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NEW BANACH SPACES DEFINED BY THE DOMAIN OF RIESZ-FIBONACCI MATRIX

Pinar Zengin Alp and Emrah Evren Kara

Abstract. The main object of this study is to introduce the spaces $c_0(\widehat{F}^q)$ and $c(\widehat{F}^q)$ derived by the matrix $\widehat{F}^q$ which is the multiplication of Riesz matrix and Fibonacci matrix. Moreover, we find the α-, β-, γ- duals of these spaces and give the characterization of matrix classes $(\Lambda(\widehat{F}^q), \Omega)$ and $(\Omega, \Lambda(\widehat{F}^q))$ for $\Lambda \in \{c_0, c\}$ and $\Omega \in \{\ell_1, c_0, c, \ell_\infty\}$.

1. Introduction

Let ω be the space of all real or complex valued sequences. We denote the family of all finite subsets of $\mathbb{N} = \{0, 1, 2, ...\}$ by $\mathcal{F}$ and the set of real numbers by $\mathbb{R}$.

Any vector subspace of ω is called a sequence space. Further c_0, c and ℓ_∞ are the spaces of all convergent to zero, convergent and bounded sequences, respectively and the norm on these spaces is given by $\|u\|_\infty = \sup_j |u_j|$. Moreover, cs_0, cs and bs are the spaces of sequences which constituted convergent to zero, convergent and bounded series, respectively.

Let $A = (a_{ij})$ be an infinite matrix with $a_{ij} \in \mathbb{R}$ for all $i, j \in \mathbb{N}$. A defines a matrix mapping from a sequence space X into another sequence space Y if for every sequence $u = (u_j) \in X$, the sequence $Au = (A_i(u))$, the $A-$transform of u, is in Y, where

$$A_i(u) = \sum_j a_{ij} u_j$$

is a convergent series for each $i \in \mathbb{N}$. Also, (X, Y) is the class of all infinite matrices defined from X into Y.

The sequence space

$$X_A = \{u = (u_j) \in \omega : Au \in X\}$$

is called as the matrix domain of an infinite matrix A in the sequence space X.

In the literature [e.g. [3, 5, 13, 14, 16, 19, 22, 23, 27, 31–33, 36]] several authors have studied to construct a new sequence space by means of the matrix domain of a particular limitation method.

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In mathematics, the sequence (f_i) of Fibonacci numbers is defined by the following formula

$$f_0 = f_1 = 1 \text{ and } f_i = f_{i-1} + f_{i-2}, \quad i \geq 2.$$

There are many fascinating features and applications of the sequence of Fibonacci numbers. For instance, the golden ratio which is famous in sciences and arts is obtained by the ratio sequence of Fibonacci numbers.

Recently, Kara [18] defined the Fibonacci matrix $\widehat{F} = (\widehat{f}_{ij})$ by using the Fibonacci numbers as

$$\widehat{f}_{ij} = \begin{cases} -\frac{f_{i+1}}{f_i} & , \ j = i-1 \\ \frac{f_i}{f_{i+1}} & , \ j = i \\ 0 & , \ 0 \leq j < i-1 \text{ or } j > i \end{cases}$$

for all $i, j \in \mathbb{N}$, where f_i is the i-th Fibonacci number $(i \in \mathbb{N})$.

Following Kara et al. [6] the sequence spaces $c_0(\widehat{F})$ and $c(\widehat{F})$ are introduced as follows:

$$c_0(\widehat{F}) = \left\{ u = (u_i) \in \omega : \lim_{i \to \infty} \left(\frac{f_i}{f_{i+1}} u_i - \frac{f_{i+1}}{f_i} u_{i-1} \right) = 0 \right\}$$

and

$$c(\widehat{F}) = \left\{ u = (u_i) \in \omega : \exists l \in \mathbb{C} \ni \lim_{i \to \infty} \left(\frac{f_i}{f_{i+1}} u_i - \frac{f_{i+1}}{f_i} u_{i-1} \right) = l \right\}.$$

Kara and İlkhan [20] have given a new matrix $T = (t_{ij})$ as

$$t_{ij} = \begin{cases} t_i & , \ j = i \\ -\frac{1}{t_i} & , \ j = i-1 \\ 0 & , \ 0 \leq j < i-1 \text{ or } j > i, \end{cases}$$

where $t_i > 0$ for all $i \in \mathbb{N}$ and $t = (t_i) \in c \backslash c_0$. If we choose $t_i = \frac{f_i}{f_{i+1}}$ $(i \in \mathbb{N})$, we have the matrix $\widehat{F}$. By using the matrix T, Kara and İlkhan [21] have defined the spaces $c_0(T)$ and $c(T)$.

Subsequently, various Banach spaces have been established by defining different matrices with the help of Fibonacci numbers. Several authors have examined some topological and geometric properties of these new spaces [7–9, 11, 15, 25, 26].

The triangle limitation matrix $R = (r_{ij})$ of Riesz mean is defined by

$$r_{ij} = \begin{cases} \frac{q_j}{Q_i} & , \ 0 \leq j \leq i \\ 0 & , \ j > i, \end{cases}$$

where (q_j) is a sequence of positive numbers and $Q_i = \sum_{j=0}^{i} q_j$ for all $i \in \mathbb{N}$.By using these matrices, Altay and Başar [2] introduced the Riesz sequence spaces of non-absolute type. More work on Riesz mean and other related topics can be found in the literature [10, 12, 24, 34].

The theory of sequence spaces plays a fundamental role in summability theory, which is a wide field of functional analysis and has many applications. For example, the solvability of infinite systems of differential equations in classical Banach sequence spaces has been investigated by several authors [1, 17, 28, 30] in recent times by using the measure of non-compactness. In this paper we introduce and work on some new Banach spaces by using Riesz and Fibonacci matrices.

