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FINANCIAL MODELS INDUCED FROM AUXILIARY INDICES AND TWITTER DATA

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Abstract. As we know, some indices and data are strong influence to the price movement of some assets now, but not to another assets and in future. Thus we define some asset models for several time intervals; intraday, weekly, monthly, and yearly asset models. We define these asset models by using Brownian motion with volatility and Poisson process, and several deterministic functions(index function, twitter data function and big-jump simple function etc). In our asset models, these deterministic functions are the positive or negative levels of auxiliary indices, of analyzed data, and for imminent and extreme state(for example, financial shock or the highest popularity in the market). These functions determined by indices, twitter data and shocking news are a kind of one of speciality of our asset models. For reasonableness of our asset models, we introduce several real data, figurers and tables, and simulations. Perhaps from our asset models, for short-term or long-term investment, we can classify and reference many kinds of usual auxiliary indices, information and data.

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1. Introduction

As we know, since Black-Scholes asset model, many kinds of asset models were introduced. Also they have particular speciality and originality each and all. But now a day according to development of computer, mobile cellular phone and technic, we can use so-called big-data and information easily. Thus for more accurate prediction, representation, and calculation of financial fluctuation and movement, we define another several kinds of asset models containing or referring maximizing returns and minimizing risk also. To get and select useful data simply, we can use some word in nine sentiments(anger, hate, dislike, fear, love, shame, sadness, hope, joy, c.f. [11]) or six mood state(calm, alert, sure, vital, kind, happy, c.f. [1], [2]) which we call it as the representation of instinct of human and we know already. We can use internet with many kind of net works and also various news and twitter data can be got in the stock markets with many kinds of indices and from stock-company service news easily.

Some indices and data are strong influence in now, but not influence another asset models and in future. Thus we need some special asset models for various time intervals, and thus define intraday, weekly, monthly, and yearly asset models. Therefore we can use some data and indices usefully to get more exact asset price movement and models in intraday or weekly asset models(c.f. [2], [8]), even if some data are need not in monthly and yearly asset models which are a little long term models.

To define our new financial asset models, we start from the construction of Lévy processes and define asset models as an exponential Lévy processes which are defined by the solutions of stochastic differential equations(SDE) derived from Lévy processes classified by jump-type semimartingales basically. But our asset models contain several deterministic functions which represent many kinds of index and data levels. To introduce and divide various functions which are changed according to time, we would like adopt a little strange SDEs which are useful to understand spacial character according to time interval. Our main asset model is represented by the following SDE for $0 \leq t \leq T$, where T is a terminal time in a time interval which is in the stopping time of semimartingale. Time interval D is the first one day, W is one week, M is one month, and Y is one year or more which are in T.

$$\begin{aligned} dS_t &= S_{t-}[(\mu_t dt + \sigma_t dW_t)I_{\{|\gamma_t(x)|<\infty\}} \\ &\quad +(-0.15)I_{\{\gamma_t(x)=-\infty\}} + (1.15)I_{\{\gamma_t(x)=\infty\}}]I_{\{t\in D\}} \\ &\quad +S_{t-}[(\mu_t^{(W)}dt + \sigma_t^{(W)}dW_t + \gamma_t^{(W)}dN_t)I_{\{t\in W\}} \\ &\quad +(\mu_t^{(M)}dt + \sigma_t^{(M)}dW_t + \gamma_t^{(M)}dN_t)I_{\{t\in M\}} \\ &\quad +(\mu_t^{(Y)}dt + \sigma_t^{(Y)}dW_t)I_{\{t\in Y\}}], \end{aligned}$$

where W_t is a Brownian motion and N_t is a compound Poisson process. Further, μ_t is a deterministic function made by daily auxiliary indices and twitter data and σ_t is the volatility of Brownian motion W_t. Further, $\mu_t^{(W)}$ is a function of predictable indices and of level(or volume) of (twitter) data concerning weekly asset prices, $\sigma_t^{(W)}$ is another volatility of Brownian motion. Functions $\gamma_t^{(W)}$ and $\gamma_t^{(M)}$ are simple functions consisted by financial or non-financial shocking news index level. The compositions of $\mu_t^{(M)}$ and $\mu_t^{(Y)}$ are consisted by many kinds of non-shocking financial or non-shocking non-financial indices, and $\sigma_t^{(M)}$ and $\sigma_t^{(Y)}$ are volatilities which of months and years. We adopt following form of equation as the solution of above SDE;

$$\begin{aligned} S_t &= S_0[\exp(\mu_t t + \sigma_t \tilde{W}_t)I_{\{|\gamma_t(x)|<\infty\}} \\ &\quad +(-0.15)I_{\{\gamma_t(x)=-\infty\}} + (0.15)I_{\{\gamma_t(x)=\infty\}}]I_{\{t\in D\}} \\ &\quad +S_0\exp\{\mu_t^{(W)}t + \sigma_t^{(W)}\tilde{W}_t + \gamma_t^{(W)}\tilde{N}_t\}I_{\{t\in W\}} \\ &\quad +S_0\exp\{\mu_t^{(M)}t + \sigma_t^{(M)}\tilde{W}_t + \gamma_t^{(M)}d\tilde{N}_t\}I_{\{t\in M\}} \\ &\quad +S_0\exp\{\mu_t^{(Y)}t + \sigma_t^{(Y)}\tilde{W}_t\}I_{\{t\in Y\}}, \end{aligned}$$

where $\tilde{W}_t$ is an adjusted Brownian motion by Girsanov theorem, and $\tilde{N}_t$ is an another jump-type martingale.

From our investing, we can insist that fluctuation and change of asset price is caused by not the special character of given asset models but various information, indices and data which are given momentarily. Thus, we use functions μ_t, $\mu_t^{(i)}(i = W, M, Y)$ and $\gamma_t^{(i)}(i = W, M)$ for simple useful asset models and portfolio also. Particularly, we are not account in detail the noise parts W_t, $\tilde{W}_t$, N_t and $\tilde{N}_t$ also because nobody can predict these parts.

We are not use and need not expectation, conditional probability, various particular density functions, heavy tail or leptokultic, etc. Thus, our deterministic comparison methods of functions μ_t, $\mu_t^{(i)}$, and $\gamma_t^{(i)}$ are more simple and easy relatively to understand our asset models. Thus, at each checking time, we can put or call some assets by new rank easily. Further, we are not account a difficult stopping time notion which is defined and used in local martingale. Thus, we can use our model simply even if we are not know high mathematical theory background.

