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REPRODUCING KERNEL HILBERT SPACE BASED ON SPECIAL INTEGRABLE SEMIMARTINGALES AND STOCHASTIC INTEGRATION

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Abstract. In this paper, we consider the integral of a stochastic process with respect of a sequence of square integrable semimartingales. By this integrals, we construct a reproducing kernel Hilbert space and study the correspondence between this space with the concepts of arbitrage and viability in mathematical finance.

1. Introduction

Motivated by a general problem in mathematical finance, based on the relation of *No Free Lunch* and *No Arbitrage*, we propose to study the reproducing kernel Hilbert space introduced by stochastic integration with respect to a sequence of special semimartingales. The case of stochastic integration with respect to a sequence of special semimartingales is a particular case of a theory of cylindrical stochastic integration, first studied in literature by Mikulevicius and Rozovskii [24, 25] and developed in the following by several research [6–14, 16, 20, 21, 23]. Infact, we can see a sequence of martingales as a cylindrical martingale with values in the set of all real-valued sequences.

A semimartingale P is a special semimartingale if it can be decomposed into $P = M + A$ where M is a local martingale and A a process with predictable finite variation, with $A_0 = 0$. Such a decomposition is then unique and say canonical. While the finite variation part A is easy to use and have suitable properties, our challenge is to play with the local martingale part M. A fundamental research in this area is done by Mémin [26] based on some results due to Dellacherie [15] and Choulli [4, 5]. The basic idea is that "by making use of an appropriate change in probability, it is possible to replace the integral with respect to a semimartingale with an integral with respect to the sum of a square integrable martingale and a predictable process with integrable variation". That is a powerful and interesting result, allow us to break the integrator such that the integral with respect to a sequence of semimartingales would be replaced with the sum of an integral with respect to a sequence of square integrable martingales

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and an integral with respect to a sequence of predictable processes with integrable variation.

The process is a motivation for making a reproducing kernel Hilbert space with respect to these integratins. A particular case is studied before by Kardaras [22]. This research is a generalization of his research.

2. Preliminaries and Results

We start this section with some definitions, notations and profitable theorems. Let $T \in \mathbb{R}^{*+}$ and $(\Omega, \mathscr{F}, \mathbb{P}, (\mathscr{F})_{0\leq t\leq T})$ be a probability filter space satisfying the usual conditions. For a càdlàg process Y, Y^* is a process such that

$$Y_t^* = \sup_{0\leq s\leq t} |Y_s|$$

We denote by $\mathcal{S}^2_{loc}(\mathbb{P})$ the set of semi-martingales Y such as Y^* is locally square integrable and write $\mathcal{S}^2_{loc}$ when the probability function $\mathbb{P}$ is significant respect to context. In this case, the space of the local martingales and locally square integrable are denote by $\mathscr{M}$ and $\mathscr{M}^2_{loc}$ respectively. Moreover, the set of integrable integrating martingales is denoted by $\mathscr{M}$.

Let $\mathbb{Q}$ be a probability law on the filter space $(\Omega, \mathscr{F}, \mathbb{P}, (\mathscr{F})_{0\leq t\leq T})$ and $M \in \mathscr{M}_{loc}(\mathbb{Q})$. We designate by $\mathscr{L}^2_{loc}(M, \mathbb{Q})$ the set of predictable processes ξ with value in $\mathbb{R}^d$ such that the growing process $\int_0^t \xi'_s d\langle M\rangle_s \xi_s$ is locally integrable where ξ'_s be the vector transpose from of ξ_s.
If $X = (X_t)_{O\leq t\leq T}$ is a semimartingale with value in $\mathbb{R}^d$, a predictable process dimensional is said to be X-integrable if the sequence of processes converges for the topology of semimartingales.

It is good to recall that a real valued stochastic process X is called a semimartingale if it can be decomposed as the sum of a local martingale and an adapted finite-variation process. Semimartingales are "good integrators", forming the largest class of processes with respect to which the Itô integral.

Definition 2.1. A real process Z is called martingale density for X if $Z, ZX \in \mathscr{M}_{loc}$ and $Z_0 = 1$ be $\mathbb{P}$ a.s. If in addition Z is strictly positive, Z is called strict martingale density for X. Moreover, let $X \in \mathcal{S}^2_{loc}$ has the canonical decomposition $X = X_0 + M + A$, B be a increasing predictable process such that $\langle M^i\rangle \ll B$, $i = 1, \ldots, d$ and σ the symmetric matrix defined by $\sigma^{ij} = \dfrac{d\langle M^i, M^j\rangle}{dB}$. We say that X satisfies the conditions of embedded structures if there exists $\lambda \in \mathscr{L}^2_{loc}(M)$ such that $dA = \sigma\lambda dB$. In this case we set $\hat{Z} = \mathscr{E}(-\lambda.M)$ and the process Z will be called "minimum martingale density for X.

