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A PREPAYMENT-RISK-NEUTRAL PRICING MODEL FOR MORTGAGE-BACKED SECURITIES

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Abstract. In this paper, we investigate a pricing model for mortgage-backed securities (MBSs) of a pay-through type of collateral mortgage obligation (CMO), embedded call options, which can be exercised by the intermediary, and pass-through MBSs. We suggest a prepayment-risk-neutral pricing model, applying a reduced-form prepayment rate model, and then compute and investigate the appropriate prices and spreads in the coupon rates between CMOs and PT MBSs. We believe that this study contributes in that it provides a sophisticated pricing model for MBSs, especially to the financial markets which are not advanced enough to finance with a simple type of MBSs.

1. Introduction

In many countries, houses are some of the most expensive assets; thus, people usually use mortgages to buy one. Financial institutions that provide such mortgages are usually financed by investors, and mortgage-backed securities (MBSs, henceforth) and covered bonds are financial instruments that are widely used as a means of financing mortgage loans in particular. MBSs were developed in the United States and are currently used mainly in the United States, Japan, and the Republic of Korea (Korea, henceforth), while covered bonds are used in European countries. In most MBSs, the cash flows to the investors are variable, depending on the borrowers' repayments, while most covered bonds have very stable cash flows, like ordinary bonds.

We propose a pricing model for these MBSs of which cash flows are stochastic in this study. The pricing of MBSs is very important, especially in emerging countries without advanced financial markets. It is difficult for the financial intermediaries in such countries to find effective tools to finance long-term mortgages except for MBSs, whereas the intermediaries in countries with advanced financial markets are able to finance with various means, such as long-term bonds. Therefore, from the relatively advanced countries in Asia, such as Korea, Japan, and Singapore, to the emerging countries, such as Indonesia, Malaysia, and Thailand, most countries wishing to enable

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long-term mortgages tend to design their own MBSs, taking the U.S. MBSs as their benchmark ([19]).

Unfortunately, the pricing of MBSs is a very difficult task. The U.S. and certain countries in Europe have very advanced long-term bond markets and thus, the financial intermediaries in those countries are willing to issue long-term covered bonds and simple long-term MBSs, whose durations or maturities are close to their long-term mortgages. In other words, those intermediaries do not have to pay high risk and term premiums to finance in the long term with a relatively simple product. However, to avoid paying a high premium, the intermediaries in emerging countries without developed long-term financial markets must construct a very complex structure of MBSs with various maturities and credit priorities. Therefore, this study makes a significant contribution in that it provides a sophisticated pricing model for those MBSs to the financial markets in emerging countries that wish to enhance housing welfare using their financial markets but are not advanced enough to finance with MBSs with a relatively simple structure.

MBSs can be divided into pass-through MBSs and pay-through MBSs, depending on the intermediary's role in the cash flows to the MBS investors. Pass-through MBSs pay the cash flows generated by the underlying mortgage pool directly to the investors through the intermediary. Pay-through MBSs' cash flows to investors can be adjusted by the intermediary, depending on the MBS structure and contracts. Most pricing studies have been conducted on pass-through MBSs since pass-through MBS is the dominant type of MBS between the two types, especially after the global financial crises, and pay-through MBSs are not appropriate for study because the structure of each one issued by various intermediary varied widely.

The cash flow from the underlying MBS mortgage pool is uncertain because it depends on the borrower's decision about the prepayment. Prepayment models have been developed as structural models and statistical models, and in recent years, the reduced-form prepayment model has been in the spotlight.

The structural model is an endogenous approach that reflects the optimal decision of the borrower by modeling the borrower's prepayment as their call option ([6], [7]). While this structural model presents economically meaningful results, its empirical estimations are mostly very inaccurate.

The statistical model basically aims to quantitatively investigate the relationship between the prepayment rate, macroeconomic variables, e.g., interest rates and house prices, and individual mortgage characteristics, e.g., loan ages, seasonal effect, loan-to-value ratio, and household composition ([17], [11], [5], [2]). However, if we apply the econometric prepayment models introduced in these statistical studies directly to MBS pricing, they are under the actual probability measure, and do not match the risk-neutral interest-rate models generally applied to the pricing. Thus, the risk premiums on the prepayment risk in these studies are not properly estimated ([1]). Therefore, methodologies that apply an option-adjusted spread (OAS, henceforth), which acts as a virtual spread to the discount rate, have usually been applied to the MBS pricing in this statistical area ([3], [4], [14], [15], [2], [8]).