Section 2 is a preliminary section. In this section, we introduce basic stochastic theories to the construction of asset models from starting Poisson processes. In section 3, we define and explain our asset models in various terms of time. In section 4, we introduce various useful indices and data. In section 5, we make and use prediction box models and simulation graphs. In section 6, we summarize and explain our result, and denote our opinions.

2. Preliminary

We assume a well-defined probability space $(\Omega, \mathbf{F}, P)$. Let us think a random walk in which the particle can in a single transition either stay in i or move to one of the adjacent state $i-1$ and $i+1$, if it is in state i now. Specifically for a random variable ξ_t, if $\xi_t = i$, then for real numbers p_i, q_i and $r_i \geq 0$, we get

$$P\{\xi_{t+\Delta t} = i+1 | \xi_t = i\} = p_i,$$
$$P\{\xi_{t+\Delta t} = i-1 | \xi_t = i\} = q_i,$$

and

$$P\{\xi_{t+\Delta t} = i | \xi_t = i\} = r_i$$

with the obvious modifications holding for $i = 0$.

2.1. Poisson processes. A Poisson process $\{N_t(\lambda), t > 0\}$ with intensity λ is defined by using the number of jump times:

$$N_t(\lambda) = \sharp\{k : T_k \in [0, t]\}, \quad t > 0,$$

where $T_k = \sum_{i=1}^{k} \tau_i(\lambda)$, and $\{\tau_i(\lambda)\}$ is an independent identically distributed(iid) sequence of exponentially distributed random variables $P(\tau_i \in [t, t + dt)) = \lambda e^{-\lambda t} dt$, $t > 0$. This inter-arrival time process $\{\tau_n(\lambda)\}_{n=1}^{\infty}$ is a counting process. Thus we know that the distribution of Poisson process $N_t(\lambda)$ is following:

$$P\{N_t(\lambda) = n\} = e^{-\lambda t}\frac{(\lambda t)^n}{n!}, \quad n = 1, 2, 3, \cdots$$

A compound Poisson process $\{X_t, t > 0\}$ with intensity λ and jump size distribution F is a continuous time random walk which is defined as a process;

$$\begin{aligned} X_t &\equiv \xi_1 + \xi_2 + \xi_3 + \cdots + \xi_{N_t(\lambda)} \\ &:= R_{N_t(\lambda)} = \Sigma_{i=1}^{N_t(\lambda)}\xi_i, \quad t > 0, \quad X_0 = 0, \end{aligned}$$

where jump sizes ξ_i are *iid* sequence of random variables with distribution F and $\{\xi_n\}$ is independent of $\{N_t(\lambda)\}$.

Further, we can get that the compound Poisson process $\{X_t(\lambda), t > 0\}$ has following properties;

(1). $X_t(\lambda)$ describes the position of a random walk after $N_t(\lambda)$ steps.

(2). If random walk $\xi_i \equiv 1$, for all $i = 1, 2, 3, \cdots$, then $X_t = N_t(\lambda), t > 0$.

(3). The jump sizes $(\xi_i)_{i\geq 1}$ occur as a Poisson intensity rate λ.

A compensated compound Poisson process $\{\tilde{X}_t, t > 0\}$ is defined as a compound Poisson process adjusted to be martingale:

$$\begin{aligned} \tilde{X}_t &:= X_t - E[X_t] \\ &= \Sigma_{i=1}^{N_t(\lambda)}\xi_i - \lambda t E[\xi_1], \quad t > 0. \end{aligned}$$

From the definition of compound Poisson process, we know that

$$\begin{aligned} E[X_t] &= E[\Sigma_{i=1}^{N_t(\lambda)}\xi_i] \\ &= \lambda t E[\xi_1] = \lambda t \int_{-\infty}^{\infty} x dF(x). \end{aligned}$$

2.2. Random measures. Let $X = \{X_t, t \geq 0\}$ be a stochastic process. A cádlág stochastic process X_t is a process of which sample paths are right continuous with left limits. We can characterize cádlág process X_t as by using following several properties. Let $\Delta X_t := X_t - X_{t-}$.

(1) The set $\{t \in [0, \infty) | \Delta X_t (:= X_t - X_{t-}) \neq 0\}$ is at most countable.

(2) The set $\{t \in [0, \infty) | \Delta X_t > \epsilon (> 0)\}$ is finite.

(3) If we define a random variable J_X on $[0, \infty) \times R$ as

$$J_X([t_1, t_2] \times A) := \sharp\{s \in [t_1, t_2] | \Delta X_t \neq 0, \quad \Delta X_s \in A\}$$

then J_X is a jump measure of of cádlág process X_t. If we define a Borel measure ν_X as, for a Borel set A,

$$\nu_X(A) := E[J_X([0,1] \times A)]$$

then this measure ν_X describe the jump rates and the jump sizes of the cádlág process X_t.

2.3. Lévy measures. To construct diverse Lévy processes, we define Lévy measures. Lévy measure ν is defined by a Borel measure on R satisfying $\nu(\{0\}) = 0$ and

$$\nu(\{0\}) = 0 \quad and \quad \int_{-\infty}^{\infty} (1 \wedge x^2)\nu(dx) < \infty.$$

2.4. Composing poisson processes. By using Lévy measure ν with $\nu(R) < \infty$, we can construct compound Poisson processes and compensated compound Poisson processes. Let ν be a Lévy measure on R with $\nu(R) = \lambda < \infty$, and let $\{\xi_n\}$ be the iid sequence of random variables with the common jump size distribution $\rho = (1/\lambda)\nu$ such that $\{\xi_n\}$ and $\{N_t(\lambda)\}$ are independent each other. The process $\{X_t, t \geq 0\}$ defined by

$$\begin{aligned} X_t &\equiv \xi_1 + \xi_2 + \xi_3 + \cdots + \xi_{N_t(\lambda)} \\ &= \Sigma_{i=1}^{N_t(\lambda)} \xi_i, \quad t > 0, \quad X_0 = 0, \end{aligned}$$

is called a compound Poisson process with intensity λ associated to ν.

