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STRONG CONVERGENCE OF PATHS FOR NONEXPANSIVE SEMIGROUPS IN BANACH SPACES

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Abstract. Let E be a uniformly convex Banach space with a uniformly Gâteaux differentiable norm, C be a nonempty closed convex subset of E and $f : C \to C$ be a fixed bounded continuous strong pseudocontraction with the coefficient $\alpha \in (0,1)$. Let $\{\lambda_t\}_{0<t<1}$ be a net of positive real numbers such that $\lim_{t\to 0}\lambda_t = \infty$ and $S = \{T(s) : 0 \le s < \infty\}$ be a nonexpansive semigroup on C such that $F(S) \neq \emptyset$, where $F(S)$ denotes the set of fixed points of the semigroup. Then sequence $\{x_t\}$ defined by $x_t = tf(x_t) + (1 - t)\frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds$ converges strongly as $t \to 0$ to $\bar{x} \in F(S)$, which solves the following variational inequality $\langle (f - I)\bar{x}, p - \bar{x}\rangle \le 0$ for all $p \in F(S)$.

1. Introduction and preliminaries

Let E be a Banach space with the dual E^*. We denote by J the normalized duality mapping from E to 2^{E^*} defined by

$$Jx = \{f^* \in E^* : \langle x, f^*\rangle = \|x\|^2 = \|f^*\|^2\},$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. Let $U_E = \{x \in E : \|x\| = 1\}$. E is said to be *Gâteaux differentiable* if the limit $\lim_{t\to 0}\frac{\|x+ty\|-\|x\|}{t}$ exists for all $x, y \in U_E$. In this case, E is said to be *smooth.* In a smooth Banach space, the normalized duality mapping is single valued. In the work, we use j to denote the single valued normalized duality mapping. The norm of E is said to be *uniformly*

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Gâteaux differentiable if for each $y \in U_E$, the limit is attained uniformly for each $x \in U_E$.

E is said to be *uniformly convex* if for any $\epsilon \in (0,2]$ there exists $\delta > 0$ such that, for all $x, y \in U_E$,

$$\|x - y\| \geq \epsilon \quad \text{implies} \quad \|x + y\| \leq 2(1 - \delta).$$

It is known that a uniformly convex Banach space is reflexive and strictly convex.

Let C be a nonempty closed convex subset of E and $T : C \to C$ be a nonlinear mapping. A point $x \in C$ is said to be a *fixed point* of T if $Tx = x$. Denote by $F(T)$ the set of fixed points of T; that is, $F(T) = \{x \in C : Tx = x\}$. Recall the following definitions.

(1) T is said to be *contractive* if there exists a constant $\alpha \in (0,1)$ such that

$$\|Tx - Ty\| \leq \alpha \|x - y\|, \quad \forall x, y \in C;$$

(2) T is said to be *strongly pseudocontractive* if there exists a constant $\alpha \in (0,1)$ and $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y) \rangle \leq \alpha \|x - y\|^2, \quad \forall x, y \in C;$$

(3) T is said to be *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in C.$$

One classical way to study nonexpansive mappings is to use contractions to approximate a nonexpansive mapping; see [3,8-10,14]. More precisely, take $t \in (0,1)$ and define a contraction $T_t : C \to C$ by

$$T_t x = tu + (1 - t)Tx, \quad x \in C, \tag{1.1}$$

where $u \in C$ is a fixed point. Banach's contraction mapping principle guarantees that T_t has a unique fixed point x_t in C. In the case that T enjoys a nonempty fixed point set, Browder [3] proved that if E is a Hilbert space, then $\{x_t\}$ does converges strongly to the fixed point of T that is nearest to u. Reich [10] extended Browder's result to the setting of Banach space and proved that if E is a uniformly smooth Banach space, then $\{x_t\}$ converges strongly to a fixed point of T and the limit defines the unique sunny nonexpansive retraction from C onto $F(T)$.