In this context and to compensate for these shortcomings, reduced-form prepayment models assume a process suitable for the existing prepayment-rate model and model the prepayment process as being influenced by macroeconomic variables ([18],

[12], [13]). These reduced-form prepayment models are mostly based on a dual stochastic process; they classify the effect of the macroeconomic variables and the prepayment rate itself and provide the prepayment rate at a specific point in time as a product of the two processes. In this study, a prepayment-rate model similar to the ones presented by [12] and [13] is adopted.

In Korea, a collateral mortgage obligation, which is closer to a pay-through MBS than a pass-through MBS, is the main. The pass-through MBSs issued in the United States are mostly issued in a single tranche (portion) with a single underlying mortgage pool, while the Korea's collateral mortgage obligation (KCMO, henceforth) issued by the Korea Housing-Finance Corporation (KHFC, henceforth), which is the only intermediary issuing MBSs in Korea, are multiple-tranche MBSs with a single underlying mortgage pool. Another characteristic of KCMO is that long-term tranches with a maturity of five years or more are attached with call options. KHFC recently began to issue pass-through (KPT, henceforth) MBSs; however, it is difficult to view these MBSs as conventional pass-through MBSs since they have multiple tranches. Dividing the cash flows from a single underlying mortgage pool into multiple tranches complicates the MBS price valuation.

In this study, we suggest and investigate a prepayment-risk-neutral pricing model for these Korean MBSs, KCMOs and KPT MBSs issued by KHFC, with a simpler structure of one tranche, rather than multiple tranches, owing to the complex pricing of a multiple-tranche MBS. We then compute the appropriate prices and spreads in the coupon rates between the KCMOs and KPT MBSs issued in Korea, although they can be said to be virtual commodities.

This paper is organized as follows. Section 2 describes our pricing model of the two MBSs. Section 3 presents the numerical results of the MBS pricing and the comparative statics on the price. Section 4 offers the conclusion.

2. The Model

2.1. KCMOs and KPT MBSs. KHFC is a state-run enterprise in Korea, founded in 2004, which provides long-term fixed-rate mortgages and issues MBSs, for which the underlying assets are the mortgages it provides. This agency has regularly issued MBSs almost twice a month, 20∼30 times a year since the global financial crisis. Most MBSs issued by KHFC are in a pay-through type of KCMO, as in Table 1.

Tranche	Amount (1Bil KRW)	Maturity	Coupon	Interest Payout	Principal Payout	Call Option Exercise
I-1	87.9	1Y	2.005%	Quarterly	At maturity	
I-2	190	2Y	2.250%	Quarterly	At maturity	
I-3	160	3Y	2.415%	Quarterly	At maturity	
I-4	250	5Y	2.820%	Quarterly	At maturity	After 3M
I-5	140	7Y	2.910%	Quarterly	At maturity	After 2Y
I-6	100	10Y	2.956%	Quarterly	At maturity	After 3Y
I-7	50	15Y	2.936%	Quarterly	At maturity	After 4Y
I-8	10	20Y	2.916%	Quarterly	At maturity	After 5Y

Table 1. KHFC MBS 2018-13 Structure: KCMO type

Table 1 presents the structure of an MBS issued by KHFC in June 2018. It consists of eight tranches with different maturities to attract Korean investors with different maturity preferences. Nevertheless, the tranches branch from a single underlying mortgage pool and, thus, share the risk of uncertain cash flows from the pool. Call options are embedded on the I-4 tranche and after. These options may only be exercised sequentially starting from the I-4 tranche; for example, the I-5 tranche becomes callable only after the I-4 tranche is fully repaid.

The funds available for exercising the call option are financed only from the payments of the borrowers in the underlying mortgage pool. KHFC cannot exercise the option with any other money, e.g., from other mortgage pools underlying other MBSs or portfolio rebalancing. Because of this single underlying mortgage pool with its multiple-tranche structure, it is very difficult for investors to predict future cash flows, which makes it difficult to evaluate the value of KHFC MBSs.

In November 2016, KHFC began issuing another type of MBS, the pass-through MBS. Table 2 describes the structure of KHFC MBS 2018-12, a KPT MBS issued by KHFC.