2.5. Composing Lévy processes. More generally, a Lévy measure ν with $\nu(R) = \infty$ can construct a pure jump Lévy process. By using a Lévy measure ν on R such that $\nu(R) < \infty$ from the condition (16), we have already constructed a pure jump process $Y = Y_t, t \geq 0$ (associated to finite Lévy measure ν). Now, we think a Lévy measure on R such that

$$\nu(R) = \infty \quad and \quad \int_{-1}^{1} |x|\nu(dx) = \infty.$$

We introduce the structure of Lévy processes. The Lévy Ito decomposition (i.e., diffusion + pure jump) method. A Lévy process $Z_t, t \geq 0$ with the Lévy measure ν is defined as

$$Z_t = X_t + Y_t, \quad t \geq 0,$$

where (a) $X_t, t \geq 0 = at + \sigma B_t, t \geq 0$ is a Brownian motion (diffusion process with drift a and diffusion σ).

(b) $Y_t, t \geq 0$ is a pure jump Lévy process associated to Lévy measure ν.

(c) $X_t, t \geq 0$ and $Y_t, t \geq 0$ are independent.

3. Asset models

Since Black-Scholes model, very many articles studied and introduced the asset models and we met very many kinds of asset models. But, if we think about the conditions of asset models empirically, we can get two stylized phenomena results which are one is asset returns are not normally distributed, they are fat tailed and skewed, simply leptokultic and another one is implied volatilities are constant neither across strike, nor across maturity, they are smile shaped. If we think these points mainly, Lévy process has a good framework that can capture these empirical phenomena. In order to reflect these empirical phenomena in asset models, many complicate Lévy measure has been proposed.

On a well-defined probability space $(\Omega, \mathbf{F}, P)$, as we see in prior section, we can construct and define various Lévy processes. By using these Lévy processes we can define various types of return models, and various asset models of the form exponential Lévy processes. Further, we can define same asset models by using stochastic differential equations and their solutions.

As an sample, we write a simple course which introduce some construction of asset model. First, we determined a Lévy measure $\mu(dx)$, then we can construct a Lévy process $Z_t, t \geq 0$ by using characteristic function of Z_t, or using Brownian motion and pure jump Lévy process associated to Lévy measure μ, i.e., $Z_t = B_t + M_t$. Thus, we adapt our return model as a derivative

$$dY_t = \nu_t dt + \sigma_t dB_t + \gamma_t dM_t,$$

basically where $dZ_t = dB_t + dM_t$. Let Q be an equivalent martingale measure to P, and M_t be a compensated compound Poisson process which is a P-martingale $N_t - \lambda t$, where λ is a intensity number of Poisson process N_t. Then $M_t^Q := N_t - \int_0^t \lambda(1 + \Psi_s)ds$ is a Q-local martingale. Let T be a stopping time which is used in local martingale. We think a basic SDE model and the solution of it, for time $t \in T$;

$$dS_t = S_{t-}[\mu_t dt + \sigma_t dW_t + \gamma_t dM_t],$$

where σ_t is the volatility of asset prices defined by an SDE

$$d\sigma_t = \delta(\sqrt{\sigma_t}, t)\sigma_t dt + \rho(\sqrt{\sigma_t}, t)\sigma_t d\tilde{W}_t,$$

where $\tilde{W}_t$ is another Brownian motion which is independent with W_t. We can get a solution of the form as exponential Lévy process.

3.1. Intraday asset model. As we know, many trades of stocks occur in same price. But we can predict that asset prices are up and down sometimes frequently and are absorbed in maximum or minimum prices. These up and down, so-called, derive the volatility of asset model, and the (absorbing) states maximum and minimum which can be represented by using Poisson processes because of their scarceness. In general, asset price movement models are represented by smooth parts and noise parts. The smooth part is linked like the drift parameter of diffusion process and also noise part is influenced from the Brownian motion and its volatility.

On the other hand, we write(represent) a term by using indicator $I_{\{|\gamma_t(x)|<\infty\}}$, and extreme events by using indicators $I_{\{\gamma_t(x)=-\infty\}}$ and $I_{\{\gamma_t(x)=\infty\}}$ where

$$\gamma_t(x) = \frac{1}{\Delta t}[\Delta S_t | S_t = x], \quad \Delta S_t = S_t - S_{t-\Delta t}.$$

We would like use this representation, because this notation is more easy to understand than using jump-type stochastic processes, Poisson processes mainly.

Let T be a stopping time which is used in local martingale. For time t which is $t \in D \subset T$, we define an intraday asset model as following;

$$\begin{aligned} dS_t &= S_{t-}[(\mu_t dt + \sigma_t dW_t)I_{\{|\gamma_t(x)|<\infty\}} \\ &\quad +(-0.15)I_{\{\gamma_t(x)=-\infty\}} + (0.15)I_{\{\gamma_t(x)=\infty\}}]I_{\{t\in D\}}, \end{aligned} \tag{1}$$

where μ_t is a function constructed by daily auxiliary indices and (twitter)data with news. For example, deposit received(DR), exchange rate (ExR) etc are indices and, as twitter data, the instinct levels of human constructed by nine sentiments(anger, hate, dislike, fear, love, shame, sadness, hope, joy) (c.f., In [2] and [3]), or six mood which are constructed by calm, alert, sure, vital, kind, happy. Also there are many kinds of useful news. Further, so-called Elliott wave is important information in this model, and we can include the Hurst exponent for sudden drops which is using in fractional Brownian motion. The function γ_t represent shocking news.

If we attach more in μ_t, we can use the prior market points(another pre-markets indices) because the prior day another market data inference to next market. As we know from Table 3 and Table 4 in below, a prior market (Dow index) influence to KOSPI about almost 7 percent(about 2/3 ratio) up in prediction sometimes. In this model, we know that the price movement of one day are restricted in interval $[0.85S_0, 1.15S_0]$ where price S_0 is the price at starting time. We define the solution of above SDE asset model as following; for $t \in D$,

$$\begin{aligned} S_t &= S_0[\exp(\mu_t t + \sigma_t \tilde{W}_t) I_{\{|\gamma_t(x)|<\infty\}} \\ &\quad +(-0.15) I_{\{\gamma_t(x)=-\infty\}} + (0.15) I_{\{\gamma_t(x)=\infty\}}] I_{\{t\in D\}}. \end{aligned}$$

3.2. Weekly asset model. When we think weekly asset model, first we must think about maximum up and minimum down jumps in prior day. In general, after big jump, there is a decreasing or increasing wave which is extinct little by little in another wave slowly(c.f. Elliott wave, comparison with Elliott wave and Brownian motion trace, etc). Further, in this model, Hurst exponent is also useful yet as intraday asset model. If there is no big-jump, this model is almost similar as the Black-Scholes asset model but not same.

