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ON COEFFICIENTS OF NILPOTENT POLYNOMIALS IN SKEW POLYNOMIAL RINGS

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Abstract. We observe the basic structure of the products of coefficients of nilpotent (left) polynomials in skew polynomial rings. This study consists of a process to extend a well-known result for semi-Armendariz rings. We introduce the concept of *α-skew n-semi-Armendariz ring*, where α is a ring endomorphism. We prove that a ring R is α-rigid if and only if the n by n upper triangular matrix ring over R is $\bar{\alpha}$-skew n-semi-Armendariz. This result are applicable to several known results.

1. Introduction

Throughout this paper all rings are associative with identity unless otherwise stated. A ring is called *reduced* if it has no nonzero nilpotent elements. Let α be an endomorphism of a ring R. A skew polynomial ring with an indeterminate x over R, written by $R[x;\alpha]$, means the polynomial ring $R[x]$ with a new multiplication $xr = \alpha(r)x$ for $r \in R$. In this situation each element of $R[x;\alpha]$ is called (left) polynomial.

An endomorphism α is called *rigid* by Krempa [10] when $a\alpha(a) = 0$ implies $a = 0$ for $a \in R$. It is trivial that rigid endomorphisms are

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injective. Hong et al. [6] called a ring *α-rigid* if it has a rigid endomorphism α of R and they showed that α-rigid rings are reduced and α is a monomorphism.

For a reduced ring R Armendariz [3, Lemma 1] proved that $a_ib_j = 0$ for all i, j whenever $f(x)g(x) = 0$ where $f(x) = \sum_{i=0}^{m} a_ix^i$, $g(x) = \sum_{j=0}^{n} b_jx^j$ are in $R[x]$ ... (*). Rege et al. [13] called a ring (not necessarily reduced) *Armendariz* if it satisfies (*). Reduced rings are Armendariz by [3, Lemma 1]. The structure of Armendariz rings was observed by many authors containing Anderson et al. [1], Hirano [4], Huh et al. [7], Kim et al. [9], Lee et al. [12], Rege et al. [13], etc. Due to Hong et al. [5], a ring R is called a *skew Armendariz ring with an endomorphism* α (or simply an *α-skew Armendariz ring*) provided that for $p = \sum_{i=0}^{m} a_ix^i, q = \sum_{j=0}^{n} b_jx^j \in R[x; \alpha]$, $pq = 0$ implies $a_i\alpha^i(b_j) = 0$ for all i, j. Every α-rigid ring is α-skew Armendariz by [5, Corollary 4]. Jeon et al. [8] called a ring *n-semi-Armendariz* provided that if $f(x) = a_0 + a_1x + \cdots + a_mx^m$ in $R[x]$ satisfies $f(x)^n = 0$ then $a_{i_1}a_{i_2}\cdots a_{i_n} = 0$ for any choice of a_{i_j}'s in $\{a_0, \cdots, a_m\}$ where $j = 1, \ldots, n$ (of course $n \geq 2$). A ring is called *semi-Armendariz* if it is n-semi-Armendariz for all $n \geq 2$. Armendariz rings are semi-Armendariz by [2, Proposition 1], but the converse need not hold since the 2 by 2 upper triangular matrix ring over a reduced ring is semi-Armendariz by [8, Theorem 1.2].

In the following we extend the concept of semi-Armendariz rings to skew polynomial rings. One can see details related to semi-Armendariz rings in [8]. In this note we will call a ring R *α-skew n-semi-Armendariz* provided that $f(x) = a_0 + a_1x + \cdots + a_mx^m$ in $R[x; \alpha]$ satisfies $f(x)^n = 0$ then

$$a_{i_1}\alpha^{i_1}(a_{i_2})\cdots\alpha^{i_1+\cdots+i_{n-1}}(a_{i_n}) = 0$$

for any choice of a_{i_j}'s in $\{a_0, \cdots, a_m\}$ where $j = 1, \ldots, n$ (of course $n \geq 2$). A ring is called *α-skew semi-Armendariz* if it is α-skew n-semi-Armendariz for all $n \geq 2$. Every α-skew Armendariz ring is α-skew semi-Armendariz by Lemma 2(4) to follow, but the converse need not hold by the following example.

