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RIEMANN SOLITONS ON CERTAIN TYPE OF KENMOTSU MANIFOLD

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Abstract. The object of the present paper is to investigate the nature of Riemann solitons on generelized D-conformally deformed Kenmotsu manifold with hyper generalized pseudo symmetric curvature conditions.

1. Introduction

Let the symbols ∇ and ∇^d stand for the Riemann connection and the generalized D-conformally deformed connection respectively. Also, let R, S, Q, r and R^d, S^d, Q^d, r^d respectively stand for curvature tensor, Ricci tensor, Ricci operator, scalar curvature with respect to ∇ and ∇^d respectively. In this study, we consider an almost contact metric manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ that consists of a $(1,1)$-tensor field ϕ, a vector field ξ and a 1-form η called respectively the structure endomorphism, the characteristic vector field and the contact form. In a recent paper, the authors ([2]) has introduced a new type of space called hyper generalized weaky symmetric manifold. Then the authors studied ([8]) hyper generalized pseudo Q-symmetric semi-Riemanian manifold. In Section 3 of this paper we extend this concept to generalized D-conformally deformed structure of a $(2n+1)$-dimensional Kenmotsu manifold.

A $(2n+1)$-dimensional Kenmotsu manifold is said to be hyper generalized pseudo symmetric (which will be abbreviated hereafter as $[H(GPS)_n, \nabla]$) if it admits the equation

$$\begin{aligned}
&(\nabla_X \bar{R})(Y,U,V,\ W)\\
=\ &2\alpha(X)\bar{R}(Y,U,V,\ W)+\alpha(Y)\bar{R}(X,U,V,\ W)\\
&+\alpha(U)\bar{R}(Y,X,V,\ W)+\alpha(V)\bar{R}(Y,U,X,\ W)\\
&+\alpha(W)\bar{R}(Y,U,V,\ X)+2\beta(X)(g\wedge S)(Y,U,V,W)\\
&+\beta(Y)(g\wedge S)(X,U,V,W)+\beta(U)\ (g\wedge S)(Y,X,V,W)\\
&+\beta(V)\ (g\wedge S)(Y,U,X,W)+\beta(W)\ (g\wedge S)(Y,U,V,X), \qquad (1)
\end{aligned}$$

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where

$$(g \wedge S)(Y,U,V,W) = g(Y,W)S(U,V) + g(U,V)S(Y,W) - g(Y,V)S(U,W) - g(U,W)S(Y,V), \tag{2}$$

and α, β being non-zero 1-forms defined as $\alpha(X) = g(X,\theta_1)$ and $\beta(X) = g(X,\theta_2)$.

Ricci flow was first introduced by R. S. Hamilton ([17]) in 1982 which generalizes the notion of Riemann flow ([20], [19]). Keeping the tune with Ricci soliton, Hirica and Udriste ([18]) introduced and studied Riemann soliton. The Riemann flow is an evolution equation for metrics on a Riemannian manifold defined as follows

$$\frac{\partial}{\partial t}G(t) = -2R(g(t)), \qquad t \in [0, I],$$

where $G = \frac{1}{2} g \circledast g$, $\circledast$ is the Kulkarni-Nomizu product and R is the Riemann curvature tensor associated to the metric g. For $(0,2)$-tensors A and B, the Kulkarni-Nomizu product $(A \circledast B)$ is given by

$$(A \circledast B)(Y,U,V,Z) = A(Y,Z)B(U,V) + A(U,V)B(Y,Z) - A(Y,V)B(U,Z) - A(U,Z)B(Y,V). \tag{3}$$

Recently, the present authors studied the Riemann solitons in the frame of $(LCS)_n$-manifolds ([4]) and α-cosymplectic manifolds ([5]). The Riemann soliton is a smooth manifold M together with Riemannian metric g that satisfies

$$2R + (g \circledast \pounds_W g) = 2\kappa(g \circledast g), \tag{4}$$

where W is a potential vector field, $\pounds_W$ denotes the Lie-derivative along the vector field W and κ is a constant. A Riemann soliton is called expanding, steady and shrinking when $\kappa < 0$, $\kappa = 0$ and $\kappa > 0$ respectively.

We organize our present paper as follows: After Introduction, in Section 2, we briefly recall some known results for Kenmotsu manifolds and generalized D-conformally deformed of a Kenmotsu manifold and established some properties of the deformed Kenmotsu manifold. In Section 3, we discuss the properties of a generalized D-conformally deformed Kenmotsu manifold under hyper generalized pseudo symmetric curvature condition equipped with Riemann solitions. Finally, e determine a necessary condition for shrinking, steady and expanding of the soliton.