Also this model has very complicate non-noise terms. We know that if base interest rate(BIR) is down, asset price is up, and rate is up then prise down in general. Total money supply(MS) is increasing, then also asset price is increasing. Further, the increasing of quantity increasing rate of currency circulation influence to GDP, inflation and the increasing of asset prices. Thus, the basic interest rate and the money supply are basic elements of asset price movements.

Let N_t be a compound Poisson process. We define our weekly asset model as following;

$$dS_t = S_{t-}(\mu_t^{(W)} dt + \sigma_t^{(W)} dW_t + \gamma_t^{(W)} dN_t) I_{\{t\in W\}}, \tag{2}$$

where $\mu_t^{(W)}$ is the function of indices which are derived from financial factors and also is the levels volume of data concerning assets. The $\sigma_t^{(W)}$ is the volatility of Brownian motion. Further, we put as $\mu_t^{(W)} = \alpha_t^{(W,1)} + \beta_t^{(W,1)}$, $\alpha_t^{(W,1)} = \mu_{PD}^{(W)} + \mu_{PW}^{(W)}$ and $\gamma_t^{(W)} = \alpha_t^{(W,2)} + \beta_t^{(W,2)}$, where the upper index (W) means the first one week. Function $\alpha_t^{(W,1)}$ is a function made by smooth and non-shocking asset price movement factor index points which are in basic financial areas. Function $\beta_t^{(W,1)}$ is also a function composed by smooth and non-shocking asset price movement factor index points which are of basic non-financial areas. In here, $\mu_{PD}^{(W)}$ is the function of data of prior day in prior week which are useful to next day market, and $\mu_{PW}^{(W)}$ is the function of prior week data which are useful to next week market(c.f. [4], [8])(c.f. In [8], also we can see that sentiments have seven days periods). Function $\gamma_t^{(W)}$ is a simple function represented by shocking news(or index), $\alpha_t^{(W,2)}$ is got from economic shocking news, and $\beta_t^{(W,2)}$ is got from non-economic shocking news(or index).

For example, if we want to get asset model simply and good portfolio in haste or by using simple factors, we may put as $\alpha^{(W,1)}$ is a function constructed by r_t, m_t, d_t etc, where r_t is the (predictive) correlation level point of interest rate, m_t is the level point of quantity increasing rate of currency circulation and d_t is the level point of dividend. If we think more financial factors, we can think the business barometer, the balance of international payment, the exchange rate, and the raw materials prices, etc.

The function $\beta_t^{(W,1)}$ can be constructed by non-financial factors; the predictable government policy, and many kinds of predictable social movements etc. Further, $\alpha_t^{(W,2)}$ is a function composed by factors; shocking asset price movement factors which are in basic financial areas and main basic economical factors. For example they are some inside factors of company(merger and acquisition, new product news etc). The function $\beta_t^{(W,2)}$ is determined by many kinds of shocking non-financial factors, for example, international war, earthquake, Sewoll-ho event, and the labor management dispute, political and social factors etc. Thus simple function $\gamma_t^{(W)} = \alpha_t^{(W,2)} + \beta_t^{(W,2)}$ is represented by big-shocking news and events.

If we think integral form of above model, we can adopt it as

$$S_t = S_0 \exp\{\mu_t^{(W)} t + \sigma_t^{(W)} \tilde{W}_t + \gamma_t^{(W)} d\tilde{N}_t\} I_{\{t \in W\}}. \tag{3}$$

3.3. Monthly asset model. For this model, we must think many factors which influence to movements of asset prices. First we define an asset model as following; for $t \in M$,

$$dS_t = S_{t-}(\mu_t^{(M)} dt + \sigma_t^{(M)} dW_t + \gamma_t^{(M)} dN_t) I_{\{t \in M\}}. \tag{4}$$

In this model, function $\mu_t^{(M)}$ is consisted by several delicate factors as prior weekly asset model, but there are many kinds of indices which are useful in long period models. Even if we can see many factors in Section 4 and Section 5 in below, these factors can be distinguished roughly by functions consisted by financial factors $\alpha_t^{(M,1)}$, and non-financial factors $\beta_t^{(M,1)}$. Thus, we define as $\mu_t^{(M)} = \alpha_t^{(M,1)} + \beta_t^{(M,1)}$.

In this model, the $\gamma_t^{(M)}$ is the level of national war and trouble(NWT), political mistakes(PM), social shocks, economic shocks, moral shock of company and industry investment news. Also, the big jumps are not distinguished, because in a little long term shocking jumps look like small relatively. Non-financial factors are consisted by political factors and social factors etc.

Particularly, from this asset model, the interest rate, the level point of quantity increasing rate of currency circulation(MS) and the index of the CLI(Composite Leading Indicator) are very an important factors. In this asset model, we need not many kinds of twitter data because they are not continue long-lived in general. The volatility $\sigma_t^{(M)}$ of Brownian motion is decreasing for important role a little.

As an example, if we think simply, we may define as $\alpha_t^{(M,1)}$ is a function of r_t, m_t, d_t also. In general, in prior model, if the interest rate is down, then asset price is increased. But, for a little long time model $\alpha_t^{(M,1)}$, its influence is low. Further, the excessive increasing of quantity increasing rate of currency circulation derive inflation, and the value of asset is decreased. But asset price is increasing.

The integral form of this model is following;

$$S_t = S_0 \exp\{\mu_t^{(M)} t + \sigma_t^{(M)} \tilde{W}_s + \gamma_t^{(M)} \tilde{N}_t\} I_{\{t \in M\}}. \tag{5}$$

3.4. Yearly asset model. This model is defined by as following

$$dS_t = S_{t-}(\mu_t^{(Y)} dt + \sigma_t^{(Y)} dW_t) I_{\{t \in Y\}}. \tag{6}$$

where $\mu_t^{(Y)} = \alpha_t^{(Y)} + \beta_t^{(Y)}$. The base interest rate(BIR), the level point of quantity increasing rate of currency circulation(MS) and an index of the composite leading indicator(CLI) are very important index factors in this model. Also, political plane is a very important factor.