Tranche	Amount (1Bil KRW)	Maturity	Coupon	Interest Payout	Principal Payout	Pass-Through
I-1	153.4	2Y	2.318%	Quarterly	At maturity	
I-2	210	5Y	2.837%	Quarterly	At maturity	After I-1 repayment
I-3	120	10Y	2.996%	Quarterly	At maturity	After I-2 repayment
I-4	30	20Y	2.996%	Quarterly	At maturity	After I-3 repayment

Table 2. KHFC MBS 2018-12 Structure: KPT type

One can see that this KPT MBS has fewer tranches and different call-option exercise schemes. For the KPT MBS, KHFC basically transfers the cash flows received from the borrowers to the investors as they are. With a KPT MBS, the cash flow to the I-1 tranche investors is quite similar to a conventional pass-through MBS. However, I-2 tranche and subsequent investors only receive interest payments; they must wait until the earlier tranche(s) are fully repaid to receive the principal from the borrowers' prepayments. Therefore, this KPT MBS is a modified version of the conventional pass-through MBS in the U.S. Since KPT MBSs also have multiple tranches, it is much difficult to evaluate the price.

2.2. Cash Flows to the Investors. Figure 1 shows the structure of the cash flows that the KCMO investors will receive.

$$\begin{tikzcd} \text{Borrowers} \arrow[r, "X"] & \text{KHFC} \arrow[r, shift left, "Y"] \arrow[r, shift right, "\text{Call}"'] \arrow[d, shift right, "Z"'] & \text{MBS Investors} \\ & \text{Money Market} \arrow[u, shift right, "\text{Cash}"'] & \end{tikzcd}$$

Figure 1. KCMO Cash Flows

When borrowers pay the principal and interest on their loans, the intermediary KHFC uses the cash inflows to pay the MBS investors their scheduled principal and interest, and the remaining cash is saved in short-term money-market accounts. X in Figure 1 denotes the cash flows from the borrowers, Y, the scheduled cash flows to the investors, and Z, the amount KHFC saves in money-market accounts. The equation $X = Y + Z$ naturally occurs here.

KHFC manages the cash flows saved in the money-market accounts and decides when and how many call options to exercise. Thus, it can prepay a part of or the entire outstanding MBS principal whenever it has available cash in the short-term money-market account. Since the cash available for exercising a call option must be financed in the corresponding trust account, the amount of the (potential) call option is determined by Z. Therefore, if Z is positive, i.e., additional savings are available, it is as if the Z amount of the outstanding principal of the MBS turns into an American callable bond[1] with face value Z. As a result, the value of a KCMO is equal to the sum of the present value of the scheduled interest and the value of the American callable bonds generated during the life of the bond, which can be written as follows:

$$V_c(0) = \mathbb{E}_Q\left[\sum_{t=1}^{T} P(0,t)\left[C(t,T)Z(t) + Y(t)\right]\right], \tag{1}$$

where $V_c(0)$ is the price of the KCMO at time 0, $P(0,t)$ is the price at time t of the zero-coupon bond maturing at time t, and $C(t,T)$ is the price at time t of the American callable bond maturing at time T. T is the expiration date of the MBS, and $\mathbb{E}_Q[\cdot]$ is the expectation under the risk-neutral measure.

On the other hand, if investors purchase KPT MBSs instead, their cash-flow income will appear as shown in Figure 2.

$$\text{Borrowers} \xrightarrow{\quad X \quad} \text{KHFC} \xrightarrow{\quad Y \quad} \text{MBS Investors}$$

Figure 2. KPT MBS Cash Flows

KHFC passes through (hands over) all the cash flows from the borrowers to the investors whenever it receives them, resulting in $X = Y$. The price of a KPT MBS, hence, can be written as follows:

$$V_p(0) = \mathbb{E}_Q\left[\sum_{t=1}^{T} P(0,t)X(t)\right], \tag{2}$$

where $V_p(0)$ is the price of the KPT MBS at time 0.

We now explain the model of the underlying mortgage pool and the corresponding MBSs. Suppose that all the mortgages in the pool have similar interest rates and maturities, and they are all repaid in equal monthly installments of the principal and interest. Then, the cash flow process from the borrowers can be modeled as follows.

$$X(t) = (c + p(t))\,\mathrm{Bal}_{\mathrm{mortgage}}(t-1), \tag{3}$$

where $X(t)$ is the cash flow amount from the borrowers at time t, c is the monthly repayment ratio of the pool, $p(t)$ is the prepayment rate at time t, and $Bal_{mortgage}(t)$ is the outstanding balance of the mortgage pool at the end of time t. If we assume

[1] One can say that this option is Bermudian, since KHFC pays the MBS investors on the designated days in every quarter, and the call option can only be exercised on the payment day. We simplified this limitation to model an American callable bond.

that the payment methods for all the mortgages are equal monthly installments of the principal and interest, c is obtained as

$$c = \frac{m\ dt}{1 - \left(\frac{1}{1+m\ dt}\right)^{T/dt}}, \tag{4}$$

where m is the mortgages' interest rate, dt is the time interval between each payment, and T is the maturity of the mortgages.

