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GENERALIZED (θ, ϕ)-DERIVATIONS ON BANACH ALGEBRAS

Choonkil Park† and Dong Yun Shin‡

Abstract. We introduce the concept of generalized (θ, ϕ)-derivations on Banach algebras, and prove the Cauchy-Rassias stability of generalized (θ, ϕ)-derivations on Banach algebras.

1. Introduction

Let X and Y be Banach spaces with norms $||\cdot||$ and $\|\cdot\|$, respectively. Consider $f : X \to Y$ to be a mapping such that $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in X$. Rassias [12] introduced the following inequality, that we call *Cauchy-Rassias inequality* : Assume that there exist constants $\epsilon \geq 0$ and $p \in [0, 1)$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon(||x||^p + ||y||^p)$$

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‡Corresponding Author.
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for all $x, y \in X$. Rassias [12] showed that there exists a unique $\mathbb{R}$-linear mapping $T : X \to Y$ such that

$$\|f(x) - T(x)\| \le \frac{2\epsilon}{2 - 2^p}||x||^p$$

for all $x \in X$. Beginning around the year 1980 the topic of approximate homomorphisms, or the stability of the equation of homomorphism, was studied by a number of mathematicians. Găvruta [5] generalized the Rassias' result in the following form: Let G be an abelian group and X a Banach space. Denote by $\varphi : G \times G \to [0, \infty)$ a function such that

$$\widetilde{\varphi}(x, y) = \sum_{k=0}^{\infty} 2^{-k}\varphi(2^k x, 2^k y) < \infty$$

for all $x, y \in G$. Suppose that $f : G \to X$ is a mapping satisfying

$$\|f(x + y) - f(x) - f(y)\| \le \varphi(x, y)$$

for all $x, y \in G$. Then there exists a unique additive mapping $T : G \to X$ such that

$$\|f(x) - T(x)\| \le \frac{1}{2}\widetilde{\varphi}(x, x)$$

for all $x \in G$.

Jun and Lee [7] proved the following: Denote by $\varphi : X \setminus \{0\} \times X \setminus \{0\} \to [0, \infty)$ a function such that

$$\widetilde{\varphi}(x, y) = \sum_{j=0}^{\infty} \frac{1}{3^j}\varphi(3^j x, 3^j y) < \infty$$

for all $x, y \in X \setminus \{0\}$. Suppose that $f : X \to Y$ is a mapping satisfying

$$\|2f(\frac{x + y}{2}) - f(x) - f(y)\| \le \varphi(x, y)$$

for all $x, y \in X \setminus \{0\}$. Then there exists a unique additive mapping $T : X \to Y$ such that

$$\|f(x) - f(0) - T(x)\| \le \frac{1}{3}\big(\widetilde{\varphi}(x, -x) + \widetilde{\varphi}(-x, 3x)\big)$$

for all $x \in X \setminus \{0\}$. The stability problem of functional equations has been investigated in several papers (see [4,13,14] and references therein).

Recently, the stability of derivations on other topological structures has been recently studied by a number of mathematicians; see [3,10,11].

In this paper, we introduce the concept of generalized (θ, ϕ)-derivations on Banach algebras, and prove the Cauchy-Rassias stability of generalized (θ, ϕ)-derivations on Banach algebras.

Throughout this paper, we denote by R the set of real numbers or complex numbers. Let θ, ϕ be endomorphisms of an algebra B over R. An additive mapping $D : B \to B$ is called a (θ, ϕ)-*derivation* on B if $D(xy) = D(x)\theta(y) + \phi(x)D(y)$ holds for all $x, y \in B$. An additive mapping $U : B \to B$ is called a *generalized* (θ, ϕ)-*derivation* on B if there exists a (θ, ϕ)-derivation $D : B \to B$ such that $U(xy) = U(x)\theta(y) + \phi(x)D(y)$ holds for all $x, y \in B$ (see [1, 2, 6]).

2. Generalized (θ, ϕ)-derivations on Banach algebras

Throughout this section, let B be a Banach algebra over R with norm $\| \cdot \|$.