Viscosity approximation method which was introduced by Moudafi [7] has been considered by many authors. In 2004, Xu [14] studied the following continuous scheme

$$x_t = tf(x_t) + (1-t)Tx_t, \tag{1.2}$$

where $t \in (0,1)$, f is a contraction with the coefficient $\alpha \in (0,1)$ and T is a nonexpansive self-mapping on C. He showed that $\{x_t\}$ defined by (1.2) converges strongly to a fixed point x of the mapping T, which also solves the following variational inequality

$$\langle f(x) - x, j(y-x)\rangle \leq 0, \quad \forall y \in F(T).$$

Recall that a family $S = \{T(s) : 0 \leq s < \infty\}$ of mappings from C into itself is called a *nonexpansive semigroup* on C if it satisfies the following conditions:

(c1) $T(0)x = x$ for all $x \in C$;
(c2) $T(s+t)x = T(s)T(t)x$ for all $x \in C$ and $s, t \geq 0$;
(c3) $\|T(s)x - T(s)y\| \leq \|x - y\|$ for all $x, y \in C$ and $s \geq 0$;
(c4) for all $x \in C$, $s \mapsto T(s)x$ is continuous.

In this paper, we use $F(S)$ to denote the set of fixed points of S, that is, $F(S) = \bigcap_{0 \leq s < \infty} F(T(s))$. We know that $F(S) \neq \emptyset$ if C is bounded; see [2].

Recently, Plubtieng and Punpaeng [8] studied the problem of convergence of paths for nonexpansive semigroups in Hilbert spaces. To be more precise, they proved the following result.

THEOREM PP. *Let C be a nonempty closed convex subset of a real Hilbert space and $S = \{T(s) : 0 \leq s < \infty\}$ be a nonexpansive semigroup on C such that $F(S) \neq \emptyset$. Let $\{\lambda_t\}$ be a net of positive real numbers such that* $\lim_{t\to 0} \lambda_t = \infty$. *Then for a contraction $f : C \to C$ with coefficient $\alpha \in (0,1)$, the sequence $\{x_t\}$ defined by*

$$x_t = tf(x_t) + (1-t)\frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds,$$

converges strongly to $\tilde{x}$, where $\tilde{x}$ is the unique solution in $F(S)$ of the variational inequality

$$\langle (I-f)\tilde{x}, x-\tilde{x}\rangle \le 0, \quad \forall x \in F(S).$$

The purpose of this paper is to establish a general Banach version of Theorem PP. In order to prove our main result, we need the following lemmas.

LEMMA 1.1. ([1,5,11]) *Let D be a nonempty bounded closed convex subset of a uniformly convex Banach Space E and $S = \{T(t) : 0 \le t < \infty\}$ be a nonexpansive semigroup on D. Then, for any $0 \le h < \infty$,*

$$\lim_{t\to\infty} \sup_{x\in D} \left\| \frac{1}{t}\int_0^t T(s)xds - T(h)\frac{1}{t}\int_0^t T(s)xds \right\| = 0.$$

A function $\omega : \mathbb{R}^+ \to \mathbb{R}^+$ is said to belong to Γ if it satisfies the following conditions:

(1) $\omega(0) = 0$;
(2) $r > 0 \Rightarrow \omega(r) > 0$;
(3) $t \le s \Rightarrow \omega(t) \le \omega(s)$.

LEMMA 1.2. ([13]) *Let E be a uniformly convex Banach space. Then, for any $R > 0$, there exists $\omega_R \in \Gamma$ such that*

$$\begin{aligned} &x, y \in B_R[0],\ x^* \in J(x),\ y^* \in J(y) \\ &\quad \Longrightarrow \quad \langle x-y, x^*-y^*\rangle \ge \omega_R(\|x-y\|)\|x-y\|, \end{aligned}$$

where $B_R[0] = \{x : \|x\| \le R\}$.

LEMMA 1.3. ([6]) *Let E be a Banach space, C be a nonempty closed convex subset of E and $T : C \to C$ be a continuous strong pseudocontraction. Then T has a unique fixed point in C.*

Next, let us recall the definition of means. Let S be a nonempty set and $B(S)$ the Banach space of all bounded real valued functions on S with the supremum norm. Let X be a subspace of $B(S)$ and μ an element in X^*, where X^* denotes the dual space of X. Then we denote by $\mu(f)$ the value of μ at $f \in X$. If $e(s) = 1$ for all $s \in S$, sometimes $\mu(e)$ will be denoted by $\mu(1)$. When X contains constants, a linear functional μ on X is said to be a *mean* on X if $\|\mu\| = \mu(1) = 1$. From [13], we see that μ is a mean on X if and only if

$$\inf_{s \in S} f(s) \leq \mu(f) \leq \sup_{s \in S} f(s), \quad \forall f \in X.$$

Set $A = (0,1)$, let $B(A)$ denote the Banach space of all bounded real value functions on A with supremum norm and let X be a subspace of $B(A)$.