For a KCMO, the scheduled cash-flow process to the investors can be described as follows:

$$Y(t) = y\mathrm{Bal}_{\mathrm{MBS}}(t-1)dt, \tag{5}$$

where $Y(t)$ is the amount of the scheduled interest cash flows to the investors at time t, y is the coupon rate of the MBS, and $\mathrm{Bal}_{\mathrm{MBS}}(t)$ is the outstanding balance of the MBS at the end of time t. Note that without prepayments, the MBS principal is supposed to be paid at maturity, as in Table 1. Now, we can write $Z(t)$, the KHFC savings amount, as follows:

$$Z(t) = X(t) - Y(t) = (c + p(t))\,\mathrm{Bal}_{\mathrm{mortgage}}(t-1) - y\mathrm{Bal}_{\mathrm{MBS}}(t-1)dt, \tag{6}$$

where $Z(t)$ is the amount KHFC saves at time t.

To evaluate the price of the American callable bond, we apply the LSMC (least-squares Monte Carlo) approach suggested by [16] with a stochastic short-rate model to be discussed later. The continuation value for deriving the American callable-bond price is assumed to follow a regression form, such that

$$CV(t) = \alpha_0 + \alpha_1 r(t) + \alpha_2 r(t)^2, \tag{7}$$

where $CV(t)$ is the continuation value of the American callable bond at time t, $r(t)$ is the interest rate at time t, and α_i $(i = 0, 1, 2)$ are the regression parameters.

Except that the cash flow of the MBS depends on the borrower's prepayment, MBS can be thought of as very similar to conventional bonds. The price of a call option embedded in a conventional callable bond is the difference between the price of the callable bond and the equivalent ordinary bond without a call option. Equally, the price of the call option embedded in a MBS in Korea would be the difference between the prices of KCMO and KPT MBS. However, the fundamental difference in the call option of a MBS is that for the conventional callable bonds in general, there is no restriction on the source of financing for exercising the call options, and for KCMO, the intermediary can only exercise the call options with the cash flows from the underlying mortgage pool. Therefore, the call option in a KCMO has stronger constraints than that in a conventional callable bond, and thus its price will be relatively smaller.

2.3. Interest-Rate and Prepayment-Rate Model. The short-rate model for the MBS valuation of this study is assumed to be a one-factor [9]-type model whose process is given by:

$$dr(t) = a\left(\theta_r(t) - r(t)\right)dt + \sigma_r dW_r(t), \tag{8}$$

where $r(t)$ is the short rate at time t, a is the reversion rate of the short-rate process, σ_r is the interest-rate volatility, and $W_r(t)$ is the standard Brownian motion at time t under the risk-neutral measure. The time-dependent mean-reversion level $\theta_r(t)$ is computed to fit the initial interest-rate term structure.

The prepayment model considered in this study is a reduced-form prepayment model similar to the ones proposed by [12] and [13]. In particular, we model the prepayment process $p(t)$ to have a proportional hazard rate, as follows:

$$p(t) = e^{f(\mathbf{x}(t))+p_0(t)}, \tag{9}$$

$$f(\mathbf{x}(t)) = \beta_1 \arctan\left(\beta_2(spread(t)+\beta_3)\right) + \beta_4 age(t) + \beta_5 age(t)^2 + \beta_6 age(t)^3, \tag{10}$$

$$dp_0(t) = \kappa\left(\tilde{\theta}_p - p_0(t)\right)dt + \sigma_p d\widetilde{W}_p(t), \tag{11}$$

where $p(t)$ is the prepayment rate at time t, $f(\mathbf{x}(t))$ is a function of the time-dependent covariate vector $\mathbf{x}(t)$ in a regression form as in (10), $spread(t)$ is the interest-rate spread between the mortgage rate and the market interest rate at time t, and $age(t)$ is the monthly age of the mortgage pool after issuance. We consider an arctangent function with respect to refinancing motive and a cubic term for $age(t)$, as in [13]. Modeled as a cubic function of age, $f(\mathbf{x}(t))$ and $\exp\{f(\mathbf{x}(t))\}$ have a shape of a log-normal function. In addition, $p_0(t)$ is the baseline hazard process; it basically captures the prepayment turnover components, following a [21] process as in (11). κ is the mean-reversion speed of the baseline prepayment-rate process, $\tilde{\theta}$ is the mean-reversion level, σ_p is the volatility, and $\widetilde{W}_p(t)$ is the standard Brownian motion at time t under the real-world measure.