2. Preliminaries

According to the definition of Blair ([11]), an *almost contact structure* (ϕ, ξ, η) on a $(2n+1)$-dimensional Riemannian manifold satisfies the following conditions

$$\phi^2 = -I + \eta \otimes \xi, \tag{5}$$

$$\eta(\xi) = 1, \tag{6}$$

$$\phi\xi = 0, \ \eta \circ \phi = 0, \ \text{rank } \phi = n - 1. \tag{7}$$

Moreover, if g is a Riemannian metric on M^{2n+1} satisfying

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{8}$$

$$g(X,\xi) = \eta(X), \tag{9}$$

$$g(\phi X, Y) = -g(X, \phi Y), \tag{10}$$

for any vector fields X, Y on M^{2n+1}, then the manifold M^{2n+1} ([11]) is said to admit an *almost contact metric structure* (ϕ, ξ, η, g).

Definition 2.1. [15] If in an almost contact metric structure (ϕ, ξ, η, g) on M^{2n+1}, the Riemann connection ∇ of g satisfies $(\nabla_X\phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$, for any vector fields X, Y on M^{2n+1}, then the structure is called Kenmotsu.

Proposition 2.2. [3,9,15] *If $(M^{2n+1}, \phi, \xi, \eta, g)$ is a Kenmotsu manifold, then for any vector fields X, Y, Z on M^{2n+1}, the following relations hold*

$$\nabla_X\xi = X - \eta(X)\xi, \tag{11}$$

$$(\nabla_X\eta)Y = g(X, Y) - \eta(X)\eta(Y), \tag{12}$$

$$S(X, \xi) = -2n\eta(X), \tag{13}$$

$$\eta(R(X, Y)Z) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X), \tag{14}$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \tag{15}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X. \tag{16}$$

Definition 2.3. [1] If a contact metric manifold M^{2n+1} with the almost contact metric structure (ϕ, ξ, η, g) is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$, where

$$\phi^d = \phi, \;\; \xi^d = \frac{1}{p}\xi, \;\; \eta^d = p\eta, \;\; g^d = qg + (p^2 - q)\eta \otimes \eta, \tag{17}$$

where p and q are constants such that $p \neq 0$ and $q > 0$, then the transformation is called a generalized D-conformal deformation.

Note that the generalized D-conformal deformation give rise to conformal deformation (for $p^2 = q$) and D-homothetic deformation (for $p = q =$ constant) ([16], [6], [10]). The generalized D-conformal deformation are studied by various authors in ([22], [23], [21], [24]).

The relation between the Levi-Civita connections ∇ of g and ∇^d of g^d is given by ([1])

$$\nabla^d_X Y = \nabla_X Y + \frac{(p^2 - q)}{p^2} g(\phi X, \phi Y)\xi, \tag{18}$$

for any vector fields X, Y on M^{2n+1}.

In view of (17), (18) and definition of Riemannian curvature tensor, Ricci tensor, scalar curvature, we get the following:

Proposition 2.4. [1] *If a Kenmotsu structure (ϕ, ξ, η, g) on M^{2n+1} is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$ under a generalized D-conformal deformation, then R, R^d, S, S^d, r and r^d are related by*

$$R^d(X, Y)Z = R(X, Y)Z + \frac{(p^2 - q)}{p^2}[g(\phi Y, \phi Z)X - g(\phi X, \phi Z)Y], \tag{19}$$

$$S^d(X, Y) = S(X, Y) + \frac{2n(p^2 - q)}{p^2} g(\phi X, \phi Y), \tag{20}$$

$$r^d = \frac{r}{q} + \frac{2n(2n+1)(p^2-q)}{p^2}, \tag{21}$$

for any vector fields X, Y, Z on M^{2n+1}.