LEMMA 1.4. ([13]) *Let C be a nonempty closed convex subset of a Banach space E. Suppose that the norm of E is uniformly Gâteaux differentiable. Let $\{x_t\}$ be a bounded set in E and $z \in C$. Let μ_t be a mean on X. Then $\mu_t \|x_t - z\|^2 = \min_{y \in C} \|x_t - y\|^2$ if and only if $\mu_t \langle y - z, J(x_t - z)\rangle \leq 0$ for all $y \in C$.*

2. Main results

THEOREM 2.1. *Let E be a uniformly convex Banach space with a uniformly Gâteaux differentiable norm, C be a nonempty closed convex subset of E and $f : C \to C$ be a fixed bounded continuous strong pseudocontraction with the coefficient $\alpha \in (0,1)$. Let $\{\lambda_t\}_{0<t<1}$ be a net of positive real numbers such that $\lim_{t \to 0} \lambda_t = \infty$ and $S = \{T(s) : 0 \leq s < \infty\}$ be a nonexpansive semigroup on C such that $F(S) \neq \emptyset$. Then $\{x_t\}$ defined by*

$$x_t = tf(x_t) + (1-t)\frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds, \tag{2.1}$$

where $t \in (0,1)$ converges strongly as $t \to 0$ to $\bar{x} \in F(S)$, which solves the following variational inequality

$$\langle f(\bar{x}) - \bar{x}, j(p - \bar{x})\rangle \leq 0, \quad \forall p \in F(S).$$

Proof. For $t \in (0,1)$, define a mapping $T_t^f : C \to C$ by

$$T_t^f x = tf(x) + (1-t)\frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x ds.$$

Then $T_t^f : C \to C$ is a continuous strong pseudocontraction for each $t \in (0,1)$. Indeed, for each $x, y \in C$, we have

$$\begin{aligned}
&\langle T_t^f x - T_t^f y, j(x-y)\rangle \\
&= t\langle f(x) - f(y), j(x-y)\rangle \\
&\quad + (1-t)\left\langle \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)xds - \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)yds, j(x-y)\right\rangle \\
&\le t\alpha\|x-y\|^2 + (1-t)\|x-y\|^2 \\
&= [1 - t(1-\alpha)]\|x-y\|^2.
\end{aligned}$$

From Lemma 1.3, we see that T_t^f has a unique fixed point x_t in C for each $t \in (0,1)$. Hence (2.1) is well defined.

Next, we show that $\{x_t\}$ is bounded. Taking $p \in F(S)$, we have

$$\begin{aligned}
\|x_t - p\|^2 &= \langle x_t - p, j(x_t - p)\rangle \\
&= t\langle f(x_t) - p, j(x_t - p)\rangle \\
&\quad + (1-t)\left\langle \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds - p, j(x_t - p)\right\rangle \\
&= t\langle f(x_t) - f(p), j(x_t - p)\rangle + t\langle f(p) - p, j(x_t - p)\rangle \\
&\quad + (1-t)\left\langle \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds - p, j(x_t - p)\right\rangle \\
&\le t\alpha\|x - p\|^2 + t\langle f(p) - p, j(x_t - p)\rangle \\
&\quad + (1-t)\|\frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds - p\|\|x_t - p\| \\
&\le [1 - t(1-\alpha)]\|x_t - p\|^2 + t\langle f(p) - p, j(x_t - p)\rangle.
\end{aligned}$$

It follows that

$$\|x_t - p\|^2 \le \frac{1}{1-\alpha}\langle f(p) - p, j(x_t - p)\rangle. \tag{2.2}$$