2. The difference sequence spaces $c_0(\widehat{F}^q)$ and $c(\widehat{F}^q)$

In this section, we introduce the spaces $c_0(\widehat{F}^q)$ and $c(\widehat{F}^q)$ derived by the matrix $\widehat{F}^q$ which is the multiplication of Riesz matrix and Fibonacci matrix. By using these infinite matrices, we define a new matrix $\widehat{F}^q = (\widehat{f}^q_{ij})$ as

$$\widehat{f}^q_{ij} = \begin{cases} \frac{q_j}{Q_i}\frac{f_j}{f_{j+1}} - \frac{q_{j+1}}{Q_i}\frac{f_{j+2}}{f_{j+1}} & , \quad j < i \\ \frac{q_i}{Q_i}\frac{f_i}{f_{i+1}} & , \quad j = i \\ 0 & , \quad j > i. \end{cases}$$

We compute the inverse $(\widehat{F}^q)^{-1} = ((\widehat{f}^q)^{-1}_{ij})$ of the matrix $\widehat{F}^q$ as

$$(\widehat{f}^q)^{-1}_{ij} = \begin{cases} \frac{f_{i+1}^2}{f_j f_{j+1}}\frac{Q_j}{q_j} - \frac{f_{i+1}^2}{f_{j+1}f_{j+2}}\frac{Q_j}{q_{j+1}} & , \quad j < i \\ \frac{Q_i}{q_i}\frac{f_{i+1}}{f_i} & , \quad j = i \\ 0 & , \quad j > i. \end{cases}$$

Now, we introduce the difference sequence spaces $c_0(\widehat{F}^q)$ and $c(\widehat{F}^q)$ by

$$c_0(\widehat{F}^q) = \left\{ u = (u_i) \in \omega : \frac{1}{Q_i}\sum_{j=0}^{i} q_j \left(\frac{f_j}{f_{j+1}}u_j - \frac{f_{j+1}}{f_j}u_{j-1} \right) \longrightarrow 0 \text{ as } i \to \infty \right\}$$

and

$$c(\widehat{F}^q) = \left\{ u = (u_i) \in \omega : \frac{1}{Q_i}\sum_{j=0}^{i} q_j \left(\frac{f_j}{f_{j+1}}u_j - \frac{f_{j+1}}{f_j}u_{j-1} \right) \longrightarrow L \text{ as } i \to \infty \text{ for some } L \in \mathbb{R} \right\}.$$

The sequence $v = (v_i)$, which will be frequently used, is defined as the $\widehat{F}^q$-transform of the sequence $u = (u_i)$; i.e.

$$v = (v_i) = \frac{1}{Q_i}\sum_{j=0}^{i} q_j \left(\frac{f_j}{f_{j+1}}u_j - \frac{f_{j+1}}{f_j}u_{j-1} \right), \qquad (i \in \mathbb{N}).$$

As the notation of matrix domain, these spaces may be redefined as

$$c_0(\widehat{F}^q) = (c_0)_{\widehat{F}^q} \quad \text{ and } \quad c(\widehat{F}^q) = c_{\widehat{F}^q}.$$

Now we may begin with the following theorem which is essential in the text.

THEOREM 2.1. *The spaces $c_0(\widehat{F}^q)$ and $c(\widehat{F}^q)$ are Banach spaces with the norm given by* $\|u\|_{\widehat{F}^q} = \sup_i \left| \frac{1}{Q_i}\sum_{j=0}^{i} q_j \left(\frac{f_j}{f_{j+1}}u_j - \frac{f_{j+1}}{f_j}u_{j-1} \right) \right|$.

Proof. We prove the theorem for only the space $c_0(\widehat{F}^q)$. Firstly, let $\|u\|_{\widehat{F}^q} = 0$. Then, $\|\widehat{F}^q u\|_\infty = 0$ and since $\|.\|_\infty$ is a norm we have $\widehat{F}^q u = \theta$. Since $\widehat{F}^q$ is invertible, we obtain $u = \theta$.

Now, let $\alpha \in \mathbb{C}$ and $u \in c_0(\widehat{F}^q)$. Then,

$$\begin{aligned} \|\alpha u\|_{\widehat{F}^q} &= \|\widehat{F}^q(\alpha u)\|_\infty = \|\alpha \widehat{F}^q u\|_\infty \\ &= |\alpha| \|\widehat{F}^q u\|_\infty = |\alpha| \|u\|_{\widehat{F}^q}. \end{aligned}$$

Take any $u, \tilde{u} \in c_0(\widehat{F}^q)$. Then,

$$\begin{aligned} \|u+\tilde{u}\|_{\widehat{F}^q} &= \|\widehat{F}^q(u+\tilde{u})\|_\infty = \|\widehat{F}^q u + \widehat{F}^q \tilde{u}\|_\infty \\ &\le \|\widehat{F}^q u\|_\infty + \|\widehat{F}^q \tilde{u}\|_\infty = \|u\|_{\widehat{F}^q} + \|\tilde{u}\|_{\widehat{F}^q}. \end{aligned}$$

Hence, $(c_0(\widehat{F}^q), \|.\|_{\widehat{F}^q})$ is a normed space. It remains to prove the completeness of the space $c_0(\widehat{F}^q)$ Now, let (u_i) be a Cauchy sequece in $c_0(\widehat{F}^q)$. Then, (v_i) is a sequence in c_0. Clearly,

$$\begin{aligned} \|u_i - u_l\|_{\widehat{F}^q} &= \|\widehat{F}^q(u_i - u_l)\|_\infty \\ &= \|\widehat{F}^q u_i - \widehat{F}^q u_l\|_\infty = \|v_i - v_l\|_\infty, \end{aligned}$$

that is, (v_i) is a Cauchy sequence in c_0. Since $(c_0, \|.\|_\infty)$ is a Banach space, there exists $\widetilde{v} \in c_0$ such that $\lim_{i\to\infty} v_i = \widetilde{v}$ in c_0. Now, let $\widetilde{u} = (\widehat{F}^q)^{-1}\widetilde{v}$; then we have

$$\begin{aligned} \lim_{i\to\infty} \|u_i - \widetilde{u}\|_{\widehat{F}^q} &= \lim_{i\to\infty} \|\widehat{F}^q(u_i - \widetilde{u})\|_\infty \\ &= \lim_{i\to\infty} \|\widehat{F}^q u_i - \widehat{F}^q \widetilde{u}\|_\infty = \lim_{i\to\infty} \|v_i - \widetilde{v}\|_\infty = 0. \end{aligned}$$

This means that $\lim_{i\to\infty} u_i = \widetilde{u}$ in $c_0(\widehat{F}^q)$, where $\widetilde{u} \in c_0(\widehat{F}^q)$. Hence the proof is completed. □

THEOREM 2.2. *The spaces $c_0(\widehat{F}^q)$ and $c(\widehat{F}^q)$ are linearly isomorphic to c_0 and c, respectively.*