Definition 2.1. Let $\theta, \phi : B \to B$ be additive mappings. An additive mapping $D : B \to B$ is called a (θ, ϕ)-*derivation* on B if $D(xy) = D(x)\theta(y) + \phi(x)D(y)$ holds for all $x, y \in B$.

An additive mapping $U : B \to B$ is called a *generalized* (θ, ϕ)-*derivation* on B if there exists a (θ, ϕ)-derivation $D : B \to B$ such that $U(xy) = U(x)\theta(y) + \phi(x)D(y)$ holds for all $x, y \in B$.

Theorem 2.2. *Let $f, g, h, u : B \to B$ be mappings with $f(0) = g(0) = h(0) = u(0) = 0$ for which there exists a function $\varphi : B \times B \to [0, \infty)$ such that*

$$\begin{aligned}
&(1) & \widetilde{\varphi}(x, y) : &= \sum_{j=0}^{\infty} \frac{1}{2^j} \varphi(2^j x, 2^j y) < \infty, \\
&(2) & \|f(x+y) - f(x) - f(y)\| &\leq \varphi(x, y), \\
&(3) & \|g(x+y) - g(x) - g(y)\| &\leq \varphi(x, y), \\
&(4) & \|h(x+y) - h(x) - h(y)\| &\leq \varphi(x, y), \\
&(5) & \|u(x+y) - u(x) - u(y)\| &\leq \varphi(x, y), \\
&(6) & \|f(xy) - f(x)g(y) - h(x)f(y)\| &\leq \varphi(x, y), \\
&(7) & \|u(xy) - u(x)g(y) - h(x)f(y)\| &\leq \varphi(x, y)
\end{aligned}$$

for all $x, y \in B$. Then there exist unique additive mappings $D, \theta, \phi, U : B \to B$ such that

$$\|f(x) - D(x)\| \leq \frac{1}{2}\widetilde{\varphi}(x, x), \tag{8}$$

$$\|g(x) - \theta(x)\| \leq \frac{1}{2}\widetilde{\varphi}(x, x), \tag{9}$$

$$\|h(x) - \phi(x)\| \leq \frac{1}{2}\widetilde{\varphi}(x, x), \tag{10}$$

$$\|u(x) - U(x)\| \leq \frac{1}{2}\widetilde{\varphi}(x, x) \tag{11}$$

for all $x \in B$. Moreover, $D : B \to B$ is a (θ, ϕ)-derivation on B, and $U : B \to B$ is a generalized (θ, ϕ)-derivation on B.

Proof. By the Găvruta's theorem [5], it follows from (1)–(5) that there exist unique additive mappings $D, \theta, \phi, U : B \to B$ satisfying (8)–(11). The additive mappings $D, \theta, \phi, U : B \to B$ are given by

$$D(x) = \lim_{l\to\infty} \frac{1}{2^l} f(2^l x), \tag{12}$$

$$\theta(x) = \lim_{l\to\infty} \frac{1}{2^l} g(2^l x), \tag{13}$$

$$\phi(x) = \lim_{l\to\infty} \frac{1}{2^l} h(2^l x), \tag{14}$$

$$U(x) = \lim_{l\to\infty} \frac{1}{2^l} u(2^l x) \tag{15}$$

for all $x \in B$.

It follows from (6) that

$$\frac{1}{2^{2l}}\|f(2^{2l}xy) - f(2^l x)g(2^l y) - h(2^l x)f(2^l y)\| \leq \frac{1}{2^{2l}}\varphi(2^l x, 2^l y) \leq \frac{1}{2^l}\varphi(2^l x, 2^l y),$$

which tends to zero as $l \to \infty$ for all $x, y \in B$ by (1). By (12)–(14),

$$D(xy) = D(x)\theta(y) + \phi(x)D(y)$$

for all $x, y \in B$. So the additive mapping $D : B \to B$ is a (θ, ϕ)-derivation on B.