This implies that

$$\|x_t - p\| \le \frac{1}{1-\alpha}\|f(p) - p\|.$$

This shows that $\{x_t\}$ is bounded. On the other hand, for each $\tau \geq 0$, we have

$$
\begin{aligned}
&\|T(\tau)x_t - x_t\| \\
&\leq \left\| T(\tau)x_t - T(\tau)\left(\frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds\right)\right\| \\
&\quad + \left\| T(\tau)\left(\frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds\right) - \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds\right\| \\
&\quad + \left\| \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds - x_t\right\| \\
&\leq 2\left\| x_t - \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds\right\| \\
&\quad + \left\| T(\tau)\left(\frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds\right) - \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds\right\| \\
&= \frac{2t}{1-t}\|f(x_t) - x_t\| \\
&\quad + \left\| T(\tau)\left(\frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds\right) - \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds\right\|.
\end{aligned} \tag{2.3}
$$

Letting $z_0 \in F(S)$ and $M = \{z \in C : \|z - z_0\| \leq \frac{1}{1-\alpha}\|f(z_0) - z_0\|\}$, we see that M is a nonempty bounded closed convex subset of C which is $T(s)$-invariant for each $s \in [0, \infty)$ and contains $\{x_t\}$. From Lemma 1.1 and passing to $\lim_{t\to 0}$ in (2.3), we can obtain that, for all $\tau \geq 0$,

$$T(\tau)x_t - x_t \to 0 \quad \text{as } t \to 0. \tag{2.4}$$

Define $h(x) = \mu_t\|x_t - x\|^2$ for all $x \in C$, where μ_t is a mean. Then $h(x)$ is a continuous, convex and $h(x) \to \infty$ as $\|x\| \to \infty$. We see that h attains its infinimum over C (see, e.g., [11,13]). Set

$$D = \Big\{x \in C : h(x) = \inf_{y \in C} h(y)\Big\}.$$

Then D is a nonempty bounded closed convex subset of C. We see that D is singleton. Indeed, suppose that $\tilde{x}, \bar{x} \in D$ and $\tilde{x} \neq \bar{x}$. From Lemma 1.2, we see that

$$
\begin{aligned}
&\langle (x_t - \bar{x}) - (x_t - \tilde{x}), j(x_t - \bar{x}) - j(x_t - \tilde{x})\rangle \\
&> \omega_R(\|\bar{x} - \tilde{x}\|)\|\bar{x} - \tilde{x}\|, \quad \forall 0 < t < 1.
\end{aligned}
$$

It follows that

$$\mu_t\langle \tilde{x} - \bar{x}, j(x_t - \bar{x}) - j(x_t - \tilde{x})\rangle > 0. \tag{2.5}$$

On the other hand, we see from Lemma 1.4 that

$$\mu_t\langle \tilde{x} - \bar{x}, j(x_t - \bar{x})\rangle \le 0 \tag{2.6}$$

and

$$\mu_t\langle \bar{x} - \tilde{x}, j(x_t - \tilde{x})\rangle \le 0. \tag{2.7}$$

Adding up (2.6) and (2.7), we arrive at

$$\mu_t\langle \tilde{x} - \bar{x}, j(x_t - \bar{x}) - j(x_t - \tilde{x})\rangle \le 0.$$

This contradicts (2.5). This shows that $\bar{x} = \tilde{x}$. Next, we denote the single element in D by $\bar{x}$. It follows from (2.4) that

$$\begin{aligned} h(T(\tau)(\bar{x})) &= \mu_t\|x_t - T(\tau)(\bar{x})\|^2 \\ &= \mu_t\|T(\tau)(x_t) - T(\tau)(\bar{x})\|^2 \\ &\le \mu_t\|x_t - \bar{x}\|^2 \\ &= h(\bar{x}), \quad \forall \tau \ge 0. \end{aligned}$$

This implies that $\bar{x} = T(\tau)(\bar{x})$ for all $\tau \ge 0$, that is, $\bar{x} \in F(S)$.