Important basic indices group may be in function $\alpha_t^{(Y)}$, and national political plane(NPP), industry investment plane(IIP), and economic police plane are may be in $\beta_t^{(Y)}$. The integral form of this model can be defined as

$$S_t = S_0 \exp\{\mu_t^{(Y)} t + \sigma_t^{(Y)} \tilde{W}_t\} I_{\{t \in Y\}}. \tag{7}$$

In this model, the term $\gamma_t^{(Y)} dN_t$ is not needed because the influence of many shocking news are soaked into Brownian motion term $\sigma_t^{(Y)} dW_t$ which is named as a noise part.

3.5. Portfolio asset model. For portfolio, we introduce a multi-dimensional asset model which is useful when we call or put several stocks. As we know in the same conditions, some asset prices are increasing and some are decreasing. This means that same factors give positive influence to some assets and may give negative influence to another assets. Thus we need a multi-dimensional asset price model even if it is a little simple, rough and a little coarse one (See [5]). By using above models, we combine them as one and define an asset model to get best portfolio as following;

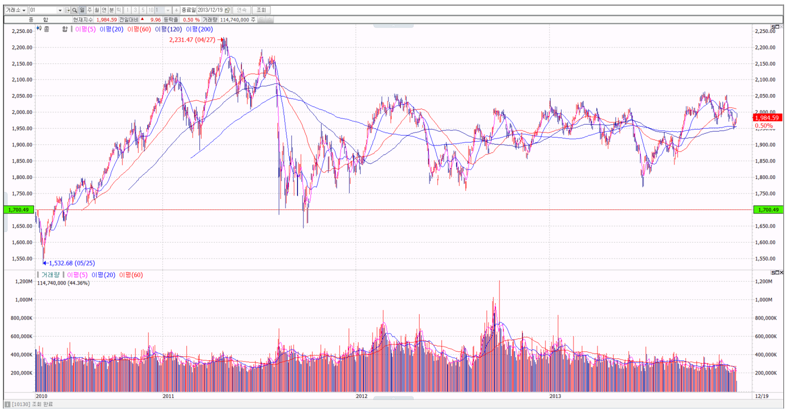
$$\begin{aligned} dS_t^i \;=\; & S_{t-}^i[(\mu_t^i dt + \sigma_t^i dW_t^i) I_{\{|\gamma_t(x)|<\infty\}} \\ & \quad +(-0.15) I_{\{\gamma_t(x)=-\infty\}} + (0.15) I_{\{\gamma_t(x)=\infty\}}] I_{\{t\in D\}} \\ & +S_{t-}^i[(\mu_t^{(W)i} dt + \sigma_t^{(W)i} dW_t^i + \gamma_t^{(W)i} dN_t^i) I_{\{t\in W\}} \\ & \quad +(\mu_t^{(M)i} dt + \sigma_t^{(M)i} dW_t^i + \gamma_t^{(M)i} dN_t^i) I_{\{t\in M\}} \\ & \quad +(\mu_t^{(Y)i} dt + \sigma_t^{(Y)i} dW_t^i) I_{\{t\in Y\}}]. \end{aligned}$$

Also the integral form of above SDE model for portfolio is following;

$$\begin{aligned} S_t^i \;=\; & S_0^i[\exp(\mu_t^i t + \sigma_t^i \tilde{W}_t^i) I_{\{|\gamma_t(x)|<\infty\}} \\ & \quad +(-0.15) I_{\{\gamma_t(x)=-\infty\}} + (0.15) I_{\{\gamma_t(x)=\infty\}}] I_{\{t\in D\}} \\ & +S_0^i \exp\{\mu_t^{(W)i} t + \sigma_t^{(W)i} \tilde{W}_t^i + \gamma_s^{(W)i} \tilde{N}_s^i\} I_{\{t\in W\}} \\ & +S_0^i \exp\{\mu_t^{(M)i} t + \sigma_t^{(M)i} \tilde{W}_t^i + \gamma_s^{(M)i} \tilde{N}_s^i\} I_{\{t\in M\}} \\ & +S_0^i \exp\{\mu_t^{(Y)i} t + \sigma_t^{(Y)i} \tilde{W}_t^i\} I_{\{t\in Y\}}. \end{aligned}$$

4. Indices and data

First we would like introduce a real graph of KOSPI. If we overlap our figures to the following graph, we can know that they have a little similar movements. Following real figure is the KOSPI real movement from the 15th of May, 2010 to the 20th of December, 2013. This figure is looks like complex and nobody's guess.



4.1. Indices. In next section, we composite some prediction for asset price movement. For good prediction, we need about ten indices over, which are given from $e-narajipyo$ which is a branch of "Statistic Korea(www.index.go.kr)", and a branch of government. If we arrange and enumerate several indices, they are as following: base interest rate, money supply, raw materials price, commodity price, exchange rate, prior market(DOW JONES) index, the increasing of lending of common people or business etc. For about index, we have no new special idea and material. Only we would like to emphasis their importance.

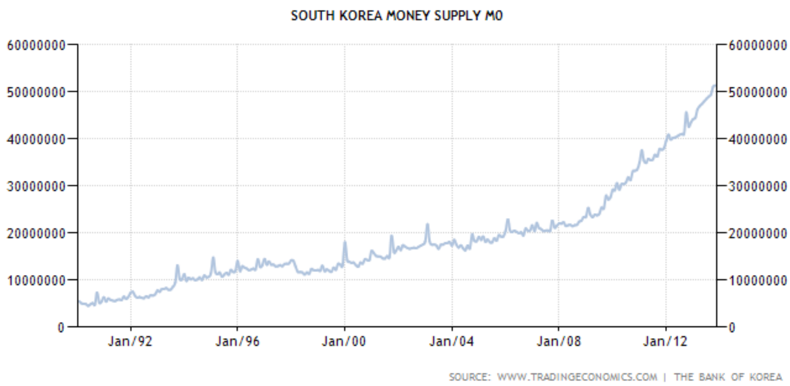
4.1.1. Basic interest rates. As we know, the base rate of central bank (and call rate also) is very important factor to the prices of stock assets. At low rate of interest, security market is booming(prosperous condition), and at high rate, is bust(recession). As we know, in 1998, r(interest rate) was almost 20 percent and KOSPI was $277pt$. By our Table 1, from February 2000 to July 2003, r is over 4.00 percent, from February 2006 to November 2008 is almost 5.00 percent. In these terms, the security market was bust. But from July 2003 to February 2006 and December 2008 to Now, the security market was prosperous. Thus we use the range of fluctuation or change of interest rates. Now, BIR is fixed at the 2.50 from the 9th of May, 2013 to the 21st of July, 2014 today.