Let R be a ring and n be a positive integer. Let $Mat_n(R)$ denote the n by n matrix ring over R and I_n be the identity of $Mat_n(R)$. We use $U_n(R)$ (resp. $L_n(R)$) to denote the n by n upper (resp. lower) triangular matrix ring over R. E_{ij} denotes the n by n matrix with (i, j)-entry 1 and zero elsewhere. Let α be an endomorphism of a ring R. We define an endomorphism $\bar{\alpha}$ of any subring in $Mat_n(R)$ by $(a_{ij}) \mapsto (\alpha(a_{ij}))$.

Example 1. Let $\mathbb{Q}(t)$ be the quotient field with an indeterminate t over $\mathbb{Q}$ and put $R = \mathbb{Q}(t)$. Define $\alpha : R \to R$ by $\frac{f(t)}{g(t)} \mapsto \frac{f(t^2)}{g(t^2)}$ then R is α-rigid and α is a monomorphism of R with $\alpha(1) = 1$. $U_2(R)$ is $\overline{\alpha}$-skew semi-Armendariz by Theorem 4 to follow. For $p = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} x$, $q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} x \in U_2[x; \bar{\alpha}]$, we have $pq = 0$ but $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \bar{\alpha}\left(\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}\right) \neq 0$. Thus $U_2(R)$ is not $\bar{\alpha}$-skew Armendariz.

2. Lemmas

Due to Lambek [11], a ring R is called *symmetric* if $rst = 0$ implies $rts = 0$ for all $r, s, t \in R$. Lambek proved that a ring R is symmetric if and only if $r_1 r_2 \cdots r_n = 0$ implies $r_{\sigma(1)} r_{\sigma(2)} \cdots r_{\sigma(n)} = 0$ for any permutation σ of the set $\{1, 2, \ldots, n\}$, where $n \geq 1$ and $r_i \in R$ for all i (see [11, Proposition 1]). This result was independently shown by Anderson and Camillo in [2, Theorem I.3]. We use this fact without mentioning.

Lemma 2. (1) *Let R be a reduced ring, n be any positive integer and $r_i \in R$ for $i = 1, \ldots, n$. Then $r_1 r_2 \cdots r_n = 0$ implies $r_{\sigma(1)} R r_{\sigma(2)} R \cdots R r_{\sigma(n)} = 0$ for any permutation σ of the set $\{1, 2, \ldots, n\}$.*

(2) *Let R be an α-rigid ring and $a_i \in R$ for $i = 1, \ldots, m$. If $a_1 \cdots a_m = 0$ then $\alpha^{n_1}(a_1) \cdots \alpha^{n_m}(a_m) = 0$ for any positive integers n_i's.*

(3) *A ring R is α-rigid if and only if $\alpha^{k_1}(a_1) \cdots \alpha^{k_m}(a_m) = 0$ (for some positive integers k_i's) implies $a_1 \cdots a_m = 0$ and R is reduced and α is a monomorphism, where $a_i \in R$ for all i.*

(4) *A ring R is α-skew Armendariz if and only if $f_1 \cdots f_n = 0$ implies $a_{1_j} \alpha^{1_j}(a_{2_j}) \cdots \alpha^{1_j + 2_j + \cdots + (n-1)_j}(a_{n_j}) = 0$, where $f_1, \ldots, f_n \in R[x; \alpha]$ and $a_{i_j} x^{i_j}$ is any term of f_i with $a_{i_j} \in R$.*

Proof. (1) It is easily checked that reduced rings are symmetric. Thus we obtain the result.

(2) From [6, Lemma 4(i)], it is true.

(3) By (2), $\alpha^{k_1}(a_1) \cdots \alpha^{k_m}(a_m) = 0$ implies $\alpha^M(a_1) \cdots \alpha^M(a_m) = \alpha^M(a_1 \cdots a_m) = 0$ where $M = max\{k_1, \ldots, k_m\}$. Thus $a_1 \cdots a_m = 0$, since α is a monomorphism.

For the converse, let $r\alpha(r) = 0$ for $r \in R$. Then $\alpha(r)\alpha^2(r) = 0$ and hence $r^2 = 0$, since α is a monomorphism. Since R is reduced, $r = 0$.