If we can find a constant λ_p for the risk-neutral prepayment adjustment, as in [13], such that

$$d\widetilde{W}_p(t) = dW_p(t) - \lambda_p dt, \tag{12}$$

we can derive the baseline prepayment-rate process under the risk-neutral measure, as follows:

$$dp_0(t) = \kappa\left(\theta_p - p_0(t)\right)dt + \sigma_p dW_p(t), \tag{13}$$

where $dW_p(t)$ is the standard Brownian motion at time t under the risk-neutral measure, and θ_p is defined as follows:

$$\theta_p \equiv \tilde{\theta}_p - \frac{\lambda_p \sigma_p}{\kappa}. \tag{14}$$

The baseline prepayment-rate process and the short-rate process are independent of each other; however, it can be seen that $f(\mathbf{x}(t))$ reflects the correlation between the interest rate and the prepayment rate through $spread(t)$. Note that the two prepayment components (10) and (11) are exponentially incorporated into $p(t)$ to ensure that $p(t)$ is non-negative.

3. Model Calibration and Empirical Results

3.1. Calibration. We primarily analyze the data of the KCMOs issued by KHFC, since it has only just begun to issue KPT MBSs, and the majority of MBSs issued by KHFC are of the KCMO type. KHFC began to issue KPT MBSs in November 2016. By the end of June 2018, it had issued only nine KPT MBSs. Given that investors usually require an additional spread for the initial market-entry premium for these new products, the KPT MBS price data are likely to be distorted. Therefore, we use the KCMO data for parameter calibration and applied the same parameters, calibrated on the prepayment rate, to compare the prices of KCMOs and KPT MBSs in a later analysis.

The model parametrization is shown in Table 3.

	Parameter	Estimate
Short-rate process	a	0.008430
	σ_r	0.005956
Regression parameters	β_1	3.961625
	β_2	93.266577
	β_3	-0.062259
	β_4	0.120272
	β_5	-0.002328
	β_6	0.000012
Baseline prepayment process	κ	2.197675
	$\tilde{\theta}_p$	0.014494
	σ_p	1.251989
	λ_p	-18.198175

Table 3. Baseline Parametrization

We used the time series of the Korean-Treasury yield curve for 2010∼2017 to calibrate the short-rate process, since KHFC uses the five-year Korean-Treasury bond yield as the index rate for its MBS coupon rate and mortgage rate. By fitting the caplet price formula of the short-rate process in (8) to the interest rate swap caplet prices, we obtained $a = 0.008430$ and $\sigma_r = 0.005956$. When we simulate the prepayment-rate process, we calculate the five-year short rate from the simulated short-rate process with the parameters in Table 3 and then compute $spread(t)$ in (10).

The calibration of the prepayment rate process is not simple because if we apply the calibration based on the empirical prepayment rate data, it is under the actual probability measure and does not match the risk-neutral measure generally applied to the pricing of financial instruments, as in this study. Therefore, recent studies on MBS pricing using stochastic prepayment rate models, such as [12] and [13], also have adopted this two-stage calibration approach that (i) calibrates the prepayment rate process with real prepayment data, and (ii) estimates the risk-neutral adjustment for the prepayment rate using the price history of MBSs with the calibrated prepayment rate process. This study follows this two-stage approach and first calibrates all the parameters in (10) and (11) with real prepayment data from KHFC, and then calibrates the risk-neutral adjustment of the prepayment, λ_p in (13) using the MBS-price time-series data.

We applied the prepayment data from 227 mortgage pools[2] issued by KHFC in 2010∼2017 to calibrate $f(\mathbf{x}(t))$ in (10) by minimizing the sum of the total squared

[2]Each mortgage pool has data for different durations. For example, KHFC MBS 2010-01, issued in January 2010, has more than 90 months of data. Therefore, the total number of observations is well over 10,000.

errors; that is,

$$\min_{\beta} \sum_{k=1}^{K} [p_k - \hat{p}_k(\beta)]^2$$

where p_k is the actual prepayment rate and $\hat{p}_k$ is the estimated prepayment rate with the regression coefficient βs computed by (10) and (11). Notice that we need to use the Korean-Treasury yield curve to compute (10) to reflect the refinancing motive. The coefficients for the refinancing component, β_2, and the aging component, β_4 and β_6, in Table 3 are estimated to be positive, which is in accordance with general intuition.