It follows from (7) that

$$\frac{1}{2^{2l}}\|u(2^{2l}xy) - u(2^l x)g(2^l y) - h(2^l x)f(2^l y)\| \leq \frac{1}{2^{2l}}\varphi(2^l x, 2^l y) \leq \frac{1}{2^l}\varphi(2^l x, 2^l y),$$

which tends to zero as $l \to \infty$ for all $x, y \in B$ by (1). Thus

$$U(xy) = U(x)\theta(y) + \phi(x)D(y)$$

for all $x, y \in B$. So the additive mapping $U : B \to B$ is a generalized (θ, ϕ)-derivation on B. □

COROLLARY 2.3. *Let $f, g, h, u : B \to B$ be mappings with $f(0) = g(0) = h(0) = u(0) = 0$ for which there exist constants $\epsilon \geq 0$ and $p \in [0, 1)$ such that*

$$\begin{aligned}
\|f(x+y) - f(x) - f(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p), \\
\|g(x+y) - g(x) - g(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p), \\
\|h(x+y) - h(x) - h(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p), \\
\|u(x+y) - u(x) - u(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p), \\
\|f(xy) - f(x)g(y) - h(x)f(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p), \\
\|u(xy) - u(x)g(y) - h(x)f(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p)
\end{aligned}$$

for all $x, y \in B$. Then there exist unique additive mappings $D, \theta, \phi, U : B \to B$ such that

$$\begin{aligned}
\|f(x) - D(x)\| &\leq \frac{2\epsilon}{2 - 2^p}\|x\|^p, \\
\|g(x) - \theta(x)\| &\leq \frac{2\epsilon}{2 - 2^p}\|x\|^p, \\
\|h(x) - \phi(x)\| &\leq \frac{2\epsilon}{2 - 2^p}\|x\|^p, \\
\|u(x) - U(x)\| &\leq \frac{2\epsilon}{2 - 2^p}\|x\|^p
\end{aligned}$$

for all $x \in B$. Moreover, $D : B \to B$ is a (θ, ϕ)-derivation on B, and $U : B \to B$ is a generalized (θ, ϕ)-derivation on B.

Proof. Defining $\varphi(x, y) = \epsilon(\|x\|^p + \|y\|^p)$ to be Th.M. Rassias upper bound in the Cauchy-Rassias inequality, and applying Theorem 2.2, we get the desired result. □

COROLLARY 2.4. *Let $\theta, \phi : B \to B$ be additive mappings. Let $f, u : B \to B$ be mappings with $f(0) = u(0) = 0$ for which there exists a function $\varphi : B \times B \to [0, \infty)$ satisfying* (1), (2), *and* (5) *such that*

$$\|f(xy) - f(x)\theta(y) - \phi(x)f(y)\| \leq \varphi(x, y), \tag{16}$$

$$\|u(xy) - u(x)\theta(y) - \phi(x)f(y)\| \leq \varphi(x, y) \tag{17}$$

for all $x, y \in B$. Then there exists a unique (θ, ϕ)-derivation $D : B \to B$ satisfying (8), *and there exists a unique generalized (θ, ϕ)-derivation $U : B \to B$ satisfying* (11).

Proof. Letting $\theta = g$ and $\phi = h$ in the statement of Theorem 2.2, we get the result. $\square$

THEOREM 2.5. *Let $f, g, h, u : B \to B$ be mappings with $f(0) = g(0) = h(0) = u(0) = 0$ for which there exists a function $\varphi : B \times B \to [0, \infty)$ satisfying* (6) *and* (7) *such that*

$$\widetilde{\varphi}(x,y) := \sum_{j=0}^{\infty} \frac{1}{3^j}\varphi(3^j x, 3^j y) < \infty, \tag{18}$$

$$\|2f(\frac{x+y}{2}) - f(x) - f(y)\| \le \varphi(x,y), \tag{19}$$

$$\|2g(\frac{x+y}{2}) - g(x) - g(y)\| \le \varphi(x,y), \tag{20}$$

$$\|2h(\frac{x+y}{2}) - h(x) - h(y)\| \le \varphi(x,y), \tag{21}$$

$$\|2u(\frac{x+y}{2}) - u(x) - u(y)\| \le \varphi(x,y) \tag{22}$$

for all $x, y \in B$. Then there exist unique additive mappings $D, \theta, \phi, U : B \to B$ such that

$$\|f(x) - D(x)\| \le \frac{1}{3}\big(\widetilde{\varphi}(x,-x) + \widetilde{\varphi}(-x,3x)\big), \tag{23}$$