(4) It suffices to show the necessity. We first compute the case of $n = 3$. Let R be an α-skew Armendariz ring and suppose that $f_1f_2f_3 = 0$ for $f_1, f_2, f_3 \in R[x;\alpha]$. We also use α for the endomorphism of $R[x;\alpha]$ defined by $\sum a_ix^i \mapsto \sum \alpha(a_i)x^i$. Then $0 = a_{1_j}\alpha^{1_j}(f_2f_3) = (\sum a_{1_j}\alpha^{1_j}(a_{2_j})x^{2_j})\alpha^{1_j}(f_3)$ and so $a_{1_j}\alpha^{1_j}(a_{2_j})\alpha^{1_j+2_j}(a_{3_j}) = 0$, where $f_2 = \sum_{2_j} a_{2_j}x^{2_j}$ and $f_3 = \sum_{3_j} a_{3_j}x^{3_j}$.

Therefore we can inductively obtain $a_{1_j}\alpha^{1_j}(a_{2_j})\cdots\alpha^{1_j+2_j+\cdots+(n-1)_j}$ $(a_{n_j}) = 0$ for $n \geq 4$, where $f_k = \sum_{k_j} a_{k_j}x^{k_j}$ for $k = 1, \ldots, n$.

□

The following is obtained naturally by definition.

Lemma 3. (1) *The class of α-skew (n-semi-)Armendariz rings is closed under subrings.*

(2) *Any direct product of α-skew n-semi-Armendariz rings is α-skew n-semi-Armendariz.*

(3) *Any direct sum of α-skew n-semi-Armendariz rings is α-skew n-semi-Armendariz.*

3. Main Theorem

For a ring R and a positive integer n define

$$N_n(R) = \{A \in U_n(R) \mid \text{ each diagonal entry of } A \text{ is zero } \}.$$

Theorem 4. *Let R be a ring, α be a monomorphism of R with $\alpha(1) = 1$, and n be a positive integer. Then the following conditions are equivalent:*

(1) *R is α-rigid;*

(2) *$U_h(R)$ is $\bar{\alpha}$-skew n-semi-Armendariz for $h = 1, 2, \cdots, n+1$;*

(3) *$U_n(R)$ is $\bar{\alpha}$-skew n-semi-Armendariz;*

(4) *$L_h(R)$ is $\bar{\alpha}$-skew n-semi-Armendariz for $h = 1, 2, \cdots, n+1$;*

(5) *$L_n(R)$ is $\bar{\alpha}$-skew n-semi-Armendariz.*

Proof. We extend the proof of [8, Theorem 1.2] to this situation. (1)⇒(2): Suppose that R is α-rigid. Then R is reduced. It suffices to prove that $U_{n+1}(R)$ is $\bar{\alpha}$-skew n-semi-Armendariz by Lemma 3(1).

Let $f(x) = A_0 + A_1x + \cdots + A_mx^m \in U_{n+1}(R)[x;\bar{\alpha}]$ with $f(x)^n = 0$ $(n \geq 2)$. Write

$$A_i = (a(i)_{uv}) \text{ for } i = 0,1,\ldots,m \text{ with } a(i)_{uv} = 0 \text{ for } u > v.$$

We will use the α-rigidness and reducedness of R without referring. From $f(x)^n = 0$, we have the system of equations

$$\sum_{s_1+s_2+\cdots+s_n=k} A_{s_1}\bar{\alpha}^{s_1}(A_{s_2})\cdots\bar{\alpha}^{\sum_{t=1}^{n-1}s_t}(A_{s_n}) = 0 \text{ for } k = 0,1,\ldots,mn.$$

From $A_0^n = 0$, we have $a(0)_{11} = \cdots = a(0)_{(n+1)(n+1)} = 0$. From $A_m\bar{\alpha}^m(A_m)\cdots\bar{\alpha}^{(n-1)m}(A_m) = 0$, we have $a(m)_{ii}\alpha^m(a(m)_{ii})\cdots\alpha^{(n-1)m}$ $(a(m)_{ii}) = 0$ for $i = 1,\ldots,n+1$; hence we get $a(m)_{ii}^n = 0$ by Lemma 2(3), entailing that $a(m)_{ii} = 0$. Thus $A_0, A_m \in N_{n+1}(R)$.