Table 1
Basic Interest Rates(2006, Feb. 9th-2014, July 25th)

Time yrmhdy	Base Rate	Rate Varian	Time yrmhdy	Base Rate	Rate Varian	Time yrmhdy	Base Rate	Rate Varian
060209	4.00	+0.25p	081027	4.75	-0.25p	110113	2.75	+0.25p
060608	4.25	+0.25p	081107	4.00	-0.75p	110310	3.00	-0.50p
060810	4.50	+0.25p	081211	3.00	-1.00p	110610	3.25	+0.25p
070712	4.75	+0.25p	090109	2.50	-0.50p	120712	3.00	-0.25p
070809	5.00	+0.25p	090212	2.00	-0.50p	121011	2.75	-0.25p
080807	5.25	+0.25p	100709	2.25	+0.25p	130509	2.50	-0.25p
081009	4.00	-0.25p	101116	2.50	+0.25p	...	...	...

Table 1 is showing The Bank of Korea, Basic Interest Rates, Form 2006, February 09th to 2014, July 25th.

4.1.2. Money supply. This index, money supply, is one of the best important index with base interest rate to predict the movement of asset prices. Also there are many economic scholars who insist that almost anytime business fluctuations occur from "the interest rate regulation" and "the revel of money supply increasing". Thus they insist that the cause of business fluctuation is not in market or the defect of capitalism but in mistake and blunder of government policy. We would like follow them. Money supply is increasing, then the prices of commodities is increasing and asset price is increasing also. But it is only price, not value.



4.1.3. Composite leading indicators(CLI). As we know, we can get indexes of CLI easily. In here, as we know, there are nine indexes(i.e., it is composed by nine indexes); inventory circulation indicator, consumer expectation index, producer's shipment index and machinery for domestic demand(excluding vessels), construction orders received(real), net batter terms of trade(price), commodity price index, opening-to-application ratio, Korea composite stock price index. It is well known that asset markets are move in almost same time with CLI.

Table 2
Index CLI and CCI
April 2013 - February 2014

Yr Mon	2013 Apr	2013 May	2013 Jun	2013 Jul	2013 Aug	2013 Sep	2013 Oct	2013 Nov	2013 Dec	2014 Jan	2014 Feb
CLI	99.5	99.7	100.3	100.5	100.7	100.4	100.8	101.1	101.5	101.6	101.5
CCI	99.9	99.8	100.1	100.1	100.3	100.1	100.2	100.2	100.4	100.7	100.6

4.1.4. Composite coincident index(CCI). As we know, there are seven groups; industrial production index, index of services(excluding wholesale and retail sale), value of construction completed(real), retail sale index, producer's shipment index for domestic market, imports(real), number of employed persons(excluding agriculture, forestry and fishing).

4.1.5. Exchange rate. We can get exchange rates easily in internet containing currency rate, popular conversion(for example, Won to Dollars), currency tools, and popular currencies etc. For example in general, if Dollar price is increasing then KOSPI is increasing because many American buy Korean stocks.

4.1.6. Others. If we say another important and main indices, we can say the prices of commodities (or wholesale price index and consumer price index etc), the exchange rate, other prior market index(c.f. Dow Jones and FTSE100, etc) and many kind of index and data of The Statistics Korea(KOSTAT). As an example, we will introduce some part of KOSPI and DOW JONES. Further, we think about DR(deposit received), and TV(trading volume).

Table 3 KOSPI Index Dec. 28th 2009 - Jun. 8th 2010								
Time yr mh dy	Conclude Index	Ratio Previous	Ups Downs	Middle Price	Maximal Price	Minimal Price	Trade Volume	Trade Money
09 12 28	1,685.59	+03.25	+0.19	1,695.19	1,695.33	1,679.56	387,992	5,815,233
09 12 29	1,672.48	-13.11	-0.78	1,674.96	1,683.05	1,662.07	317,298	4,223,295
09 12 30	1,682.77	+10.29	+0.62	1,670.19	1,682.77	1,661.11	326,034	4,282,374
10 01 04	1,696.14	+13.37	+0.79	1,681.71	1,696.14	1,681.71	295,646	4,358,418
10 01 05	1,690.62	-05.52	-0.33	1,701.62	1,702.39	1,686.45	407,629	6,821,590
10 01 06	1,705.32	+14.70	+0.87	1,697.88	1,706.89	1,696.10	425,407	6,386,635
10 01 07	1,683.45	-21.87	-1.28	1,702.92	1,707.90	1,683.45	461,562	7,493,146
10 01 08	1,695.26	+11.81	+0.70	1,694.06	1,695.26	1,668.84	379,138	6,960.330
....	....	...	.....	......	......	......	.....	.....

Table 3 is showing the real trades piece of KOSPI from December 28th, 2009 to June 22nd, 2010.

Table 4 DOW JONSE Index Dec. 29th 2009 - Jun. 8th 2010							
Time yr mh dy	Conclude Index	Ratio Previous	Ups Downs	Middle Price	Maximal Price	Minimal Price	Trade Volume
09 12 29	10,545.41	-001.67	-0.02	10,547.83	10,580.33	10,544.28	092,887,099
09 12 30	10,548.51	+003.10	+0.03	10,544.36	10,550.70	10,505.66	110,160,283
09 12 31	10,428.05	-120.46	-1.14	10,548.51	10,555.01	10,423.13	137,941,788
10 01 04	10,583.96	+155.91	+1.50	10,340.69	10,604.97	10,430.69	179,781,702
10 01 05	10,572.02	-011.94	-0.11	10,584.56	10,584.56	10,522.52	188,542,224
10 01 06	10,573.68	+001.66	+0.02	10,564.72	10,594.99	10,546.55	186,042,927
10 01 07	10,606.86	+033.18	+0.31	10,571.11	10,612.37	10,505.21	217,391,185
10 01 08	10,618.19	+011.33	+0.11	10,606.40	10,619.40	10,554.33	172,712,171
10 01 11	10,663.99	+045.80	+0.43	10,620.31	10,676.23	10,591.59	182,045,111
....	....	...	.....	......	......	......	.....