After deriving the time-series estimates of $f(\mathbf{x}(t))$, the difference between the prepayment data and the $f(\mathbf{x}(t))$ estimates, that is, $\ln p(t) - f(\mathbf{x}(t))$, serves as the data set for calibrating the baseline prepayment process, $p_0(t)$. Applying [10]'s method, we obtain the parameter values in Table 3, that is, we can rewrite (11) as

$$p_0(t_i) = \alpha_0 p_0(t_{i-1}) + \alpha_1 + \epsilon,$$

$$\alpha_0 \equiv e^{-\kappa(t_i - t_{i-1})}, \tag{15}$$

$$\alpha_1 = \tilde{\theta}_p(1 - e^{-\kappa(t_i - t_{i-1})}), \tag{16}$$

$$\epsilon \sim N\left(0, \sigma_p^2 \frac{1 - \exp^{-2\kappa(t_i - t_{i-1})}}{2\kappa}\right). \tag{17}$$

We can estimate α_0 and α_1, and obtain κ and $\tilde{\theta}_p$ using (15) and (16). σ_p is then estimated by computing the variance of the series of $(p_0(t_i) - \hat{\alpha}_0 p_0(t_{i-1}) + \hat{\alpha}_1)$ using (17).

To estimate λ_p, we need the KHFC MBS market price. Unfortunately, the trading volume of KHFC MBS in the Korean financial markets is too small to reflect the proper market valuation of the MBS. Instead, we apply the issuance data, which are fairly regularly priced by the initial buyers, so that λ_p presents the risk-neutral prepayment adjustment of the issuance price. Therefore, one can say that our analysis mostly explains the MBS issuance market in Korea.

Since the KHFC MBS consists of eight tranches traded in the market, eight different MBS prices exist for one mortgage pool. We take the I-4 tranche, which has a five-year maturity, for the representative tranche of the prepayment risk because it is the first tranche for which the call option can be exercised. Then, we assume the volume of the corresponding mortgage pool is the size of the weight of the I-4 tranche among the eight tranches. We look for the value of each λ_p that satisfies the prices of each I-4 tranche issued in 2016$\sim$2017 and compute the average to obtain $\lambda_p = -18.198175$.

Following Figure 3 shows the prepayment rate, i.e., the single monthly mortality rate of the MBSs studied in this paper. One can easily find that the prepayment rate series according to the loan age shows the shape of a log-normal function. The prepayment rate in the last about 10 months fluctuates much since the remaining balance of the mortgages is so small. Therefore, we only used the data for 72 months or less for the stability of the input data.

3.2. Empirical Demonstrations and Analysis. Table 4 shows the 8 KCMOs in the first half of 2018, investigated with the model of this study[3]. In the table, 'Amount'

[3]MBS 2018-01, MBS 2018-04, MBS 2018-10, and MBS 2018-12 are KPT MBSs, and thus they are excluded and valued later in Figure 6.

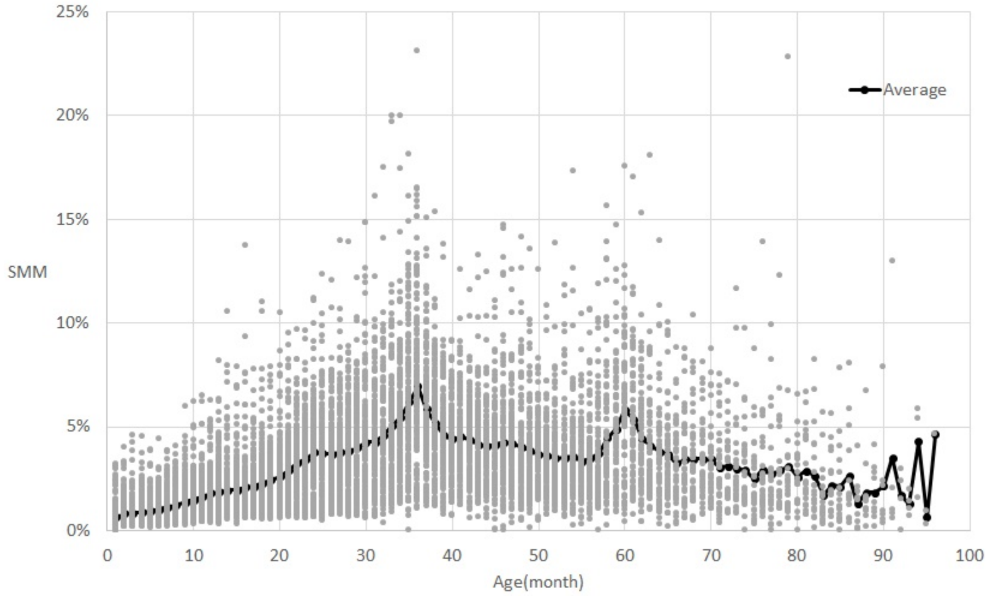

FIGURE 3. SMM with respect to Loan Age

is the issue amount of each I-4 tranche of KCMOs, and 'Total amount' is the sum of the issue amount of all tranches.