Proof. To prove this we should show the existence of a linear bijection between the spaces $c_0(\widehat{F}^q)$ and c_0, which preserves the norms. Consider the mapping $\Psi : c_0(\widehat{F}^q) \to c_0$ defined by $\Psi(u) = \widehat{F}^q u$ for all $u \in c_0(\widehat{F}^q)$. Clearly, Ψ is linear and injective. Let $u = (u_j)$ be the sequence with terms

$$u_j = \sum_{r=0}^{j} \left(\frac{f_{j+1}^2}{f_r f_{r+1}} \frac{Q_r}{q_r} - \frac{f_{j+1}^2}{f_{r+1} f_{r+2}} \frac{Q_r}{q_{r+1}} \right) v_r + \frac{f_{j+1}}{f_{j+2}} \frac{Q_j}{q_{j+1}} v_j$$

for all $j \in \mathbb{N}$, where $v = (v_j) \in c_0$. It follows that

$$
\begin{aligned}
&\frac{1}{Q_i}\sum_{j=0}^{i} q_j \left(\frac{f_j}{f_{j+1}}u_j - \frac{f_{j+1}}{f_j}u_{j-1}\right) \\
&= \frac{1}{Q_i}\sum_{j=0}^{i} q_j \frac{f_j}{f_{j+1}} \left(\sum_{r=0}^{j}\left(\frac{f_{j+1}^2}{f_r f_{r+1}}\frac{Q_r}{q_r} - \frac{f_{j+1}^2}{f_{r+1}f_{r+2}}\frac{Q_r}{q_{r+1}}\right)v_r + \frac{f_{j+1}}{f_{j+2}}\frac{Q_j}{q_{j+1}}v_j\right) \\
&- q_j \frac{f_{j+1}}{f_j}\left(\sum_{r=0}^{j-1}\left(\frac{f_j^2}{f_r f_{r+1}}\frac{Q_r}{q_r} - \frac{f_j^2}{f_{r+1}f_{r+2}}\frac{Q_r}{q_{r+1}}\right)v_r + \frac{f_j}{f_{j+1}}\frac{Q_{j-1}}{q_j}v_{j-1}\right) \\
&= \frac{1}{Q_i}\sum_{j=0}^{i} q_j \left(\sum_{r=0}^{j}\left(\frac{f_j f_{j+1}}{f_r f_{r+1}}\frac{Q_r}{q_r} - \frac{f_j f_{j+1}}{f_{r+1}f_{r+2}}\frac{Q_r}{q_{r+1}}\right)v_r + \frac{f_j}{f_{j+2}}\frac{Q_j}{q_{j+1}}v_j\right) \\
&- q_j \left(\sum_{r=0}^{j-1}\left(\frac{f_j f_{j+1}}{f_r f_{r+1}}\frac{Q_r}{q_r} - \frac{f_j f_{j+1}}{f_{r+1}f_{r+2}}\frac{Q_r}{q_{r+1}}\right)v_r + \frac{Q_{j-1}}{q_j}v_{j-1}\right) \\
&= \frac{1}{Q_i}\sum_{j=0}^{i} q_j \left(\left(\frac{Q_j}{q_j} - \frac{f_j}{f_{j+2}}\frac{Q_j}{q_{j+1}}\right)v_j + \frac{f_j}{f_{j+2}}\frac{Q_j}{q_{j+1}}v_j - \frac{Q_{j-1}}{q_j}v_{j-1}\right) \\
&= \frac{1}{Q_i}\sum_{j=0}^{i} q_j \left(\frac{Q_j}{q_j}v_j - \frac{Q_{j-1}}{q_j}v_{j-1}\right) \\
&= \frac{1}{Q_i}\sum_{j=0}^{i} (Q_j v_j - Q_{j-1}v_{j-1}) = v_i \longrightarrow 0 \quad (i \to \infty)
\end{aligned}
$$

which means that $u = (u_i) \in c_0(\widehat{F}^q)$. From this equality, we also deduce that $\|u\|_{\widehat{F}^q} = \|\Psi u\|_\infty$. It is clear here that if the spaces $c(\widehat{F}^q)$ and c are respectively replaced by the spaces $c_0(\widehat{F}^q)$ and c_0, then we obtain the fact that $c_0(\widehat{F}^q) \cong c$. This completes the proof. $\square$

Theorem 2.3. *The inclusion $c_0(\widehat{F}^q) \subset c(\widehat{F}^q)$ is strict.*

Proof. Let u be in $c_0(\widehat{F}^q)$. Hence, $\widehat{F}^q u \in c_0$. Since the inclusion $c_0 \subset c$ holds, we have $\widehat{F}^q u \in c$. Thus, $u \in c(\widehat{F}^q)$. This means that $c_0(\widehat{F}^q) \subset c(\widehat{F}^q)$. Further, let us consider the sequence $u = (u_j)$ defined as

$$
u_j = \begin{cases} 1 & , \quad j = 0, \\ \sum_{r=0}^{j-1}\left(\frac{f_{j+1}^2}{f_r f_{r+1}}\frac{Q_r}{q_r} - \frac{f_{j+1}^2}{f_{r+1}f_{r+2}}\frac{Q_r}{q_{r+1}}\right) + \frac{f_{j+1}}{f_j}\frac{Q_j}{q_j} & , \quad j > 1. \end{cases}
$$

Then, we obtain $\widehat{F}^q u = (1, 1, 1, ...) \in c \backslash c_0$. It follows that $u \in c(\widehat{F}^q) \backslash c_0(\widehat{F}^q)$; that is, the inclusion strictly holds. $\square$

3. The α-, β- and γ-duals of the spaces $c_0(\widehat{F}^q)$ and $c(\widehat{F}^q)$

In this section, we determine the α-, β- and γ-duals of the spaces $c_0(\widehat{F}^q)$ and $c(\widehat{F}^q)$. The α-, β- and γ-duals of a sequence space Λ are defined respectively by

$$\Lambda^\alpha = \left\{ a = (a_j) \in \omega : \sum_{j=1}^{\infty} |a_j u_j| < \infty \text{ for all } u = (u_j) \in \Lambda \right\},$$

$$\Lambda^\beta = \left\{ a = (a_j) \in \omega : \sum_{j=1}^{\infty} a_j u_j \text{ converges for all } u = (u_j) \in \Lambda \right\},$$

$$\Lambda^\gamma = \left\{ a = (a_j) \in \omega : \sup_i \left| \sum_{j=1}^{i} a_j u_j \right| < \infty \text{ for all } u = (u_j) \in \Lambda \right\},$$

.

Now, we quote some lemmas which are necessary to find the $\alpha-$, β- and γ-duals of the spaces $c_0(\widehat{F}^q)$ and $c(\widehat{F}^q)$.