Consider the coefficient of $f(x)^n$ of degree n. In the equality

$$\sum_{s_1+\cdots+s_n=n} A_{s_1}\bar{\alpha}^{s_1}(A_{s_2})\cdots\bar{\alpha}^{\sum_{t=1}^{n-1}s_t}(A_{s_n}) = 0,$$

any term (except $A_1\bar{\alpha}(A_1)\cdots\bar{\alpha}^{(n-1)}(A_1)$) contains $\bar{\alpha}^s(A_0)$ (for some s) as a factor, and so it is contained in $N_{n+1}(R)$ from $A_0 \in N_{n+1}(R)$. Consequently $A_1\bar{\alpha}(A_1)\cdots\bar{\alpha}^{(n-1)}(A_1) \in N_{n+1}(R)$, and so we get $A_1 \in N_{n+1}(R)$ by the same computation as A_m.

Next we proceed by induction on $i = 0,1,\ldots,m-1$. Consider the coefficient of $f(x)^n$ of degree ni. In the equality

$$\sum_{s_1+\cdots+s_n=ni} A_{s_1}\bar{\alpha}^{s_1}(A_{s_2})\cdots\bar{\alpha}^{\sum_{t=1}^{n-1}s_t}(A_{s_n}) = 0,$$

any term (except $A_i\bar{\alpha}(A_i)\cdots\bar{\alpha}^{(n-1)}(A_i)$) contains $\bar{\alpha}^s(A_j)$ (for some s) with $j < i$ as a factor, and so it is contained in $N_{n+1}(R)$ by induction hypothesis. Consequently $A_i\bar{\alpha}(A_i)\cdots\bar{\alpha}^{(n-1)}(A_i) \in N_{n+1}(R)$ and then $A_i \in N_{n+1}(R)$ by the same computation as A_m. Whence we have

$$a(i)_{11} = a(i)_{22} = \cdots = a(i)_{(n+1)(n+1)} = 0$$

for $i = 0,1,\ldots,m$ and it follows that

$$A_{s_1}\bar{\alpha}^{s_1}(A_{s_2})\cdots\bar{\alpha}^{\sum_{t=1}^{n-1}s_t}(A_{s_n}) = (a(s_1)_{12}\alpha^{s_1}(a(s_2)_{23}) \\ \cdots\alpha^{\sum_{t=1}^{n-1}s_t}(a(s_n)_{n(n+1)}))E_{1(n+1)}$$

for any choice of s_i's. But this equality is equivalent to the system of equations

$$(*)\qquad \sum_{s_1+s_2+\cdots+s_n=k} a(s_1)_{12}\alpha^{s_1}(a(s_2)_{23})\cdots\alpha^{\sum_{t=1}^{n-1}s_t}(a(s_n)_{n(n+1)})=0$$

for $k=0,1,\ldots,mn$. For the case of $k=1$, if we multiply the equation

$$\sum_{s_1+s_2+\cdots+s_n=1} a(s_1)_{12}\alpha^{s_1}(a(s_2)_{23})\cdots\alpha^{\sum_{t=1}^{n-1}s_t}(a(s_n)_{n(n+1)})=0$$

on the right side by $a(0)_{12}\cdots a(0)_{(i-1)i}a(0)_{(i+1)(i+2)}\cdots a(0)_{n(n+1)}$, then from $a(0)_{12}\cdots\ a(0)_{n(n+1)}=0$ and Lemma 2(1) we obtain

$$\begin{aligned}(a(0)_{12}\cdots a(0)_{(i-1)i}a(1)_{i(i+1)}\alpha(a(0)_{(i+1)(i+2)})\cdots\alpha(a(0)_{n(n+1)}))&\\ (a(0)_{12}\cdots a(0)_{(i-1)i}a(0)_{(i+1)(i+2)}\cdots a(0)_{n(n+1)})&=0\end{aligned}$$

for $i=1,\ldots,n$ since every other term contains $a(0)_{i(i+1)}$ for $i=1,2,\ldots,n$ as factors. It then follows that

$$(a(0)_{12}\cdots a(0)_{(i-1)i}a(1)_{i(i+1)}\alpha(a(0)_{(i+1)(i+2)})\cdots\alpha(a(0)_{n(n+1)}))^2=0$$

by Lemma 2(1, 2), and so

$$a(0)_{12}\cdots a(0)_{(i-1)i}a(1)_{i(i+1)}\alpha(a(0)_{(i+1)(i+2)})\cdots\alpha(a(0)_{n(n+1)})=0.$$