Now we shall bring out some properties of a generalized D-conformally deformed structure (ϕ^d, ξ^d, η^d, g^d) of a Kenmotsu manifold M^{2n+1} as follows:

PROPOSITION 2.5. *Under a generalized D-conformal deformation of a Kenmotsu structure (ϕ, ξ, η, g) on M^{2n+1} is transformed into (ϕ^d, ξ^d, η^d, g^d), then for any vector fields X, Y, Z on M^{2n+1}, we have*

$$\phi^d = -I + \eta^d \otimes \xi^d, \tag{22}$$

$$\eta^d(\xi^d) = 1, \tag{23}$$

$$\phi^d \xi^d = 0, \ \eta^d \circ \phi^d = 0, \tag{24}$$

$$g^d(\phi^d X, \phi^d Y) = g^d(X,Y) - \eta^d(X)\eta^d(Y), \tag{25}$$

$$g^d(X, \xi^d) = \eta^d(X), \tag{26}$$

$$\nabla^d_X \xi^d = \frac{1}{p}[X - \eta^d(X)\xi^d], \tag{27}$$

$$(\nabla^d_X \eta^d)Y = \frac{1}{p}[g^d(X,Y) - \eta^d(X)\eta^d(Y)], \tag{28}$$

$$S^d(X, \xi^d) = -\frac{2n}{p^2}\eta^d(X), \tag{29}$$

$$\eta^d(R^d(X,Y)Z) = \frac{1}{p^2}[g^d(X,Z)\eta^d(Y) - g^d(Y,Z)\eta^d(X)], \tag{30}$$

$$R^d(\xi^d, X)Y = \frac{1}{p^2}[\eta^d(Y)X - g^d(X,Y)\xi^d], \tag{31}$$

$$R^d(X,Y)\xi^d = \frac{1}{p^2}[\eta^d(X)Y - \eta^d(Y)X]. \tag{32}$$

Now using (28) and (29), we obtain

$$(\nabla^d_X S^d)(Y, \xi^d) = -\frac{1}{p}[\frac{2n}{p^2}g^d(X,Y) + S^d(X,Y)], \tag{33}$$

for any vector fields X ,Y and Z on M^{2n+1}.

3. Main results

In the beginning, we shall define a hyper generalized pseudo symmetric space on a generalized D-conformally deformed structure (ϕ^d, ξ^d, η^d, g^d) of a Kenmotsu manifold M^{2n+1}.

3.1. Hyper generalized pseudo symmetric deformed Kenmotsu manifold.

Definition 3.1. A generalized D-conformally deformed structure (ϕ^d, ξ^d, η^d, g^d) of a Kenmotsu manifold M^{2n+1} is said to be hyper generalized pseudo symmetric if it satisfies the condition

$$\begin{aligned}
& (\nabla^d_X \bar{R}^d)(Y,U,V,W) \\
= \ & 2\alpha^d(X)\bar{R}^d(Y,U,V,\ W) + \alpha^d(Y)\bar{R}^d(X,U,V,\ W) \\
& +\alpha^d(U)\bar{R}^d(Y,X,V,\ W) + \alpha^d(V)\bar{R}^d(Y,U,X,\ W) \\
& +\alpha^d(W)\bar{R}^d(Y,U,V,\ X) + 2\beta^d(X)(g^d \wedge S^d)(Y,U,V,W) \\
& +\beta^d(Y)(g^d \wedge S^d)(X,U,V,W) + \beta^d(U)\ (g^d \wedge S^d)(Y,X,V,W) \\
& +\beta^d(V)\ (g^d \wedge S^d)(Y,U,X,W) + \beta^d(W)\ (g^d \wedge S^d)(Y,U,V,X). \qquad (34)
\end{aligned}$$

where

$$\begin{aligned}
(g^d \wedge S^d)(Y,U,V,W) \ = \ & g^d(Y,W)S^d(U,V) + g^d(U,V)S^d(Y,W) \\
& -g^d(Y,V)S^d(U,W) - g^d(U,W)S^d(Y,V), \qquad (35)
\end{aligned}$$

and A^d_i are non-zero 1-forms defined by $A^d_i(X) = g^d(X,\sigma_i)$, for $i = 1,2$.

In this section, we consider a Kenmotsu manifold (M^{2n+1}, g) $n \geq 1$ which is hyper generalized pseudo symmetric. Now, making use of (35) in (34), we find