Lemma 3.1. *[35] Let $A = (a_{ij})$ be an infinite matrix.*
$A = (a_{ij}) \in (c_0, c) \Leftrightarrow$

$$\sup_i \sum_j |a_{ij}| < \infty \tag{1}$$

and

$$\lim_{i\to\infty} a_{ij} \text{ exists for each } j \in \mathbb{N} \tag{2}$$

hold.
$A = (a_{ij}) \in (c, c) \Leftrightarrow$ (1), (2) and

$$\lim_{i\to\infty} \sum_j a_{ij} \text{ exists} \tag{3}$$

hold.
$A = (a_{ij}) \in (c_0, \ell_\infty) = (c, \ell_\infty) \Leftrightarrow$ (1) holds.
$A = (a_{ij}) \in (c_0, \ell_1) = (c, \ell_1) \Leftrightarrow$

$$\sup_{K\in\mathcal{F}} \sum_j \left| \sum_{i\in K} a_{ij} \right| < \infty, \tag{4}$$

holds.

Lemma 3.2. *Let $a = (a_i) \in \omega$ and the matrix $B = (b_{ij})$ be defined by $B_i = a_i(\widehat{F}^q)^{-1}$, that is,*

$$b_{ij} = \begin{cases} 0 & , \quad j > i \\ a_i(\widehat{f}^q)^{-1}_{ij} & , \quad 0 \le j \le i \end{cases}$$

for all $i, j \in \mathbb{N}$. Then, $a \in (\Lambda(\widehat{F}^q))^\alpha$ if and only if $B \in (\Lambda, \ell_1)$, where $\Lambda \in \{c_0, c\}$.

Proof. Let $v = (v_i)$ be the $\widehat{F}^q$-transform of a sequence $u = (u_i) \in \omega$; that is, $v_i = \widehat{F}^q_i(u)$ for all $i \in \mathbb{N}$. Then, we have

$$a_i u_i = a_i(\widehat{F}^q)^{-1}_i(v) = B_i(v)$$

for all $i \in \mathbb{N}$. Thus, we obtain that $au = (a_i u_i) \in \ell_1$ with $u \in \Lambda(\widehat{F}^q)$ if and only if $Bv \in \ell_1$ with $v \in \Lambda$. This implies that $a \in (\Lambda(\widehat{F}^q))^\alpha$ if and only if $B \in (\Lambda, \ell_1)$, where $\Lambda \in \{c_0, c\}$. Hence, the proof is completed. $\square$

LEMMA 3.3. *[4, Theorem 3.1] Let $C = (c_{ij})$ be defined with a sequence $a = (a_j) \in \omega$ and the inverse matrix $S = (s_{ij})$ of the triangle matrix $T = (t_{ij})$ by*

$$c_{ij} = \begin{cases} 0 & , \quad j > i \\ \sum_{r=j}^{i} a_r s_{rj} & , \quad 0 \leq j \leq i \end{cases}$$

for all $i, j \in \mathbb{N}$. Then,

$$(\Lambda(T))^\beta = \{a = (a_j) \in \omega : C \in (\Lambda, c)\},$$

$$(\Lambda(T))^\gamma = \{a = (a_j) \in \omega : C \in (\Lambda, \ell_\infty)\},$$

where $\Lambda \in \{c_0, c\}$.

THEOREM 3.4. *Let define the following sets:*

$$\hat{d}_1 = \left\{ a = (a_j) \in \omega : \sup_{K \in \mathcal{F}} \sum_j \left| \sum_{i \in K} \left(\frac{f_{i+1}^2}{f_j f_{j+1}} \frac{Q_j}{q_j} - \frac{f_{i+1}^2}{f_{j+1} f_{j+2}} \frac{Q_j}{q_{j+1}} \right) a_i \right| < \infty \right\},$$

$$\hat{d}_2 = \left\{ a = (a_j) \in \omega : \sup_i \sum_j \left| \sum_{r=j}^{i} \left(\frac{f_{r+1}^2}{f_j f_{j+1}} \frac{Q_j}{q_j} - \frac{f_{r+1}^2}{f_{j+1} f_{j+2}} \frac{Q_j}{q_{j+1}} \right) a_r \right| < \infty \right\},$$

$$\hat{d}_3 = \left\{ a = (a_j) \in \omega : \lim_{i \to \infty} \sum_{r=j}^{i} \left(\frac{f_{r+1}^2}{f_j f_{j+1}} \frac{Q_j}{q_j} - \frac{f_{r+1}^2}{f_{j+1} f_{j+2}} \frac{Q_j}{q_{j+1}} \right) a_r \text{ exists for each } j \in \mathbb{N} \right\}$$

and

$$\hat{d}_4 = \left\{ a = (a_j) \in \omega : \lim_{i \to \infty} \sum_{j=0}^{i} \sum_{r=j}^{i} \left(\frac{f_{r+1}^2}{f_j f_{j+1}} \frac{Q_j}{q_j} - \frac{f_{r+1}^2}{f_{j+1} f_{j+2}} \frac{Q_j}{q_{j+1}} \right) a_r \text{ exists for each } j \in \mathbb{N} \right\}.$$

Then, we have
(a) $(c_0(\widehat{F}^q))^\alpha = (c(\widehat{F}^q))^\alpha = \hat{d}_1$.
(b) $(c_0(\widehat{F}^q))^\beta = \hat{d}_2 \cap \hat{d}_3$ and $(c(\widehat{F}^q))^\beta = \hat{d}_2 \cap \hat{d}_3 \cap \hat{d}_4$.
(c) $(c_0(\widehat{F}^q))^\gamma = (c(\widehat{F}^q))^\gamma = \hat{d}_2$.

Proof. The proof is obtained by combining Lemmas 3.1, 3.2 and 3.3. $\square$

4. Characterization of certain classes of matrices

In the final section, we give the characterization of the classes $(\Lambda(\widehat{F}^q), \Omega)$ and $(\Omega, \Lambda(\widehat{F}^q))$, where $\Lambda \in \{c_0, c\}$ and $\Omega \in \{\ell_1, c_0, c, \ell_\infty\}$.

Throughout this section, we write $a(i,j) = \sum_{r=0}^{i} a_{rj}$ for given an infinite matrix $A = (a_{ij})$, where $i, j \in \mathbb{N}$. Now, we give a lemma which is necessary together with Lemma 3.1 to characterising the classes $(\Lambda(\widehat{F}^q), \Omega)$.