Table 4 is showing the real trades piece of DOW JONES from December 29th, 2009 to June 22nd, 2010. As we know in Table 3 and Table 4, the price movements of prior market(Table 4 of DOW JONES) influence to KOSPI. If we use above tables, we can improve accuracy about 7 percent.

4.2. Data. From many kind of data, we can get and make many kinds of useful material. Thus, when we talk about "data", we mean big-data which are not yet index. But also there is no level and standard to determine index and data. Thus we mean data are useful materials which are not from public institutions or body. As some examples, we enumerate as following; twitter mood for six mood, nine sentiments for trade of asset, twitter reaction or response for the word of debt, the word of future, the word of stock return, the industry investment information, and sport sentiments etc.

4.2.1. Twitter six mood. As we see in [1] and [2], we know that the accuracy of Dow Jones Industrial Average prediction can be significantly improved by the inclusion of specific public mood dimensions but not others. They find an accuracy of 87.6 percents in predicting the daily up and down changes in the closing values of the DJIA and a reduction of the mean average percentage error by more than 6 percents(See Figure 2 and Table 1 in [1]). We can use these words also.

4.2.2. Nine sentiments for trade Of asset. As we see in [11], we know that the finding from the autocorrelation analysis proved autocorrelation and period of the sentiments while the results from the principle component analysis reported that the nine sentiments could be connected with positivity and negativity(See the Tables in [11]).

Further, we think about Twitter In Debt, In Future Word, Number and Industry Investment, and Sports Sentiments And Stock Return also.

4.3. Shocking events. For about function gamma which is in our asset models, we have several idea for useful materials. We count about the national war and trouble, political mistakes, labor management dispute, social big shocking events, economic political news, moral shocking for company owner, and industry investment national plan news etc.

5. Prediction and simulation

To get more accurate simulation if possible, we need many kinds of index, data, and event information. But we will use indexes in our simulations, BIR(Base Interest Rate), MS(money Supply of Korea Bank), CLI(Composite Leading Indicator), ExR(Exchange Rate with Dollar), CCI(Composite Coincident Indicator), OR(Others of Prior Market Index), DR(Deposit Received), and TV(Trading Volume), etc. Also, as useful and important indices, we can use Primary Material Price Indices. They are BDI(Baltic Dry Index), CRBI(Commodity Research Bear index), and LMEI(Loden Metal Exchange Index).

We can use twitter data, for example SM(Six Mood; calm, alert, sure, vital, kind, happy) and NS(Nine Sentiment; anger, hate, dislike, fear, love, shame, sadness, hope, joy) which are arranged from so-called big data. Further we can check TR(Twitter in Return), TD(Twitter in Debt), TF(Twitter in Future word), and TII(Twitter in Industry Investment) which are arranged from and by cellular smart phone, etc.

Further we can check big news and use big and important events; NWT(National War and Trouble) which is almost 0 point, PM(Political Mistake), SN(Social big News from mass communications), EPM(Economic Police Mistake), MSC(Moral shock of Company), SK(Social big Shocking events) and IIN(Industry Investment News), etc.

Finally, we must count volatility(σ). But this volatility is given by price movements, we will not denote tn tables. Thus we will use minimum 10 factors. Further we assume that $-0.15 < \mu, \delta < +0.15$ in following Tables.

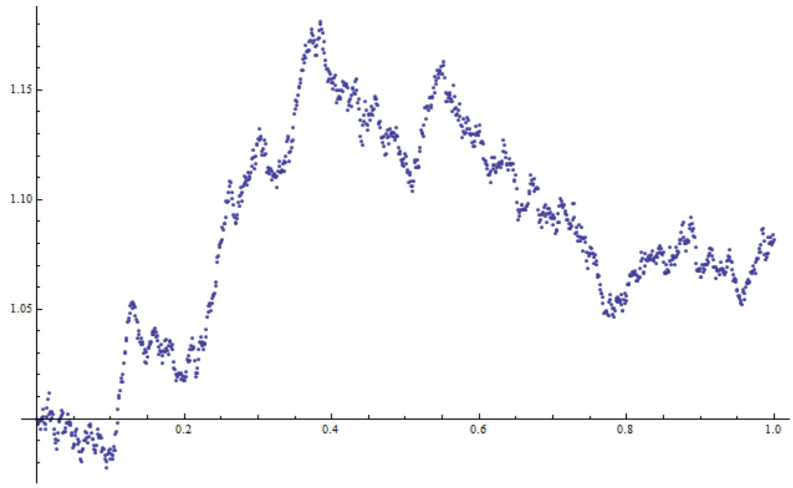
5.1. Intraday asset model prediction box. First, we will show figure 5.1 of one day KOSPI index movement which is of the 18th February, 2014.

Foot Note: KOSPI start 1946.88, Maximum 1951.06, Minimum 1935.60, Final 1946.91, △ 0.55(+0.035 percent), Total Volume was 200447.

Table 5
Sample Prediction Box Of Intraday
Feb. 18th in 2014.

Material	BIR	MS	CLI	ExR	CCI	OR	TV	DR	...	Value
Value	2.50	20,000	101.5	1080.50	100.6	·	20044	27229	...	...
UpDown	0	0	0	-4.20	+0.60	+0.79	-01504	-0002	...	...
Apply	0	0	0	+ 0.42	+0.60	+0.79	-1.50	-0.20	...	+0.10

From the above useful indices, various (twitter) data, big news and big events(c.f. Table 5), we can predict future asset price movements rawly. Following Graph is a simulation by using Mathematica and several materials. It is also important that each index value is of same fixed time on one day for comparison in next day, for example, at A.M. 10:00.