$$\begin{aligned}
& (\nabla_X \bar{R}^d)(Y,U,V,\ W) \\
= \ & 2\alpha^d(X)\bar{R}^d(Y,U,V,\ W) + \alpha^d(Y)\bar{R}^d(X,U,V,\ W) \\
& +\alpha^d(U)\bar{R}^d(Y,X,V,\ W) + \alpha^d(V)\bar{R}^d(Y,U,X,\ W) \\
& +\alpha^d(W)\bar{R}^d(Y,U,V,\ X) + 2\beta^d(X)[g^d(Y,W)S^d(U,V) \\
& +g^d(U,V)S^d(Y,W) - g^d(Y,V)S^d(U,W) \\
& -g^d(U,W)S^d(Y,V)] + \beta^d(Y)[g^d(X,W)S^d(U,V) \\
& +g^d(U,V)S^d(X,W) - g^d(X,V)S^d(U,W) \\
& -g^d(U,W)S^d(X,V)] + \beta^d(U)\ [g^d(Y,W)S^d(X,V) \\
& +g^d(X,V)S^d(Y,W) - g^d(Y,V)S^d(X,W) \\
& -g^d(X,W)S^d(Y,V)] + \beta^d(V)\ [g^d(Y,W)S^d(U,X) \\
& +g^d(U,X)S^d(Y,W) - g^d(Y,X)S^d(U,W) - g^d(U,W)S^d(Y,X)] \\
& +\beta^d(W)[g^d(Y,X)S^d(U,V) + g^d(U,V)S^d(Y,X) \\
& -g^d(Y,V)S^d(U,X) - g^d(U,X)S^d(Y,V)]. \qquad (36)
\end{aligned}$$

Now, contracting over Y and W in (36), we get

$$\begin{aligned} & (\nabla^d_X S^d)(U,V) \\ = \ & 2\alpha^d(X)S^d(U,V) + \alpha^d(U)S^d(X,V) \\ & +\alpha^d(R^d(X,U)V) + 2\beta^d(Q^d X)g^d(U,V) \\ & +\alpha^d(R^d(X,V)U) + \alpha^d(V)S^d(X,U) \\ & +2\beta^d(X)[2nS^d(U,V) + r^d g^d(U,V)] \\ & +\beta^d(U)\ [(2n-2)S^d(X,V) + r^d g^d(X,V)] \\ & +\beta^d(V)\ [(2n-2)S^d(X,U) + r^d g^d(X,U)] \\ & -\beta^d(Q^d U)g^d(X,V) - \beta^d(Q^d V)g^d(X,U). \end{aligned} \tag{37}$$

Now setting $V = \xi^d$ and using (29), (31), (32) in the foregoing equation, we obtain

$$\begin{aligned} & (\nabla^d_X S^d)(U,\xi^d) \\ = \ & -\frac{2n}{p^2}[2\alpha^d(X)\eta^d(U) + \alpha^d(U)\eta^d(X)] + \alpha^d(\xi^d)S^d(X,U) \\ & +\frac{1}{p^2}[g^d(X,U)\alpha^d(\xi^d) - 2\eta^d(U)\alpha^d(X) + \eta^d(X)\alpha^d(U)] \\ & +2\beta^d(X)(r^d - \frac{4n^2}{p^2})\eta^d(U) + \beta^d(U)(r^d - \frac{4n(n-1)}{p^2})\eta^d(X) \\ & +\beta^d(\xi^d)\ [2(n-1)S^d(U,X) + r^d g^d(U,X)] \\ & +2\beta^d(Q^d X)\eta^d(U) - \beta^d(Q^d U)\eta^d(X) + \frac{2n}{p^2}\beta^d(\xi^d)g^d(U,X) \end{aligned} \tag{38}$$

which yields by using (33)

$$\begin{aligned} & -\frac{1}{p}[\frac{2n}{p^2}g^d\,(X,U) + S^d\,(X,U)] \\ = \ & [-\frac{2(2n+1)}{p^2}\alpha^d(X) + 2\beta^d(X)(r^d - \frac{4n^2}{p^2}) + 2\beta^d(Q^d X)]\eta^d(U) \\ & +[-\frac{(2n-1)}{p^2}\alpha^d(U) + (r^d - \frac{4n(n-1)}{p^2})\beta^d(U) - \beta^d(Q^d U)]\eta^d(X) \\ & +\frac{1}{p^2}\alpha^d(\xi^d)g^d(X,U) + \alpha^d(\xi^d)S^d(X,U) + \frac{2n}{p^2}\beta^d(\xi^d)g^d(U,X) \\ & +\beta^d(\xi^d)\ [2(n-1)S^d(U,X) + r^d g^d(U,X)]. \end{aligned} \tag{39}$$