$$\|g(x) - \theta(x)\| \le \frac{1}{3}\big(\widetilde{\varphi}(x,-x) + \widetilde{\varphi}(-x,3x)\big), \tag{24}$$

$$\|h(x) - \phi(x)\| \le \frac{1}{3}\big(\widetilde{\varphi}(x,-x) + \widetilde{\varphi}(-x,3x)\big), \tag{25}$$

$$\|u(x) - U(x)\| \le \frac{1}{3}\big(\widetilde{\varphi}(x,-x) + \widetilde{\varphi}(-x,3x)\big) \tag{26}$$

for all $x \in B$. Moreover, $D : B \to B$ is a (θ, ϕ)-derivation on B, and $U : B \to B$ is a generalized (θ, ϕ)-derivation on B.

Proof. By the Jun and Lee's theorem [7, Theorem 1], it follows from (18)–(22) that there exist unique additive mappings $D, \theta, \phi, U : B \to B$ satisfying (23)–(26). The additive mappings $D, \theta, \phi, U : B \to B$ are given by

$$D(x) = \lim_{l\to\infty} \frac{1}{3^l} f(3^l x), \tag{27}$$

$$\theta(x) = \lim_{l\to\infty} \frac{1}{3^l} g(3^l x), \tag{28}$$

$$\phi(x) = \lim_{l\to\infty} \frac{1}{3^l} h(3^l x), \tag{29}$$

$$U(x) = \lim_{l\to\infty} \frac{1}{3^l} u(3^l x), \tag{30}$$

for all $x \in B$.

It follows from (6) that

$$\frac{1}{3^{2l}} \|f(3^{2l}xy) - f(3^l x)g(3^l y) - h(3^l x)f(3^l y)\| \le \frac{1}{3^{2l}} \varphi(3^l x, 3^l y) \le \frac{1}{3^l} \varphi(3^l x, 3^l y),$$

which tends to zero as $l \to \infty$ for all $x, y \in B$ by (18). By (27)–(30),

$$D(xy) = D(x)\theta(y) + \phi(x)D(y)$$

for all $x, y \in B$. So the additive mapping $D : B \to B$ is a (θ, ϕ)-derivation on B.

It follows from (7) that

$$\frac{1}{3^{2l}} \|u(3^{2l}xy) - u(3^l x)g(3^l y) - h(3^l x)f(3^l y)\| \le \frac{1}{3^{2l}} \varphi(3^l x, 3^l y) \le \frac{1}{3^l} \varphi(3^l x, 3^l y),$$

which tends to zero as $l \to \infty$ for all $x, y \in B$ by (18). Thus

$$U(xy) = U(x)\theta(y) + \phi(x)D(y)$$

for all $x, y \in B$. So the additive mapping $U : B \to B$ is a generalized (θ, ϕ)-derivation on B. $\square$

Corollary 2.6. *Let $f, g, h, u : B \to B$ be mappings with $f(0) = g(0) = h(0) = u(0) = 0$ for which there exist constants $\epsilon \ge 0$ and*

$p \in [0,1)$ *such that*

$$\begin{aligned}
\|2f(\frac{x+y}{2}) - f(x) - f(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p), \\
\|2g(\frac{x+y}{2}) - g(x) - g(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p), \\
\|2h(\frac{x+y}{2}) - h(x) - h(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p), \\
\|2u(\frac{x+y}{2}) - u(x) - u(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p), \\
\|f(xy) - f(x)g(y) - h(x)f(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p), \\
\|u(xy) - u(x)g(y) - h(x)f(y)\| &\leq \epsilon(\|x\|^p + \|y\|^p)
\end{aligned}$$

for all $x, y \in B$. *Then there exist unique additive mappings* $D, \theta, \phi, U : B \to B$ *such that*

$$\begin{aligned}
\|f(x) - D(x)\| &\leq \frac{3+3^p}{3-3^p}\epsilon\|x\|^p, \\
\|g(x) - \theta(x)\| &\leq \frac{3+3^p}{3-3^p}\epsilon\|x\|^p, \\
\|h(x) - \phi(x)\| &\leq \frac{3+3^p}{3-3^p}\epsilon\|x\|^p, \\
\|u(x) - U(x)\| &\leq \frac{3+3^p}{3-3^p}\epsilon\|x\|^p
\end{aligned}$$

for all $x \in B$. *Moreover,* $D : B \to B$ *is a* (θ, ϕ)*-derivation on* B*, and* $U : B \to B$ *is a generalized* (θ, ϕ)*-derivation on* B.