Theorem 2.2. *[4] Suppose that X has a strict martingale density Z and that:*

$$\begin{cases} X \text{ is continuous} & (2.1a) \\ \text{or} \\ \begin{cases} X \in \mathcal{S}^2_{loc} \\ Z \in \mathscr{M}^2_{loc} \end{cases} & (2.1b) \end{cases}$$

Then the following assertions are equivalent:

(i) X satisfied the structural conditions.
(ii) There is a single local martingale $L \in \mathscr{M}_{loc}$ strongly orthogonal to each $M^i, i = 1, \ldots, d$, such that:

$$Z = \mathscr{E}(-\lambda.M + L)$$

Corollary 2.3. *[4] With the same assumptions as Theorem 2.2, we have:*
(i) $\alpha^i \in \mathscr{L}^2_{loc}(M)$ for $i = 1, \ldots, d$
(ii) a) If (2.1a) satisfies we have $Z = \mathscr{E}(-\lambda.M)\mathscr{E}(L)$
b) If (2.1b) holds we have $L \in \mathscr{M}^2_{loc}$.

Now, it is good to have a brief view of reproducing kernel Hilbert spaces. These spaces have wide applications, including complex analysis, harmonic analysis and mathematical finance. See [1,3,17–19,27,28].

A *reproducing kernel Hilbert space* (RKHS) is a Hilbert space $\mathscr{H}$ of functions, say f, on a fixed set X such that every linear functional (induced by $x \in X$),

$$E_x(f) := f(x), \quad f \in \mathscr{H}. \tag{2.2}$$

is continuous in the norm of $\mathscr{H}$.

Hence, by Riesz' representation theorem, there is a corresponding $h_x \in \mathscr{H}$ such that

$$E_x f = \langle f, h_x \rangle_{\mathscr{H}} \tag{2.3}$$

where $\langle \cdot, \cdot \rangle_{\mathscr{H}}$ denotes the inner product in $\mathscr{H}$. Setting

$$K(x,y) = \langle h_y, h_x \rangle_{\mathscr{H}}, \quad (x,y) \in X \times X$$

we get a *positive definite* kernel, i.e., $\forall n \in \mathbb{N}$, $\forall \{\alpha_i\}_1^n$, $\forall \{x_i\}_1^n$, $\alpha_i \in \mathbb{C}$, $x_i \in X$, we have

$$\sum_i \sum_j \alpha_i \overline{\alpha}_j K(x_i, x_j) \geq 0. \tag{2.4}$$

Conversely, if K is given positive definite, i.e., satisfying (2.4), then by [2], there is a RKHS such that (2.3) holds.

Given a positivie definite kernel K, we may take $\mathscr{H}(K)$ to be the completion of

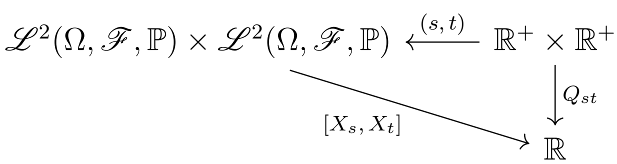
$$\psi = \sum_i \alpha_i K(\cdot, x_i) \tag{2.5}$$

in the norm

$$\|\psi\|^2_{\mathscr{H}(K)} = \sum_i \sum_j \alpha_i \overline{\alpha}_j K(x_i, x_j), \tag{2.6}$$

A well-known example of a RKHS is the space of random variables respect to covariance function as the kernel function.

As a specific case, let $X = (X_t; t \in \mathbb{R}^+)$ be a semimartingales. The covariance matrix Q with entrees $Q_{st} = [X_s, X_t]$ is defined such that Q_{st} holds in the following diagram:



It is well known that this matrix is positive definite in matrix sense and by the theorem, corresponding to any positive definite matrix, there exist a RKHS.