Product	Issue Date	Amount (1Bil KRW)	Coupon	Total amount (1Bil KRW)
MBS 2018-02	01/26	2,600	2.799%	10,536
MBS 2018-03	02/09	2,400	2.869%	9,626
MBS 2018-05	03/09	1,900	2.905%	7,609
MBS 2018-06	03/23	2,400	2.877%	9,968
MBS 2018-08	04/06	1,800	2.757%	7,239
MBS 2018-09	04/20	2,300	2.767%	9,345
MBS 2018-11	05/11	1,800	2.888%	7,343
MBS 2018-13	05/29	2,500	2.82%	9,882

TABLE 4. Issuance Details of I-4 tranches of KCMOs

Figure 4 presents the valuation result of the MBS prices in Table 4 applied to the prepayment-risk-neutral model of this study. The unit of prices in the figure is the relative price to the par-bond price computed as price $= V/$face value, and a price of 1 means that the value of MBS is the same as the original principal.

As shown in the figure, our model generates price-evaluation results that are fairly close to the real price data. Moreover, it can be seen that the estimated price is higher than the actual price in Q1 and vice versa for Q2. This may be because the investors in the Korean market expected the interest rates to rise in the second quarter. Figure 5 shows several series of Korean treasury-bond yields.

Now, we move to the comparative analysis of the KCMO MBS and KPT MBS prices, taking five KPT MBSs issued in the first half of 2018 as our analysis objectives. We also assume that the KCMO MBSs and the KPT MBSs have the similar

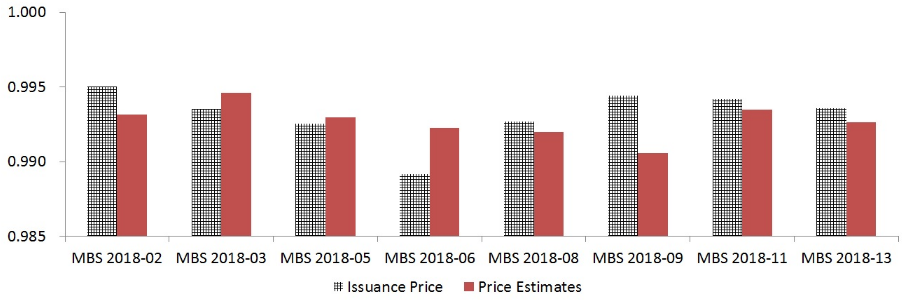

Figure 4. Price-Estimation Results

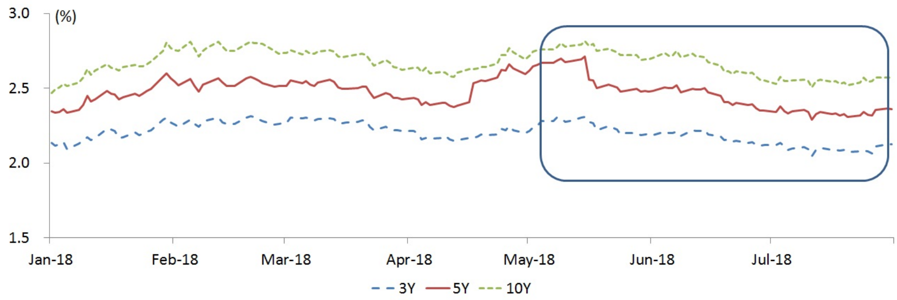

Figure 5. Korean Treasury-Bond Yield

underlying mortgage pool and same maturities and coupon rates. Figure 6 represents the estimated prices and fair-coupon spread between the two types of MBS.

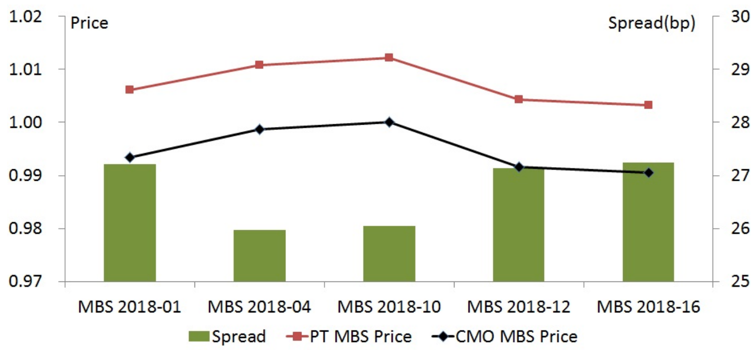

Figure 6. KCMO and KPT MBS Prices

The prices of the KPT MBSs are definitely higher than those of the KCMO MBSs, owing to the value of the call options embedded in the KCMO MBSs. The difference between the prices of the two MBSs is the call option value, around 1%. In addition, the fair-spread amounts are purely from the call-option value, determined by converting the option prices into spreads. The spread is the proper difference between the coupon rates of the two types of MBS when they have the same underlying mortgage pool and maturity. This is computed by dividing the difference between the prices of the two MBSs by the duration of each MBS. In Figure 6, those spreads are likely to be around 25∼28 basis points (BPs).