Lemma 4.1. *[35] Let $A=(a_{ij})$ be an infinite matrix.*
$A=(a_{ij})\in(c_0,c_0)$ if and only if (1) and

$$\lim_{i\to\infty} a_{ij}=0 \text{ for each } j\in\mathbb{N} \tag{5}$$

hold.
$A=(a_{ij})\in(c,c_0)$ if and only if (1), (5) and

$$\lim_{i\to\infty}\sum_j a_{ij}=0 \tag{6}$$

hold.

Now, we have a theorem which is required to characterize the classes of matrices from $c_0(\widehat{F}^q)$ or $c(\widehat{F}^q)$ to classical sequence spaces.

Theorem 4.2. *Let $\Lambda\in\{c_0,c\}$ and Ω be a sequence space. Then, we have $A=(a_{ij})\in(\Lambda(\widehat{F}^q),\Omega)$ if and only if*

$$D^{(i)}=\left(d_{lj}^{(i)}\right)\in(\Lambda,c)\ for\ each\ fixed\ i\in\mathbb{N}, \tag{7}$$

$$D=(d_{ij})\in(\Lambda,\Omega), \tag{8}$$

where

$$d_{lj}^{(i)}=\begin{cases} 0 & , \ j>l \\ \frac{f_{j+1}}{f_j}\frac{Q_j}{q_j}a_{ij}+\sum_{r=j+1}^{l}\left(\frac{f_{r+1}^2}{f_jf_{j+1}}\frac{Q_j}{q_j}-\frac{f_{r+1}^2}{f_{j+1}f_{j+2}}\frac{Q_j}{q_{j+1}}\right)a_{ir} & , \ 0\le j\le l\end{cases}$$

and

$$d_{ij}=\frac{f_{j+1}}{f_j}\frac{Q_j}{q_j}a_{ij}+\sum_{r=j+1}^{\infty}\left(\frac{f_{r+1}^2}{f_jf_{j+1}}\frac{Q_j}{q_j}-\frac{f_{r+1}^2}{f_{j+1}f_{j+2}}\frac{Q_j}{q_{j+1}}\right)a_{ir}$$

for all $i,j,l\in\mathbb{N}$.

Proof. The proof can be given using a similar technique as for Theorem 4.1 of [29]. So, we omit the proof. □

Now, we list some conditions:

$$\sup_l\sum_{j=0}^{l}\left|d_{lj}^{(i)}\right|<\infty \text{ for each fixed } i\in\mathbb{N}, \tag{9}$$

$$\lim_{l\to\infty} d_{lj}^{(i)} \text{ exists for each fixed } i,j\in\mathbb{N}, \tag{10}$$

$$\lim_{l\to\infty}\sum_{j=0}^{l} d_{lj}^{(i)} \text{ exists for each fixed } i\in\mathbb{N}. \tag{11}$$

Theorem 4.3. *Let $A=(a_{ij})$ be an infinite matrix. Then, we have:*
(a) $A=(a_{ij})\in(c_0(\widehat{F}^q),\ell_1)\Leftrightarrow$ (9) and (10) are true, and (4) is true for d_{ij}.
(b) $A=(a_{ij})\in(c_0(\widehat{F}^q),c_0)\Leftrightarrow$ (9) and (10) are true, and (1) and (5) are true for d_{ij}.

(c) $A = (a_{ij}) \in (c_0(\widehat{F}^q), c) \Leftrightarrow$ (9) *and* (10) *are true, and* (1) *and* (2) *are true for* d_{ij}.

(d) $A = (a_{ij}) \in (c_0(\widehat{F}^q), \ell_\infty) \Leftrightarrow$ (9) *and* (10) *are true, and* (1) *is true for* d_{ij}.

Proof. (a)From Theorem (4.2), $A = (a_{ij}) \in (c_0(\widehat{F}^q), l_1)$ if and only if $D^{(i)} = (d_{lj}^{(i)}) \in (c_0, c)$ for each $i \in \mathbb{N}$ and $D = (d_{ij}) \in (c_0, l_1)$. It follows from Lemma (3.1) that $D^{(i)} = (d_{lj}^{(i)}) \in (c_0, c)$ if and only if (9) and (10) hold and $D = (d_{ij}) \in (c_0, l_1)$ if and only if (4) holds for d_{ij}.

(b)From Theorem (4.2), $A = (a_{ij}) \in (c_0(\widehat{F}^q), c_0)$ if and only if $D^{(i)} = (d_{lj}^{(i)}) \in (c_0, c)$ for each $i \in \mathbb{N}$ and $D = (d_{ij}) \in (c_0, c_0)$. It follows from Lemma (3.1) that $D^{(i)} = (d_{lj}^{(i)}) \in (c_0, c)$ if and only if (9) and (10) hold and from Lemma (4.1) that $D = (d_{ij}) \in (c_0, c_0)$ if and only if (1) and (5) hold for d_{ij}.

(c)From Theorem (4.2), $A = (a_{ij}) \in (c_0(\widehat{F}^q), c)$ if and only if $D^{(i)} = (d_{lj}^{(i)}) \in (c_0, c)$ for each $i \in \mathbb{N}$ and $D = (d_{ij}) \in (c_0, c)$. It follows from Lemma (3.1) that $D^{(i)} = (d_{lj}^{(i)}) \in (c_0, c)$ if and only if (9) and (10) hold and $D = (d_{ij}) \in (c_0, c)$ if and only if (1) and (2) hold for d_{ij}.

(d)From Theorem (4.2), $A = (a_{ij}) \in (c_0(\widehat{F}^q), l_\infty)$ if and only if $D^{(i)} = (d_{lj}^{(i)}) \in (c_0, c)$ for each $i \in \mathbb{N}$ and $D = (d_{ij}) \in (c_0, l_\infty)$. It follows from Lemma (3.1) that $D^{(i)} = (d_{lj}^{(i)}) \in (c_0, c)$ if and only if (9) and (10) hold and $D = (d_{ij}) \in (c_0, l_\infty)$ if and only if (1) holds for d_{ij}. □

THEOREM 4.4. *Let* $A = (a_{ij})$ *be an infinite matrix. Then, we have:*

(a) $A = (a_{ij}) \in (c(\widehat{F}^q), \ell_1) \Leftrightarrow$ (9), (10) *and* (11) *are true, and* (4) *is true for* d_{ij}.