Kardaras in [22] extended the above definition to a collection of semimartingales as follows:

Definition 2.4. [22] A collection $Q \sim (Q_t^{ij}; (i,j) \in I \times I) \in \mathfrak{F}^{I\times I}$ of adapted, continuous processes of finite variation will be called an stochastic aggregate kernel on $I \times I$, if, for each fixed pair $(i,j) \in I \times I$, $Q_{ij} = Q_{ji}$ holds and for every $0 \le s \le t$ and every finite subspace J of I, we have

$$\sum_{(i,j)\in J\times J} z_i(Q_t^{ij} - Q_s^{ij})z_j \ge 0, \quad (z_i)_{i\in J} \in \mathbb{R}^J. \tag{2.7}$$

where $\mathfrak{Fin}(I)$ is any finite subset of I and $\mathfrak{F}$ is the set of all adapted and right-continuous scalar processes B of finite first variation on compact time intervals, with $B(0) = 0$.

Indeed, he introduces an extended concept of positiveness. Moreover, he is constructed matrix Q by a collection of continuous semimartingales, that is, he just studied the condition 2.1a. In more general case of structural conditions, the similar results can be gained and we study them under conditions 2.1b. That is because the continuity is a strong condition on a collection of semimartingales. Instead, we select the collection $X = (X^i; i \in I)$ with martingale density Z such that $X^i \in \mathcal{S}^2_{loc}$, $X^i_0 = 0$ and $Z^i \in \mathscr{M}^2_{loc}(P)$ for each $i \in I$ and forming the matrix Q of covariances. The main reason that we need the structural conditions is to define an inner product on the space, introduced by the function space of integral operator with respect to the matrix Q.

Let $X \sim (X^i;\, i \in I)$ be a family of $\mathcal{S}^2_{loc}$ with the Doob decomposition:

$$X^i = A^i + M^i, \quad i \in I,$$

and $Q \sim (Q^{ij})$ via $Q^{ij} := \langle M^i, M^j\rangle$, for all $(i,j) \in I\times I$. In this way, Q is a covariance function. Now we can considering the columns of matrix Q as a process. So, for any $\xi = (\xi^i)$ be a family of predictable process belongs $\mathscr{L}^2_{loc}(M)$ the following process are well-defined:

$$F = \int_o^{\cdot} \sum_{j\in I} \xi^j(t)dQ^{Ij}(t) \quad \text{where} \quad F^i = \int_o^{\cdot} \sum_{j\in I} \xi^j(t)dQ^{ij}(t) \quad i \in I \tag{2.8}$$

In this way, we have

$$dF := \sum_{j\in I} \xi^j dQ^{Ij} \quad \text{and} \quad \|dF\|^2_{dQ} := \sum_{i\in I} \xi^i dF^i = \sum_{(i,j)\in I\times I} \xi^i dQ^{ij}\xi^j, \tag{2.9}$$

Let $\mathscr{H}(dQ)$ be the space of all linear predictable combination of the columns of the matrix dQ. Clearly, $dF \in \mathscr{H}(dQ)$. We can define the following bilinear form on $\mathscr{H}(dQ)$ as:

$$\langle dF, dG\rangle_{\mathscr{H}(dQ)} = \sum_{(i,j)} \xi_i(t)dQ^{ij}(t)\zeta^j(t),$$

for any

$$dF = \sum_{(i,j)} \xi^i(t) dQ^{Ij} \quad and \quad dG = \sum_{(i,j)} \zeta^i(t) dQ^{Ij}(t). \tag{2.10}$$

In this case, we can construct the Hilbert space

$$\mathscr{K}(Q) := \left\{ F \in \mathfrak{F} \;\middle|\; dF \in \mathscr{H}(dQ),\ \int_0^{\cdot} \|dF^J(t)\|^2_{\mathscr{H}(dQ^J)} < \infty \right\}. \tag{2.11}$$

LEMMA 2.5. *$\mathscr{K}(Q)$ is a reproducing kernel Hilbert space.*

Proof. Matrix Q is positive definite in the sense of stochastic matrices (2.7). Moreover, for any $F \in \mathscr{K}(Q)$ we have

$$\langle Q^{Ij}, F\rangle_{\mathscr{K}(Q)} = \int_0^{\cdot} \langle dQ^{Ij}, dF\rangle_{\mathscr{K}(Q)} = F^i$$

which is the reproducing property. Therefore, Q is a reproducing kernel for $\mathscr{K}(Q)$. □

In the following, we focus on a specific forms of predictable stochastic process as integrand. Let $F \in \mathfrak{F}$. We define a process with respect to F as follows:

$$\alpha^F = (\alpha^{F,n}) := \lim_{n\to\infty} \left(dQ + \frac{1}{n} \sum_{i\in I} |dF^i| \; \mathbb{I}_{\mathbb{R}} \right)^{-1} dF \tag{2.12}$$