Putting $X = U = \xi^d$ in succession, we obtain from (39) that

$$[r^d - \frac{2n(2n-1)}{p^2}]\beta^d(\xi^d) = \frac{2n}{p^2}\alpha^d(\xi^d). \tag{40}$$

$$\begin{aligned} & -\frac{2(2n+1)}{p^2}\alpha^d(X) + 2\beta^d(X)(r^d - \frac{4n^2}{p^2}) + 2\beta^d(Q^d X) \\ = \ & [\frac{2(2n-1)}{p^2}\alpha^d(\xi^d) - 2\beta^d(\xi^d)(r^d - \frac{4n(n-1)}{p^2}) + 2\beta^d(Q^d\xi^d)]\eta^d(X). \end{aligned} \tag{41}$$

and

$$-\frac{(2n-1)}{p^2}\alpha^d(U)+(r^d-\frac{4n(n-1)}{p^2})\beta^d(U)-\beta^d(Q^dU)$$
$$= [\frac{(6n+1)}{p^2}\alpha^d(\xi^d)-(3r^d-\frac{12n^2-2n}{p^2})\beta^d(\xi^d)]\eta^d(U). \tag{42}$$

By virtue of (40), (41) and (42), the equation (39) yields

$$S^d(X,U)$$
$$= -\left(\frac{\frac{1}{p^2}\alpha^d(\xi^d)+(r^d+\frac{2n}{p^2})\beta^d(\xi^d)+\frac{2n}{p^3}}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)g^d(U,X)$$
$$-\left(\frac{\frac{(10n-1)}{p^2}\alpha^d(\xi^d)-(5r^d-\frac{20n^2-14n}{p^2})\beta^d(\xi^d)}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)\eta^d(U)\eta^d(X). \tag{43}$$

and (40) gives

$$r^d=\frac{2n}{p^2}[\frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)}+(2n-1)]. \tag{44}$$

Next using (44) in (43) we have

$$S^d(X,U)$$
$$= -\left(\frac{\frac{(2n+1)}{p^2}\alpha^d(\xi^d)+\frac{4n^2}{p^2}\beta^d(\xi^d)+\frac{2n}{p^3}}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)g^d(U,X)$$
$$+\left(\frac{\frac{1}{p^2}\alpha^d(\xi^d)+\frac{4n}{p^2}\beta^d(\xi^d)}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)\eta^d(U)\eta^d(X). \tag{45}$$

This motivate us to state:

THEOREM 3.2. *Let $(\phi^d, \xi^d, \eta^d, g^d)$ be a generalized D-conformally deformed hyper generalized pseudo symmetric Kenmotsu manifold M^{2n+1}. Then such a space is conformally flat provided $\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)\neq 0$.*

THEOREM 3.3. *The scalar curvature of a hyper generalized pseudo symmetric generalized D-conformally deformed Kenmotsu manifold is constant.*

COROLLARY 3.4. *Let $(\phi^d, \xi^d, \eta^d, g^d)$ be a generalized D-conformally deformed pseudo symmetric Kenmotsu manifold M^{2n+1}. Then such a space is conformally flat provided $p\alpha^d(\xi^d)\neq -1$.*

4. Generalized D-conformally deformed hyper generalized pseudo symmetric Kenmotsu manifold and the case of Riemann soliton

In this section, we consider a generalized D-conformally deformed Kenmotsu manifold $(\phi^d, \xi^d, \eta^d, g^d)$ admitting a Riemann soliton. Then with the aid of (3) and (4), we obtain

$$\begin{aligned}
& 2R^d(Y,U,V,Z) + g^d(Y,Z)(\pounds_{\xi^d} g^d)(U,V) \\
& + g^d(U,V)(\pounds_{\xi^d} g^d)(Y,Z) \\
& - g^d(Y,V)(\pounds_{\xi^d} g^d)(U,Z) - g^d(U,Z)(\pounds_{\xi^d} g^d)(Y,V) \\
= \; & 2\kappa \left[g^d(Y,Z) g^d(U,V) - g^d(Y,V) g^d(U,Z) \right]. \qquad (46)
\end{aligned}$$

Now by contraction over Y and Z we get

$$\frac{1}{2}(\pounds_{\xi^d} g^d)(U,V) + \frac{1}{2n-1} S^d(U,V) = \frac{2n\kappa - div(\xi^d)}{2n-1} g^d(U,V). \qquad (47)$$

and then using (47) in (46), we get

$$r^d = 2n[(2n+1)\kappa - \frac{4n}{p}]. \qquad (48)$$

Comparing (44) with (48) we have

$$(2n+1)p^2\kappa = 4np + (2n-1) + \frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)}. \qquad (49)$$

This leads to the following:

Theorem 4.1. *Assume that a Kenmotsu structure (ϕ, ξ, η, g) on M^{2n+1} is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$ under a generalized D-conformally deformation which is a hyper generalized pseudo symmetric space. Then the Riemann soliton is expanding, steady and shrinking as* $\frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)} + 2n(2p+1) <=> 1$.

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