(b) $A = (a_{ij}) \in (c(\widehat{F}^q), c_0) \Leftrightarrow$ (9), (10) *and* (11) *are true, and* (1), (5) *and* (6) *are true for* d_{ij}.

(c) $A = (a_{ij}) \in (c(\widehat{F}^q), c) \Leftrightarrow$ (9), (10) *and* (11) *are true, and* (1), (2) *and* (3) *are true for* d_{ij}.

(d) $A = (a_{ij}) \in (c(\widehat{F}^q), \ell_\infty) \Leftrightarrow$ (9), (10) *and* (11) *are true, and* (1) *is true for* d_{ij}.

Proof. (a)From Theorem (4.2), $A = (a_{ij}) \in (c(\widehat{F}^q), l_1)$ if and only if $D^{(i)} = (d_{lj}^{(i)}) \in (c, c)$ for each $i \in \mathbb{N}$ and $D = (d_{ij}) \in (c, l_1)$. It follows from Lemma (3.1) that $D^{(i)} = (d_{lj}^{(i)}) \in (c, c)$ if and only if (9), (10) and (11) hold and $D = (d_{ij}) \in (c, l_1)$ if and only if (4) holds for d_{ij}.

(b)From Theorem (4.2), $A = (a_{ij}) \in (c(\widehat{F}^q), c_0)$ if and only if $D^{(i)} = (d_{lj}^{(i)}) \in (c, c)$ for each $i \in \mathbb{N}$ and $D = (d_{ij}) \in (c, c_0)$. It follows from Lemma (3.1) that $D^{(i)} = (d_{lj}^{(i)}) \in (c, c)$ if and only if (9), (10) and (11) hold and from Lemma (4.1) that $D = (d_{ij}) \in (c, c_0)$ if and only if (1), (5) and (6) hold for d_{ij}.

(c)From Theorem (4.2), $A = (a_{ij}) \in (c(\widehat{F}^q), c)$ if and only if $D^{(i)} = (d_{lj}^{(i)}) \in (c, c)$ for each $i \in \mathbb{N}$ and $D = (d_{ij}) \in (c, c)$. It follows from Lemma (3.1) that $D^{(i)} = (d_{lj}^{(i)}) \in (c, c)$ if and only if (9) and (10) hold and $D = (d_{ij}) \in (c, c)$ if and only if (1), (2) and (3) hold for d_{ij}.

(d)From Theorem (4.2), $A = (a_{ij}) \in (c(\widehat{F}^q), l_\infty)$ if and only if $D^{(i)} = (d_{lj}^{(i)}) \in (c, c)$ for each $i \in \mathbb{N}$ and $D = (d_{ij}) \in (c, l_\infty)$. It follows from Lemma (3.1) that $D^{(i)} = (d_{lj}^{(i)}) \in (c, c)$ if and only if (9), (10) and (11) hold and $D = (d_{ij}) \in (c, l_\infty)$ if and only if (1) holds for d_{ij}. □

As a consequence of these theorems, we have the following results.

Corollary 4.5. *Let $A = (a_{ij})$ be an infinite matrix. Then, we have:*

(a) $A = (a_{ij}) \in (c_0(\widehat{F}^q), cs_0) \Leftrightarrow$ (9) and (10) are true, and (1) and (5) are true for $d(i,j)$.

(b) $A = (a_{ij}) \in (c_0(\widehat{F}^q), cs) \Leftrightarrow$ (9) and (10) are true, and (1) and (2) are true for $d(i,j)$.

(c) $A = (a_{ij}) \in (c_0(\widehat{F}^q), bs) \Leftrightarrow$ (9) and (10) are true, and (1) is true for $d(i,j)$.

Corollary 4.6. *Let $A = (a_{ij})$ be an infinite matrix. Then, we have:*

(a) $A = (a_{ij}) \in (c(\widehat{F}^q), cs_0) \Leftrightarrow$ (9), (10) and (11) are true, and (1), (5) and (6) are true for $d(i,j)$.

(b) $A = (a_{ij}) \in (c(\widehat{F}^q), cs) \Leftrightarrow$ (9), (10) and (11) are true, and (1), (2) and (3) are true for $d(i,j)$.

(c) $A = (a_{ij}) \in (c(\widehat{F}^q), bs) \Leftrightarrow$ (9), (10) and (11) are true, and (1) is true for $d(i,j)$.

Now, we give a lemma which is necessary together with Lemma 3.1 and Lemma 4.1 for characterising the classes $(\Omega, \Lambda(\widehat{F}^q))$.

Lemma 4.7. *[35] Let $A = (a_{ij})$ be an infinite matrix.*

$A = (a_{ij}) \in (\ell_1, c_0) \Leftrightarrow$ (5) and

$$\sup_{i,j} |a_{ij}| < \infty, \tag{12}$$

hold.

$A = (a_{ij}) \in (\ell_\infty, c_0) \Leftrightarrow$ (5) and

$$\lim_{i\to\infty} \sum_j |a_{ij}| = 0 \tag{13}$$

hold.

$A = (a_{ij}) \in (\ell_1, c) \Leftrightarrow$ (2) and (12) hold.

$A = (a_{ij}) \in (\ell_\infty, c) \Leftrightarrow$ (2) and

$$\lim_{i\to\infty} \sum_j |a_{ij}| = \sum_j \left| \lim_{i\to\infty} a_{ij} \right| \tag{14}$$

hold.

Before characterizing the matrix classes from classical spaces to $c_0(\widehat{F}^q)$ or $c(\widehat{F}^q)$, we prove a theorem. We change the roles of the spaces $\Lambda(\widehat{F}^q)$ and Λ with Ω in Theorem 4.2.

Theorem 4.8. *Let $A = (a_{ij})$ and $B = (b_{ij})$ be two infinite matrices and*

$$b_{ij} = \sum_{r=0}^{i-1} \left(\frac{q_r}{Q_i} \frac{f_r}{f_{r+1}} - \frac{q_{r+1}}{Q_i} \frac{f_{r+2}}{f_{r+1}} \right) a_{rj} + \frac{q_i}{Q_i} \frac{f_i}{f_{i+1}} a_{ij} \tag{15}$$

holds for all $i, j \in \mathbb{N}$. Then, $A \in (\Omega, \Lambda(\widehat{F}^q))$ if and only if $B \in (\Omega, \Lambda)$, where $\Lambda \in \{c_0, c\}$ and $\Omega \in \{\ell_1, c_0, c, \ell_\infty\}$.