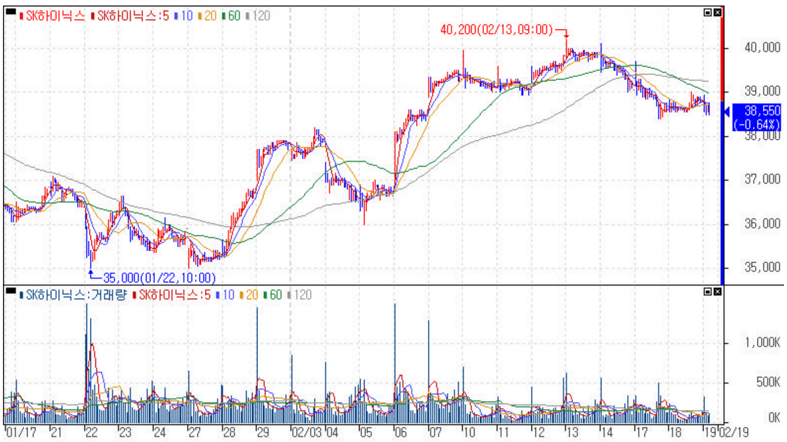


Foot Note: The 18th of February, 2014 is an ordinary day with no big events. In this above Graph, we used ExR, CCI, OR, TV, and DR mainly(cf. above Table 5). The start slope was +0.10 at 0, and was no changed. We used Mathematica 6.

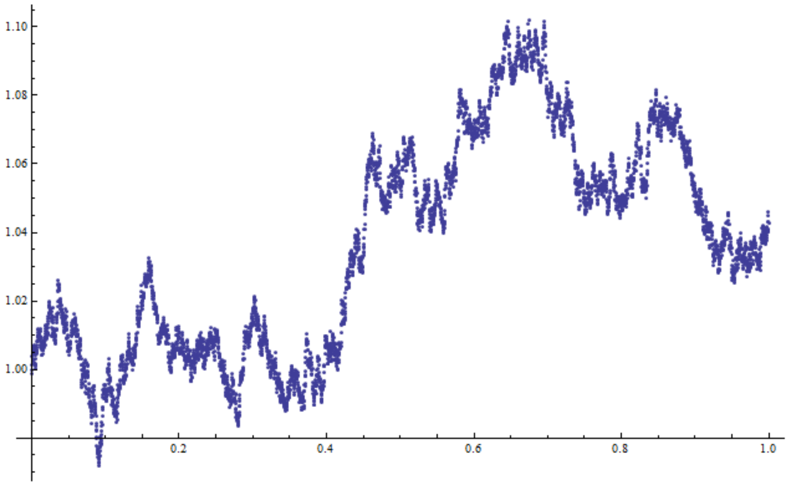
5.2. Weekly asset model prediction box. This figure 5.2 is of KOSPI index movement which is of from the 12th to the 18th in February, 2014.

Foot Note: KOSPI was start at 1932.07 on the 12th, Maximum 1955.04(Feb. 17th, A.M. 9:00), Minimum 1924.39(Feb. 13th, P.M. 2:45), Final 1946.91.

5.3. Monthly asset model prediction box. This figure 5.3 is of one month SK Hynix price movement which is of from the 17th January to the 18th February, 2014.

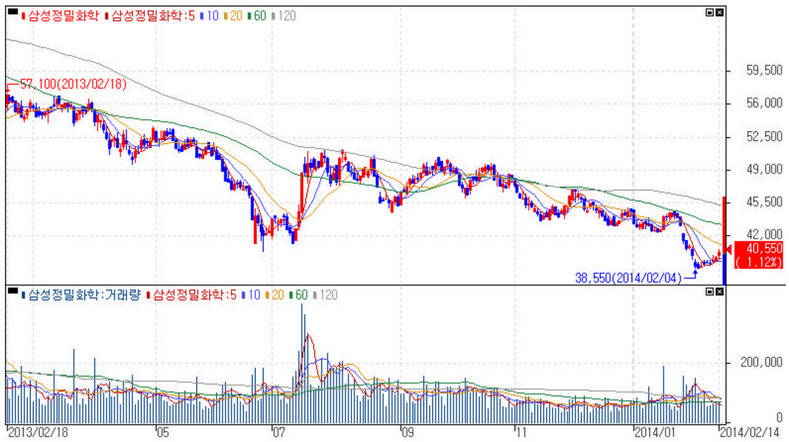


The following Graph 2 is the simulation of Figure 5.3.



Foot note: For this Graph 2, we start at 1.0 and give slope +0.1 on the first(on Jan. 17th Friday), give slope -0.1, give slope +0.2, give slope -0.3, and give slope +0.1 for each one week(5 days) interval by using Mathematica 6.

5.4. Yearly asset model prediction box. This figure is of Samsung Fine Chemicals price movement which is of from the 18th February, 2013 to the 18th February, 2014.



5.5. Postscript. It is very regretful that we are not study for special stocks in subsection 5.1 and 5.2. As we know, our Figures 5.1 and 5.2 are KOSPI market. For intraday asset model, Elliott wave is a little useful graph in statistics even if it is not founded by rational theory, and Hurst exponent is useful to prediction on a sudden drop of stock price even if the background theory is a little difficult to understand and to get exponent.

When we think and deal with a special stock, it is very important that we must not change the sorts of materials and check time of every each day, because they are very useful now on today or but not in tomorrow and next tomorrow data for comparison at same time.

As we know from tables and figures, these indices, data and stock price movement primary factors of intraday, weekly, monthly and yearly asset models are different. Moveover, we would like express that, for the prediction of asset price movements, is it necessary that we purchase big-data, information(ex, include zirasi etc.) and many kind twitter data? We can make and get import and valuable data and information by arrange daily data if possible, at same time on every day.

6. Summary

For Intraday and Weekly asset models, we need many kind of data, delicate information and useful news. But for a little ling term asset models, as example, for Monthly and Yearly asset models, basic financial indices are more useful. If we arrange big financial main indices which are influence to price of assets are basic interest rate(BIR) and money supply(MS). Add to above two factors, we can arrange as composite leading indication(CLI), composite coincident indicator(CCI) and industry investment data(IID) of primary material price index(PMPI) etc. If we add important factors more for prediction of asset price movements, they are politics and economic policy. But, nowadays the development of internet and communication, international financial information also influence to conclusion of asset market prices. Finally, we add some import factors that the ability and morality of management or owner of company influence to the stock price of company.

We would like say a word. Now, today is the 21st of July, 2014. We know from news that government want to down BIR. As we can check several important indices and data, we can predict the future of financial market. If there are no big chocking events or accidents, KOSPI market will be brisk for a while.

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