It is easy to see that α^F is a predictable process and we have

$$F = \int_o^{\cdot} \sum_{j\in I} \alpha^{Fj}(t) dQ^{Ij}(t) \tag{2.13}$$

With respect to α^F, we define a non decreasing, $[0,\infty]$-valued following process

$$\int_0^T \|dF\|^2_{\mathscr{H}(dQ)} := \lim_{n\to\infty} \int_0^T \langle \alpha^{F,n}(t), dF(t)\rangle_{\mathbb{R}^I}, \quad T \in \mathbb{R}^+$$

With the above notation we can define the following spaces:

$$\mathscr{H}(Q) := \left\{ F \in \mathfrak{F} \;\middle|\; dF \in \mathscr{H}(dQ),\ \sup_J \int_0^T \|dF^J(t)\|^2_{\mathscr{H}(dQ^J)} < \infty, \quad \forall\, T \in \mathbb{R}^+ \right\}.$$

where $\mathscr{H}(dQ^J)$ is any finite dimensional subspace of $\mathscr{H}(dQ)$ and dF^J is the restriction of dF to dQ^J. On $\mathscr{H}(Q)$ we can define the following bilinear form

$$\langle F, G\rangle_{\mathscr{H}(Q)} := \int_0^{\cdot} \langle dF, dG\rangle_{\mathscr{H}(dQ)}$$

Clearly, $\mathscr{H}(Q)$ is a subspace of $\mathscr{K}(Q)$. With the restricted kernel function, $\mathscr{H}(Q)$ is also a reproducing kernel Hilbert space. In $\mathscr{H}(Q)$, we need to define $\langle dF(t), dM(t)\rangle_{dQ(t)}$. For doing that, let $\mathcal{S}(M)$ be the space of all local semimartingales L that are stochastic integral respect to $M$of the form

$$L = \int_0^{\cdot} \sum_{i\in I} \alpha^i dM^i$$

which vanish in zero and such that

$$\int_0^T \sum_{i,j\in I} \alpha_t^i dQ_t^{ij} \alpha_t^j < \infty \quad for\ all \quad T \in \mathbb{R}^+ \tag{2.14}$$

We first provide the following lemma.

Lemma 2.6. *Every $F \in \mathscr{H}(Q)$ has unique representation of the form $F := (\langle L, M^i \rangle)$ if and only if $L \in \mathcal{S}(M)$ satisfying in* (2.14) .

Proof. Let $F \in \mathscr{H}(Q)$ has a representation of the form $F := (\langle L, M^i \rangle)$. By the (2.13) there exists predictable process α^F such that

$$\int_0^T \sum_{i,j \in I} \alpha_t^{F^i} dQ_t^{ij} \alpha_t^{F^j} = \int_0^T \|dF(t)\|^2_{\mathscr{H}(dQ^J)} < \infty \tag{2.15}$$

Set $L = \int_0^\cdot \sum_{i \in I} \alpha^{Fi} dM^i$. Conversely, let $L \in \mathcal{S}(M)$ satisfying in (2.14) and set $F := (\langle L, M^i \rangle)$. In this way $\int_0^\cdot \|dF\|^2_{\mathscr{H}(dQ)}$ equals (2.15). □

Similar to lemma 2.6, for every $F \in \mathscr{H}(Q)$ there exist a predictable process $\alpha^F = (\alpha^{Fi})$ as in (2.12) which satisfies in (2.14). Moreover, since $F \in \mathscr{H}(Q)$ by (2.9) the relation (2.15) holds. Define

$$M^F := \int_0^\cdot \sum_{i \in I} \alpha^{F^i} dM^i \tag{2.16}$$

In this way, $\langle M^F, M^i \rangle_{\mathscr{H}(Q)} = F^i$ for all $i \in I$ and

$$\langle M^F, M^F \rangle_{\mathscr{H}(Q)} = \int_0^\cdot \|dF(t)\|^2_{\mathscr{H}(dQ)} < \infty$$

Therefore, $M^F \in \mathcal{S}(M)$. By lemma 2.6 the above definition is well-defined and we have $F = \langle M^F, M \rangle$. Therefore, we can write

$$dM^F = \sum_{i \in I} \alpha^{F^i} dM^i = \langle dF, dM \rangle_{\mathscr{H}(dQ)}$$

and consequently

$$M^F = \int_0^\cdot \langle dF(t), dM(t) \rangle_{\mathscr{H}(dQ)} \tag{2.17}$$

Clearly M^F is a semimartingale.
In another hand, we have a nice result of the structural condition.