Proof. Let $u = (u_j) \in \Omega$. By the aid of (15), we have

$$\sum_{j=0}^{l} b_{ij} u_j = \sum_{j=0}^{l} \left(\sum_{r=0}^{i-1} \left(\frac{q_r}{Q_i} \frac{f_r}{f_{r+1}} - \frac{q_{r+1}}{Q_i} \frac{f_{r+2}}{f_{r+1}} \right) a_{rj} + \frac{q_i}{Q_i} \frac{f_i}{f_{i+1}} a_{ij} \right) u_j$$
$$= \frac{1}{Q_i} \sum_{r=0}^{i} q_r \left(\frac{f_r}{f_{r+1}} \sum_{j=0}^{l} a_{rj} u_j - \frac{f_{r+1}}{f_r} \sum_{j=0}^{l} a_{r-1,j} u_j \right)$$

for all $i, l \in \mathbb{N}$ which yields as $l \to \infty$ that $(B_i(u)) = (\widehat{F}_i^q(Au))$. Therefore, we conclude that $Au \in \Lambda(\widehat{F}^q)$ for $u \in \Omega$ if and only if $Bu \in \Lambda$ for $u \in \Omega$, where $\Lambda \in \{c_0, c\}$ and $\Omega \in \{\ell_1, c_0, c, \ell_\infty\}$. Hence the proof is completed. □

THEOREM 4.9. *Let $A = (a_{ij})$ and $B = (b_{ij})$ satisfy the relation in* (15). *Then, we have:*

(a) $A = (a_{ij}) \in (\ell_1, c_0(\widehat{F}^q)) \Leftrightarrow$ (5) *and* (12) *are true for b_{ij}.*

(b) $A = (a_{ij}) \in (c_0, c_0(\widehat{F}^q)) \Leftrightarrow$ (1) *and* (5) *are true for b_{ij}.*

(c) $A = (a_{ij}) \in (c, c_0(\widehat{F}^q)) \Leftrightarrow$ (1), (5) *and* (6) *are true for b_{ij}.*

(d) $A = (a_{ij}) \in (\ell_\infty, c_0(\widehat{F}^q)) \Leftrightarrow$ (5) *and* (13) *are true for b_{ij}.*

Proof. (a) From Theorem (4.8), $A = (a_{ij}) \in (l_1, c_0(\widehat{F}^q))$ if and only if $B = (b_{ij}) \in (l_1, c_0)$. It follows from Lemma (4.7) that $B = (b_{ij}) \in (l_1, c_0)$ if and only if (5) and (12) hold for b_{ij}.

(b)From Theorem (4.8), $A = (a_{ij}) \in (c_0, c_0(\widehat{F}^q))$ if and only if $B = (b_{ij}) \in (c_0, c_0)$. It follows from Lemma (4.1) that $B = (b_{ij}) \in (c_0, c_0)$ if and only if (1) and (5) hold for b_{ij}.

(c)From Theorem (4.8), $A = (a_{ij}) \in (c, c_0(\widehat{F}^q))$ if and only if $B = (b_{ij}) \in (c, c_0)$. It follows from Lemma (4.1) that $B = (b_{ij}) \in (c, c_0)$ if and only if (1), (5) and (6) hold for b_{ij}.

(d)From Theorem (4.8), $A = (a_{ij}) \in (l_\infty, c_0(\widehat{F}^q))$ if and only if $B = (b_{ij}) \in (l_\infty, c_0)$. It follows from Lemma (4.7) that $B = (b_{ij}) \in (l_\infty, c_0)$ if and only if (5) and (13) hold for b_{ij}.

□

THEOREM 4.10. *Let $A = (a_{ij})$ and $B = (b_{ij})$ satisfy the relation in* (15). *Then, we have:*

(a) $A = (a_{ij}) \in (\ell_1, c(\widehat{F}^q)) \Leftrightarrow$ (2) *and* (12) *are true for b_{ij}.*

(b) $A = (a_{ij}) \in (c_0, c(\widehat{F}^q)) \Leftrightarrow$ (1) *and* (2) *are true for b_{ij}.*

(c) $A = (a_{ij}) \in (c, c(\widehat{F}^q)) \Leftrightarrow$ (1), (2) *and* (3) *are true for b_{ij}.*

(d) $A = (a_{ij}) \in (\ell_\infty, c(\widehat{F}^q)) \Leftrightarrow$ (2) *and* (14) *are true for b_{ij}.*

Proof. (a) From Theorem (4.8), $A = (a_{ij}) \in (l_1, c(\widehat{F}^q))$ if and only if $B = (b_{ij}) \in (l_1, c)$. It follows from Lemma (4.7) that $B = (b_{ij}) \in (l_1, c)$ if and only if (2) and (12) hold for b_{ij}.

(b)From Theorem (4.8), $A = (a_{ij}) \in (c_0, c(\widehat{F}^q))$ if and only if $B = (b_{ij}) \in (c_0, c)$. It follows from Lemma (3.1) that $B = (b_{ij}) \in (c_0, c)$ if and only if (1) and (2) hold for b_{ij}.

(c)From Theorem (4.8), $A = (a_{ij}) \in (c, c(\widehat{F}^q))$ if and only if $B = (b_{ij}) \in (c, c)$. It follows from Lemma (3.1) that $B = (b_{ij}) \in (c, c)$ if and only if (1), (2) and (3) hold for b_{ij}.

(d)From Theorem (4.8), $A = (a_{ij}) \in (l_\infty, c(\widehat{F}^q))$ if and only if $B = (b_{ij}) \in (l_\infty, c)$. It follows from Lemma (4.7) that $B = (b_{ij}) \in (l_\infty, c)$ if and only if (2) and (14) hold for b_{ij}. □

5. Conclusions

We have established new Banach spaces by the aid of a matrix which is the multiplication of Riesz and Fibonacci matrices. These spaces are more general than some other spaces and not a special case of the ones defined earlier. By using similar techniques in this paper, more comprehensive and general spaces can be constructed.

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References

[1] A. Alotaibi, M. Mursaleen, B. AS Alamri and S. A. Mohiuddine, *Compact operators on some Fibonacci difference sequence spaces*, J. Inequal. Appl., **2015**, 2015, Art. no. 203.

[2] Altay, B., Başar, F.: *Some Paranormed Riesz Sequence Spaces of Non-absolute Type*, Southeast Asian Bulletin of Mathematics **30** (4), (2006), 591–608.

