac{[(3+\mu)(2\lambda+\mu) + 24\xi\delta] h_2 + (1-\mu)(2\lambda+\mu) p_2}{2(2\lambda+\mu+6\xi\delta)[(\mu+1)(2\lambda+\mu) + 12\xi\delta]}.$$

And, we get

$$|a_3| \leq \frac{[(3+\mu)(2\lambda+\mu) + 24\xi\delta] |h''(0)| + |1-\mu| (2\lambda+\mu) |p''(0)|}{4(2\lambda+\mu+6\xi\delta)[(\mu+1)(2\lambda+\mu) + 12\xi\delta]}.$$

This evidently completes the proof of Theorem 2.1. □

3. Corollaries and consequences

By setting $\delta = 0$ in Theorem 2.1, we get Corollary 3.1 below.

COROLLARY 3.1. *[7, Theorem 3] Let the function $f(z)$ given by the Taylor-Maclaurin series expansion* (1) *be in the function class $\mathcal{N}_\Sigma^{h,p}(\lambda,\mu)$. Then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2 + |p'(0)|^2}{2(\lambda+\mu)^2}}, \sqrt{\frac{|h''(0)| + |p''(0)|}{2(\mu+1)(2\lambda+\mu)}}\right\}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2 + |p'(0)|^2}{2(\lambda+\mu)^2} + \frac{|h''(0)| + |p''(0)|}{4(2\lambda+\mu)}, \frac{(3+\mu)|h''(0)| + |1-\mu| |p''(0)|}{4(\mu+1)(2\lambda+\mu)}\right\}.$$

By setting $\delta = 0$, $\mu = 0$ and $\lambda = 1$ in Theorem 2.1, we get Corollary 3.2 below.

COROLLARY 3.2. *[2, Theorem 2.1] Let the function $f(z)$ given by the Taylor-Maclaurin series expansion* (1) *be in the function class $\mathcal{B}_\Sigma^{h,p}$. Then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2 + |p'(0)|^2}{2}}, \sqrt{\frac{|h''(0)| + |p''(0)|}{4}}\right\}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2 + |p'(0)|^2}{2} + \frac{|h''(0)| + |p''(0)|}{8}, \frac{3|h''(0)| + |p''(0)|}{8}\right\}.$$

By setting $\delta = 0$ and $\mu = 1$ in Theorem 2.1, we get the following consequence.

Corollary 3.3. *Let the function* $f(z)$ *given by the Taylor-Maclaurin series expansion* (1) *be in the function class* $\mathcal{B}_{\Sigma}^{h,p}(\lambda)$. *Then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{2(\lambda+1)^2}}\,,\,\sqrt{\frac{|h''(0)|+|p''(0)|}{4(2\lambda+1)}}\right\}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2+|p'(0)|^2}{2(\lambda+1)^2}+\frac{|h''(0)|+|p''(0)|}{4(2\lambda+1)}\,,\,\frac{|h''(0)|}{2(2\lambda+1)}\right\}.$$

Remark 3.4. Corollary 3.3 is an improvement of the estimates obtained by Xu *et al.* [10, Theorem 3].

By setting $\delta=0$, $\mu=1$ and $\lambda=1$ in Theorem 2.1, we get the following consequence.

Corollary 3.5. *Let the function* $f(z)$ *given by the Taylor-Maclaurin series expansion* (1) *be in the function class* $\mathcal{H}_{\Sigma}^{h,p}$. *Then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{8}}\,,\,\sqrt{\frac{|h''(0)|+|p''(0)|}{12}}\right\}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2+|p'(0)|^2}{8}+\frac{|h''(0)|+|p''(0)|}{12}\,,\,\frac{|h''(0)|}{6}\right\}.$$

Remark 3.6. Corollary 3.5 is an improvement of the estimates obtained by Xu *et al.* [9, Theorem 3].

If we set

$$h(z)=\left(\frac{1+z}{1-z}\right)^{\alpha} \qquad \text{and} \qquad p(z)=\left(\frac{1-z}{1+z}\right)^{\alpha} \qquad (0<\alpha\leq 1)$$

in Theorem 2.1, then we have Corollary 3.7 below.

Corollary 3.7. *Let the function* $f(z)$ *given by the Taylor-Maclaurin series expansion* (1) *be in the function class* $\mathcal{B}_{\Sigma}^{\mu}[\alpha,\lambda,\delta]$. *Then*

$$|a_2| \leq \min\left\{\frac{2\alpha}{\lambda+\mu+2\xi\delta}\,,\,\frac{2\alpha}{\sqrt{(\mu+1)(2\lambda+\mu)+12\xi\delta}}\right\}$$

and

$$|a_3| \leq \begin{cases} \min\left\{\frac{4\alpha^2}{(\lambda+\mu+2\xi\delta)^2}+\frac{2\alpha^2}{2\lambda+\mu+6\xi\delta}\,,\,\frac{4\alpha^2}{(\mu+1)(2\lambda+\mu)+12\xi\delta}\right\} & ,\quad 0\leq\mu<1 \\ \frac{2\alpha^2}{2\lambda+\mu+6\xi\delta} & ,\quad \mu\geq 1 \end{cases}.$$

Remark 3.8. It is worthy to note that Corollary 3.7 is an improvement of Theorem 1.2.

Remark 3.9. If we set

$$h(z)=\frac{1+(1-2\beta)z}{1-z} \qquad \text{and} \qquad p(z)=\frac{1-(1-2\beta)z}{1+z} \qquad (0\leq\beta<1)$$

in Theorem 2.1, then we can readily deduce Theorem 1.4.

Conflict of Interest. The authors declare that they have no conflict of interest.

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