Proof. Defining $\varphi(x, y) = \epsilon(\|x\|^p + \|y\|^p)$, and applying Theorem 2.5, we get the desired result. □

Corollary 2.7. *Let* $\theta, \phi : B \to B$ *be additive mappings. Let* $f, u : B \to B$ *be mappings with* $f(0) = u(0) = 0$ *for which there exists a function* $\varphi : B \times B \to [0, \infty)$ *satisfying* (18), (19), (22), (16) *and* (17). *Then there exists a unique* (θ, ϕ)*-derivation* $D : B \to B$ *satisfying* (23), *and there exists a unique generalized* (θ, ϕ)*-derivation* $U : B \to B$ *satisfying* (26).

Proof. Letting $\theta = g$ and $\phi = h$ in the statement of Theorem 2.5, we get the result. □

Theorem 2.8. *Let* $f, g, h, u : B \to B$ *be mappings with* $f(0) = g(0) = h(0) = u(0) = 0$ *for which there exists a function* $\varphi : B \times B \to$

$[0, \infty)$ *satisfying* (19)–(22), (6) *and* (7) *such that*

$$\sum_{j=0}^{\infty} 3^{2j} \varphi(\frac{x}{3^j}, \frac{y}{3^j}) < \infty \tag{31}$$

for all $x, y \in B$. *Then there exist unique additive mappings* $D, \theta, \phi, U : B \to B$ *such that*

$$\|f(x) - D(x)\| \leq \widetilde{\varphi}(\frac{x}{3}, -\frac{x}{3}) + \widetilde{\varphi}(-\frac{x}{3}, x), \tag{32}$$

$$\|g(x) - \theta(x)\| \leq \widetilde{\varphi}(\frac{x}{3}, -\frac{x}{3}) + \widetilde{\varphi}(-\frac{x}{3}, x), \tag{33}$$

$$\|h(x) - \phi(x)\| \leq \widetilde{\varphi}(\frac{x}{3}, -\frac{x}{3}) + \widetilde{\varphi}(-\frac{x}{3}, x), \tag{34}$$

$$\|u(x) - U(x)\| \leq \widetilde{\varphi}(\frac{x}{3}, -\frac{x}{3}) + \widetilde{\varphi}(-\frac{x}{3}, x) \tag{35}$$

for all $x \in B$, *where*

$$\widetilde{\varphi}(x, y) := \sum_{j=0}^{\infty} 3^j \varphi(\frac{x}{3^j}, \frac{y}{3^j})$$

for all $x, y \in B$. *Moreover,* $D : B \to B$ *is a* (θ, ϕ)*-derivation on* B, *and* $U : B \to B$ *is a generalized* (θ, ϕ)*-derivation on* B.

Proof. By the Jun and Lee's theorem [7, Theorem 7], it follows from (31) and (19)–(22) that there exist unique additive mappings $D, \theta, \phi, U : B \to B$ satisfying (32)–(35). The additive mappings $D, \theta, \phi, U : B \to B$ are given by

$$D(x) = \lim_{l \to \infty} 3^l f(\frac{x}{3^l}), \tag{36}$$

$$\theta(x) = \lim_{l \to \infty} 3^l g(\frac{x}{3^l}), \tag{37}$$

$$\phi(x) = \lim_{l \to \infty} 3^l h(\frac{x}{3^l}), \tag{38}$$

$$U(x) = \lim_{l \to \infty} 3^l u(\frac{x}{3^l}), \tag{39}$$

for all $x \in B$.