We proceed by induction on $k=0,1,\ldots,mn-1$. Let v be maximal in the set $\{s_i \mid s_1+s_2+\cdots+s_n=k\}$ where $k\in\{1,\ldots,mn-1\}$. Consider a term

$$a(s_1)_{12}\alpha^{s_1}(a(s_2)_{23})\cdots\alpha^{\sum_{t=1}^{n-1}s_t}(a(s_n)_{n(n+1)})$$

with $s_i=v$ and $s_1+s_2+\cdots+s_n=k$. Note that not all s_j's are equal by the choice of v. Multiplying $\sum_{s_1+s_2+\cdots+s_n=k} a(s_1)_{12}\alpha^{s_1}(a(s_2)_{23})\cdots \alpha^{\sum_{t=1}^{n-1}s_t}(a(s_n)_{n(n+1)})=0$ on the right side by

$$\begin{aligned}a(s_1)_{12}\cdots\alpha^{\sum_{t=1}^{i-2}s_t}(a(s_{i-1})_{(i-1)i})\alpha^{\sum_{t=1}^{i}s_t}(a(s_{i+1})_{(i+1)(i+2)})&\\ \cdots\alpha^{\sum_{t=1}^{n-1}s_t}(a(s_n)_{n(n+1)}),&\end{aligned}$$

we have

$$\begin{aligned}(a(s_1)_{12}\alpha^{s_1}(a(s_2)_{23})\cdots\alpha^{\sum_{t=1}^{n-1}s_t}(a(s_n)_{n(n+1)}))&\\ (a(s_1)_{12}\cdots\alpha^{\sum_{t=1}^{i-2}s_t}(a(s_{i-1})_{(i-1)i})&\\ \alpha^{\sum_{t=1}^{i}s_t}(a(s_{i+1})_{(i+1)(i+2)})\cdots\alpha^{\sum_{t=1}^{n-1}s_t}(a(s_n)_{n(n+1)}))&=0\end{aligned}$$

by induction hypothesis and Lemma 2(1, 2) since every other term (after multiplying) contains

$$\alpha^{h_1}(a(t_1)_{12}), \cdots, \alpha^{h_n}(a(t_n)_{n(n+1)})$$

(for some h_i's), with $t_1 + \cdots + t_n \leq k-1$, as factors. Thus we have

$$(a(s_1)_{12}\alpha^{s_1}(a(s_2)_{23}) \cdots \alpha^{\sum_{t=1}^{n-1} s_t}(a(s_n)_{n(n+1)}))^2 = 0$$

by Lemma 2(1, 2), entailing $a(s_1)_{12}\alpha^{s_1}(a(s_2)_{23}) \cdots \alpha^{\sum_{t=1}^{n-1} s_t}(a(s_n)_{n(n+1)}) = 0$. Next take such v in the remaining terms and apply the same computation method.

Proceeding in this manner we finally get to $a(u_1)_{12}\alpha^{u_1}(a(u_2)_{23}) \cdots \alpha^{\sum_{t=1}^{n-1} u_t}(a(u_n)_{n(n+1)}) = 0$ for any choice of u_i's such that $u_1 + u_2 + \cdots + u_n = k$ and not all u_i's are equal. In this situation, if k is divisible by n then we finally have $a(\frac{k}{n})_{12}\alpha^{\frac{k}{n}}(a(\frac{k}{n})_{23}) \cdots \alpha^{\frac{k(n-1)}{n}}(a(\frac{k}{n})_{n(n+1)}) = 0$. Thus all terms in $(*)$ are zero, and consequently $a(s_1)_{12}\alpha^{s_1}(a(s_2)_{23}) \cdots \alpha^{\sum_{t=1}^{n-1} s_t}(a(s_n)_{n(n+1)}) = 0$ for any $k \in \{1, 2, \ldots, mn-1\}$ and any choice of s_i's with $s_1 + s_2 + \cdots + s_n = k$.