On the other hand, we see from Lemma 1.4 that

$$\mu_t\langle y - \bar{x}, j(x_t - \bar{x})\rangle \le 0, \quad \forall y \in C.$$

By taking $y = f(\bar{x})$, we obtain that

$$\mu_t\langle f(\bar{x}) - \bar{x}, j(x_t - \bar{x})\rangle \le 0. \tag{2.8}$$

Combining (2.2) with (2.8), we arrive at

$$\mu_t\|x_t - \bar{x}\|^2 = 0.$$

This implies that there exists a subnet $\{x_{t_\alpha}\}$ of $\{x_t\}$ such that $x_{t_\alpha} \to \bar{x}$.

Notice that

$$x_t - f(x_t) = (1-t)\bigg(\frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds - f(x_t)\bigg).$$

For any $p \in F(S)$, we see that

$$\begin{aligned}
&\langle x_t - f(x_t), j(x_t - p)\rangle \\
&= (1-t)\bigg\langle \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds - f(x_t), j(x_t - p)\bigg\rangle \\
&= (1-t)\bigg\langle \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds - x_t, j(x_t - p)\bigg\rangle \\
&\quad + (1-t)\langle x_t - f(x_t), j(x_t - p)\rangle \\
&= (1-t)\bigg\langle \frac{1}{\lambda_t}\int_0^{\lambda_t} T(s)x_t ds - p, j(x_t - p)\bigg\rangle \\
&\quad + (1-t)\langle p - x_t, j(x_t - p)\rangle + (1-t)\langle x_t - f(x_t), j(x_t - p)\rangle \\
&\le (1-t)\langle x_t - f(x_t), j(x_t - p)\rangle,
\end{aligned}$$

which implies that

$$\langle x_t - f(x_t), j(x_t - p)\rangle \le 0, \quad \forall p \in T(S). \tag{2.9}$$

In particular, we have

$$\langle x_{t_\alpha} - f(x_{t_\alpha}), j(x_{t_\alpha} - p)\rangle \le 0, \quad \forall p \in T(S). \tag{2.10}$$

It follows that

$$\langle \bar{x} - f(\bar{x}), j(\bar{x} - p)\rangle \le 0, \quad \forall p \in T(S). \tag{2.11}$$

Assume that there exists another subnet $\{x_{t_\beta}\}$ of $\{x_t\}$ such that $x_{t_\beta} \to \hat{x} \in F(S)$. From (2.11), we arrive at

$$\langle \bar{x} - f(\bar{x}), j(\bar{x} - \hat{x})\rangle \le 0. \tag{2.12}$$

In view of (2.9), we see that

$$\langle x_{t_\beta} - f(x_{t_\beta}), j(x_{t_\beta} - \bar{x})\rangle \le 0. \tag{2.13}$$

It follows that

$$\langle \hat{x} - f(\hat{x}), j(\hat{x} - \bar{x}) \rangle \leq 0. \tag{2.14}$$

Adding up (2.12) and (2.14), we obtain that

$$\langle \bar{x} - f(\bar{x}) - \hat{x} + f(\hat{x}), j(\bar{x} - \hat{x}) \rangle \leq 0.$$

This implies that

$$\|\bar{x} - \hat{x}\|^2 \leq \alpha \|\bar{x} - \hat{x}\|^2.$$

Note that $\alpha \in (0,1)$. We see that $\bar{x} = \hat{x}$. This shows that $\{x_t\}$ converges strongly to $\bar{x} \in F(S)$, which is the unique solution to the variational inequality

$$\langle f(\bar{x}) - \bar{x}, j(p - \bar{x}) \rangle \leq 0, \quad \forall p \in F(S).$$

This completes the proof. □

Remark 2.2. The viscosity approximation method considered in Theorem 2.1 is different from Moudafi's and Chen et al.'s. In [7], Moudafi considered f as a contraction. In [4], Chen et al. considered f as a Lipschitz strong pseudocontraction. In this work, we consider f as a continuous strong pseudocontraction.

Remark 2.3. Theorem 2.1 which includes the corresponding results announced in Chen and Song [5], Shioji and Takahashi [12] and Xu [14] as special cases mainly improves Theorem PP (Theorem 3.1 of Plubtieng and Punpaeng [8]) in the following aspects.

(1) Extend the space from Hilbert spaces to uniformly convex Banach spaces;

(2) Extend the mapping f from the class of contractions to the class continuous strong pseudocontractions.

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