Lemma 2.7. *Let X be a collection of stochastic process satisfying the structural condition SC and $X^i = A^i + M^i$ be the Doob decomposition. Then $A = (A^i) \in \mathscr{H}(Q)$.*

Proof. By a consequence of the structural conditions, $dA^i \ll d\langle M^i, M^i \rangle$, that is there exist α_i such that $dA^i = \alpha_i d\langle M^i, M^i \rangle$. Then

$$\int_0^T \|dA(t)\|^2_{\mathcal{H}(dQ)} = \int_0^T \alpha_i^2(t) d\langle M^i, M^i \rangle < \infty, \qquad \forall T \in \mathbb{R}^+.$$

So $A \in \mathscr{H}(Q)$. □

Now, we can define the following space:

$$\mathcal{S}(X) = \left\{ \langle F(t), A(t) \rangle_{\mathscr{H}(Q)} + \langle F(t), M(t) \rangle_{\mathscr{H}(Q)} \;\middle|\; F \in \mathscr{H}(Q) \right\}.$$

Actually, the $\mathcal{S}(X)$ is a set of semimartingales. The main reason to select X with structural conditions is to make sure that $dA(t)$ makes sense and we can define $\mathcal{S}(X)$

safely.
For any two elements $Z_1, Z_2 \in \mathcal{S}(X)$ such that:

$$Z_1 = \langle F(t), A(t)\rangle_{\mathscr{H}(Q)} + \langle F(t), M(t)\rangle_{\mathscr{H}(Q)}$$
$$Z_2 = \langle G(t), A(t)\rangle_{\mathscr{H}(Q)} + \langle G(t), M(t)\rangle_{\mathscr{H}(Q)}$$

One can define the following bilinear form:

$$\langle Z_1, Z_2\rangle_{\mathcal{S}(X)} = \langle F, G\rangle_{\mathscr{H}(Q)}$$

where F and G are as (2.10). We can show that

Corollary 2.8. *Any $Z \in \mathcal{S}(X)$ is a square summable semimartingale.*

Proof. By definition of Z, it has a decomposition of a finite variation part and and a square summable part. Therefore $Z \in \mathcal{S}^2_{loc}$ □

Lemma 2.9. *For any $Z \in \mathcal{S}(X)$, it holds that $(\langle Z, M^i\rangle;\, i \in I) \in \mathscr{H}(Q)$.*

Proof. Let $Z \in \mathcal{S}(X)$. We have seen that Z is a semimartingale, so has a decomposition $Z = A^o + M^o$ which A^o is finite variation and M^o is a local martingale. Therefor by definition of matrix Q, there exist a collection of predictable process ξ^o such that $d\langle M^o, M^i\rangle = \sum_{j\in I} \xi^{oi} dQ^{Ij} \in \mathscr{H}(dQ)$ and we have

$$\|d\langle M^o, M^i\rangle\|^2_{dQ} := \sum_{i\in I} \xi^{oi} d\langle M^{oi}, M^i\rangle = \sum_{(i,j)\in I\times I} \xi^{oi} dQ^{ij} \xi^{oj} < \infty.$$

Then

$$\sup_J \int_0^T \|d\langle M^o, M^i\rangle^J(t)\|^2_{\mathscr{H}(dQ^J)} < \infty,$$

for any finite J subset of I. The last inequality holds by the specific form of Z. □

This leads us to have a Hilbert space isomorphism.

Theorem 2.10. *Two spaces $\mathcal{S}(X)$ and $\mathscr{H}(Q)$ are isomorphic. So the space $\mathcal{S}(X)$ of extended stochastic integrals admits the representation , and is topologically isomorphic to the stochastic aggregate reproducing kernel Hilbert spaces $\mathscr{H}(Q)$.*

Proof. We claim that the mapping $\psi : A \in \mathscr{H}(Q) \to \mathcal{S}(X)$ is a an isomorphism

$$\psi(F)(\cdot) = \langle F(t), A(t)\rangle_{\mathscr{H}(Q)} + \langle F(t), M(t)\rangle_{\mathscr{H}(Q)}$$

Since $A \in \mathscr{H}(Q)$, then there exists a predictable process $\lambda^J = (\lambda^J_j; j \in J)$ such that $A = \lambda dQ^{Ij}$ and by definition, dA and therefore $\langle dF(t), dA(t)\rangle_{dQ(t)}$ make sense. So ψ would be well defined. In this case, it is easy to see that the mapping ψ is 1-1 and onto, naturally. Moreover, it preserves the spaces operations, because the integral is a linear function. The closeness of both spaces guaranties that a bijection homomorphism is an isomorphism. This isomorphism preserves the inner product and consequently the norms. So two spaces are topologically isomorphism. □

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