Now recalling that $a(s_1)_{12}\alpha^{s_1}(a(s_2)_{23}) \cdots \alpha^{\sum_{t=1}^{n-1} s_t}(a(s_n)_{n(n+1)}) = 0$ is equivalent to

$$A_{s_1}\bar{\alpha}^{s_1}(A_{s_2}) \cdots \bar{\alpha}^{\sum_{t=1}^{n-1} s_t}(A_{s_n}) = 0,$$

we obtain $A_{s_1}\bar{\alpha}^{s_1}(A_{s_2}) \cdots \bar{\alpha}^{\sum_{t=1}^{n-1} s_t}(A_{s_n}) = 0$ for any $k \in \{0, 1, 2, \ldots, mn\}$ and any choice of s_i's with $s_1 + \cdots + s_n = k$. Therefore $U_{n+1}(R)$ is $\bar{\alpha}$-skew n-semi-Armendariz.

(3)$\Rightarrow$(1): Assume on the contrary that there is $0 \neq a \in R$ with $a\alpha(a) = 0$. Let $A = (a_{ij}) \in N_n(R)$ with $a_{i(i+1)} = 1$ for all i and elsewhere zero, and $B = (b_{ij}) \in U_n(R)$ with $b_{11} = a, b_{nn} = -a$ and elsewhere zero. Then we have the following computation:

$$(\dagger) \quad AB\bar{\alpha}(A) = B\bar{\alpha}(A^h)B = B\bar{\alpha}(B) = 0, A^{n-k}B = (-a)E_{kn}, B\bar{\alpha}(A^k) = aE_{1(k+1)}$$

for $k = 1, \ldots, n-1$ and all h. Consider $f(x) = A + Bx \in U_n(R)[x; \bar{\alpha}]$. Then since $B\bar{\alpha}(A^{n-1}) = aE_{1n}$ we have

$$f(x)^n = (A^{n-1}B + B\bar{\alpha}(A^{n-1}))x = ((-a)E_{1n} + aE_{1n})x = 0$$

by $(\dagger)$ but $A^{n-1}B, B\bar{\alpha}(A^{n-1})$ are both nonzero. Thus $U_n(R)$ is not $\bar{\alpha}$-skew n-semi-Armendariz, a contradiction.

(2)$\Rightarrow$(3) is obtained from Lemma 2(2) and the proofs of (1)$\Rightarrow$(4), (4)$\Rightarrow$(5), and (5)$\Rightarrow$(1) are similar to the case of $U_n(R)$. $\square$

References

[1] D.D. Anderson, V. Camillo, *Armendariz rings and Gaussian rings*, Comm. Algebra **26** (1998), 2265–2272.
[2] D.D. Anderson, V. Camillo, *Semigroups and rings whose zero products commute*, Comm. Algebra **27** (1999), 2847–2852.
[3] E.P. Armendariz, *A note on extensions of Baer and P.P.-rings*, J. Austral. Math. Soc. **18** (1974), 470–473.
[4] Y. Hirano, *On annihilator ideals of a polynomial ring over a noncommutative ring*, J. Pure Appl. Algebra **168**, (2002), 45–52.
[5] C.Y. Hong, N.K. Kim, T.K. Kwak, *On skew Armendariz rings*, Comm. Algebra **31** (2003), 103–122.
[6] C.Y. Hong, N.K. Kim, T.K. Kwak, *Ore extensions of Baer and p.p.-rings*, J. Pure and Appl. Algebra **151** (2000), 215–226.
[7] C. Huh, Y. Lee, A. Smoktunowicz, *Armendariz rings and semicommutative rings*, Comm. Algebra **30** (2002), 751–761.
[8] Y.C. Jeon, Y. Lee, S.J. Ryu, *A structure on coefficients of nilpotent polynomials*, J. Korean Math. Soc. **47** (2010), 719–733.
[9] N.K. Kim, Y. Lee, *Armendariz rings and reduced rings*, J. Algebra **223** (2000), 477–488.
[10] J. Krempa, *Some examples of reduced rings*, Algebra Colloq. **3** (1996), 289–300.
[11] J. Lambek, *On the representation of modules by sheaves of factor modules*, Canad. Math. Bull. **14** (1971), 359-368.
[12] T.-K. Lee, T.-L. Wong, *On Armendariz rings*, Houston J. Math. **2** (2003), 583–593.
[13] M.B. Rege, S. Chhawchharia, *Armendariz rings*, Proc. Japan Acad. Ser. A Math. Sci. **73** (1997), 14–17.

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