It follows from (6) that

$$3^{2l} \|f(\frac{xy}{3^{2l}}) - f(\frac{x}{3^l})g(\frac{y}{3^l}) - h(\frac{x}{3^l})f(\frac{y}{3^l})\| \leq 3^{2l} \varphi(\frac{x}{3^l}, \frac{y}{3^l}),$$

which tends to zero as $l \to \infty$ for all $x, y \in B$ by (31). By (36)–(39),

$$D(xy) = D(x)\theta(y) + \phi(x)D(y)$$

for all $x, y \in B$. So the additive mapping $D : B \to B$ is a (θ, ϕ)-derivation on B.

It follows from (7) that

$$3^{2l}\|u(\frac{xy}{3^{2l}}) - u(\frac{x}{3^l})g(\frac{y}{3^l}) - h(\frac{x}{3^l})f(\frac{y}{3^l})\| \le 3^{2l}\varphi(\frac{x}{3^l}, \frac{y}{3^l}),$$

which tends to zero as $l \to \infty$ for all $x, y \in B$ by (31). Thus

$$U(xy) = U(x)\theta(y) + \phi(x)D(y)$$

for all $x, y \in B$. So the additive mapping $U : B \to B$ is a generalized (θ, ϕ)-derivation on B. □

Corollary 2.9. *Let $f, g, h, u : B \to B$ be mappings with $f(0) = g(0) = h(0) = u(0) = 0$ for which there exist constants $\epsilon \ge 0$ and $p \in (2, \infty)$ such that*

$$\begin{aligned}
\|2f(\frac{x+y}{2}) - f(x) - f(y)\| &\le \epsilon(\|x\|^p + \|y\|^p),\\
\|2g(\frac{x+y}{2}) - g(x) - g(y)\| &\le \epsilon(\|x\|^p + \|y\|^p),\\
\|2h(\frac{x+y}{2}) - h(x) - h(y)\| &\le \epsilon(\|x\|^p + \|y\|^p),\\
\|2u(\frac{x+y}{2}) - u(x) - u(y)\| &\le \epsilon(\|x\|^p + \|y\|^p),\\
\|f(xy) - f(x)g(y) - h(x)f(y)\| &\le \epsilon(\|x\|^p + \|y\|^p),\\
\|u(xy) - u(x)g(y) - h(x)f(y)\| &\le \epsilon(\|x\|^p + \|y\|^p)
\end{aligned}$$

for all $x, y \in B$. Then there exist unique additive mappings $D, \theta, \phi, U : B \to B$ such that

$$\begin{aligned}
\|f(x) - D(x)\| &\le \frac{3^p + 3}{3^p - 3}\epsilon\|x\|^p,\\
\|g(x) - \theta(x)\| &\le \frac{3^p + 3}{3^p - 3}\epsilon\|x\|^p,\\
\|h(x) - \phi(x)\| &\le \frac{3^p + 3}{3^p - 3}\epsilon\|x\|^p,\\
\|u(x) - U(x)\| &\le \frac{3^p + 3}{3^p - 3}\epsilon\|x\|^p
\end{aligned}$$

for all $x \in B$. Moreover, $D : B \to B$ is a (θ, ϕ)-derivation on B, and $U : B \to B$ is a generalized (θ, ϕ)-derivation on B.

Proof. Defining $\varphi(x, y) = \epsilon(\|x\|^p + \|y\|^p)$, and applying Theorem 2.8, we get the desired result. $\square$

COROLLARY 2.10. *Let $\theta, \phi : B \to B$ be additive mappings. Let $f, u : B \to B$ be mappings with $f(0) = u(0) = 0$ for which there exists a function $\varphi : B \times B \to [0, \infty)$ satisfying* (31), (19), (22), (16) *and* (17). *Then there exists a unique (θ, ϕ)-derivation $D : B \to B$ satisfying* (32), *and there exists a unique generalized (θ, ϕ)-derivation $U : B \to B$ satisfying* (35).

Proof. Letting $\theta = g$ and $\phi = h$ in the statement of Theorem 2.8, we get the result. $\square$

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Choonkil Park
Department of Mathematics
Hanyang University
Seoul 133-791, Korea
E-mail: baak@hanyang.ac.kr

Dong Yun Shin
Department of Mathematics
University of Seoul
Seoul 130-743, Korea
E-mail: dyshin@uos.ac.kr