[3] Altay, Bilal, and Feyzi Başar. *Generalization of the sequence space $l(p)$ derived by weighted mean*, Journal of Mathematical Analysis and Applications **330** (1) (2007): 174–185.

[4] Altay, B., Başar, F.: *Certain topological properties and duals of the matrix domain of a triangle matrix in a sequence space*, J. Math. Anal. Appl. **336** (1) (2007), 632–645.

[5] Aydın, C., and F. Başar, *Some new sequence spaces which include the spaces l_p and l_∞*, Demonstratio Mathematica **38** (3) (2005), 641–656.

[6] Başarır, M., Başar, F., Kara, EE., *On the spaces of Fibonacci difference null and convergent sequences*, arXiv:1309.0150v1 [math.FA], (2013).

[7] M. Candan, *A new approach on the spaces of generalized Fibonacci difference null and convergent sequences*, Math. Aeterna, **5** 2015, 191–210.

[8] M. Candan and E. E. Kara, *A study on topological and geometrical characteristics of new Banach sequence spaces*, Gulf J. Math., **3** 2015, 67–84.

[9] A. Das and B. Hazarika, *Some new Fibonacci difference spaces of non-absolute type and compact operators*, Linear Multilinear Algebra, **65** (2017), 2551–2573.

[10] S. Demiriz and C. Çakan, *Some topological and geometrical properties of a new difference sequence space*, Abstr. Appl. Anal., **2011** (2011), Art. ID 213878.

[11] S. Demiriz, E. E. Kara and M. Başarır, *On the Fibonacci almost convergent sequence space and Fibonacci core*, Kyungpook Math. J., **55** (2015), 355–372.

[12] H. B. Ellidokuzoğlu and S. Demiriz, *Euler-Riesz difference sequence spaces*, Turk. J. Math. Comput. Sci., **7** (2017), 63–72.

[13] S. Ercan , Ç.A. Bektaş, *On some sequence spaces of non-absolute type*, Kragujevac J. Math.,**38** (2014), 195–202.

[14] 14. S. Ercan , Ç.A. Bektaş, *On new convergent difference BK-spaces*, J. Comput. Anal. Appl. **23** (5) (2017), 793–801.

[15] S. Ercan and Ç. A. Bektaş, *Some topological and geometric properties of a new BK-space derived by using regular matrix of Fibonacci numbers*, Linear Multilinear Algebra, **65** (2017), 909–921.

[16] S. Ercan, *On λ_r-Convergence and λ_r-Boundedness*, J. Adv. Phys., **7** (2018), 123–129.

[17] M. İlkhan and E. E. Kara,*Remarkable applications of measure of non-compactness for infinite system of differential equations*, Appl. Appl. Math., **5** (2019), 1–12.

[18] Kara, E.E.: *Some topological and geometrical properties of new Banach sequence spaces*, J. Inequal. Appl. **38** (2013).

[19] E. E. Kara and S. Demiriz, *Some new paranormed difference sequence spaces derived by Fibonacci numbers*, Miskolc Math. Notes, **16** (2015), 907–923.

[20] Kara, EE., İlkhan, M., *On some Banach sequence spaces derived by a new band matrix*, British Journal of Mathematics & Computer Science, **9** (2) (2015), 141–159.

[21] Kara, EE., İlkhan, M., *Some properties of generalized Fibonacci sequence spaces, Linear and Multilinear Algebra*, **64** (11) (2016), 2208–2223.

[22] M. İlkhan Kara and M. A. Bayrakdar, *A study on matrix domain of Riesz-Euler totient matrix in the space of p-absolutely summable sequences*, Commun. Adv. Math. Sci., **4** (1) (2021), 14–25.

[23] M. İlkhan Kara and E. E. Kara, *Matrix transformations and compact operators on Catalan sequence spaces*, J. Math. Anal. Appl., **498** (1) (2021), Article 124925, 1–17.

[24] V. A. Khan, *On Riesz–Musielak–Orlicz sequence spaces*, Numer. Funct. Anal. Optim., **28** (2007), 883–895.

[25] V. A. Khan, R. K. A. Rababah, K. M. A. S. Alshlool, S. A. A. Abdullah and A. Ahmad, *On ideal convergence Fibonacci difference sequence spaces*, Adv. Difference Equ., **2018** (2018), Art. no. 199.

[26] V. A. Khan, K. M. A. S. Alshlool, A. A. H. Makharesh and S. A. A. Abdullah, *On spaces of ideal convergent Fibonacci difference sequence defined by Orlicz function*, Sigma J. Eng. Nat. Sci., **37** (2019), 143–154.

[27] Kızmaz, H.: *On certain sequence spaces*, Can. Math. Bull. **24**(2) (1981), 169–176.

[28] M. Kirişçi, *The application domain of infinite matrices with algorithms*, Univers. J. Math. Appl., **1** (2018), 1–9.

[29] Kirişçi, M., Başar, F., *Some new sequence spaces derived by the domain of generalized difference matrix*, Comput. Math. Appl. **60** (2010), 1299–1309.

[30] S. A. Mohiuddine and B. Hazarika, *Some classes of ideal convergent sequences and generalized difference matrix operator*, Filomat,**31** (2017), 1827–1834.

[31] Mursaleen, M., *Generalized spaces of difference sequences*, J. Math. Anal. Appl. **203** (3) (1996), 738–745.

[32] Mursaleen, M., F. Başar, and B. Altay., *On the Euler sequence spaces which include the spaces l_p and l_∞ II*, Nonlinear Analysis: Theory, Methods and Applications **65** (3) (2006), 707–717.

[33] Mursaleen, M., and Abdullah K. Noman., *On some new sequence spaces of non-absolute type related to the spaces l_p and l_∞ I*, Filomat **25** (2) (2011), 33–51.

[34] H. Polat, *Some new Pascal sequence spaces*, Fundam. J. Math. Appl., **1** (2018), 61–68.

[35] Stieglitz, M., Tietz, H., *Matrix transformationen von folgenraumen eine ergebnisübersicht*, Math. Z. **154** (1977), 1–16.

[36] T. Yaying and M. İlkhan Kara, *On sequence spaces defined by the domain of tribonacci matrix in c_0 and c*, Korean J. Math.,**29** (1) (2021), 25–40.

Pınar Zengin Alp
Department of Mathematics, Düzce University, Düzce, Turkey
E-mail: pinarzengin@duzce.edu.tr

Emrah Evren Kara
Department of Mathematics, Düzce University, Düzce, Turkey
E-mail: karaeevren@gmail.com