We now calculate comparative statistics on various parameters in Table 3 by deriving the prices of three MBSs, KHFC MBS 2018-10, KHFC MBS 2018-12, and KHFC

MBS 2018-16, which are the most recently issued. Figure 7 demonstrates the fair spreads between the coupon rates of the KCMO MBSs and KPT MBSs with respect to the interest-rate volatility.

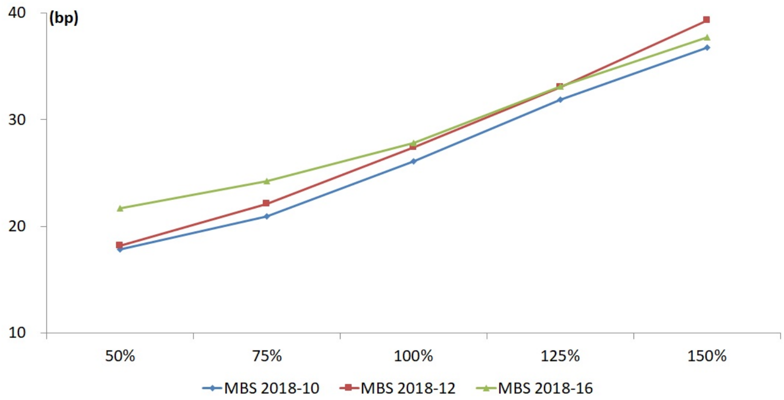

FIGURE 7. Fair Spreads with Respect to the Interest-Rate Volatility

The percent values on the x-axis are the values relative to the volatility, $\sigma_r = 0.005956$ in Table 3; i.e., 50% on the x-axis in Figure 7 means $\sigma_r = 0.5 * 0.005956$. One can easily confirm that a higher interest-rate volatility leads to a higher spread in Figure 7, since the callable bond price decreases with a higher interest-rate volatility.

We also present the changes in the fair spreads according to the change in the refinancing sensitivity, β_2, in Figure 8.

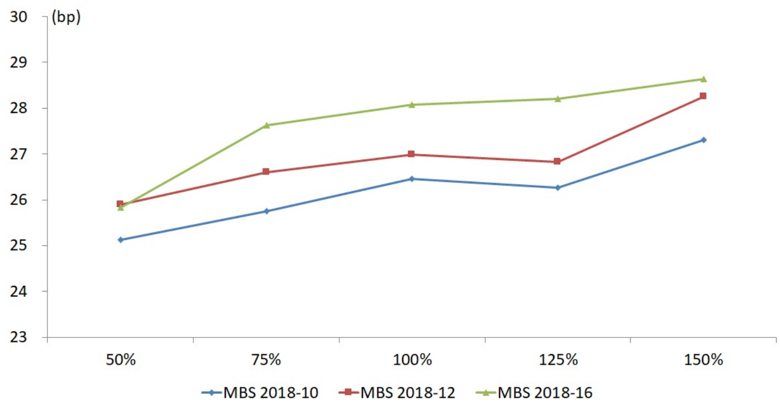

FIGURE 8. Fair Spreads with Respect to the Refinancing Sensitivity of the Prepayment

In addition, the percentage values on the x-axis denote the values relative to $\beta_2 = 93.266577$ in Table 3. If β_2 is large, the rate of change for the prepayment rate is large for a change in an interest rate of the same size; therefore, a borrower with a higher β_2 may be a refinance-sensitive borrower. As can be seen from the figure, the more sensitive borrowers in the underlying mortgage pool, the more the fair spread increases. This is because the cash flows from the mortgage pool are more volatile with more refinance-sensitive borrowers, resulting in the larger difference in the prices of KCMO MBSs and KPT MBSs. The risks of volatile cash flows are further amplified for KCMO MBSs, since the intermediary KHFC has the option of whether to pass through the cash from the borrowers.

The impact of the aging component of the prepayment rate on the fair spread is illustrated in Figure 9.

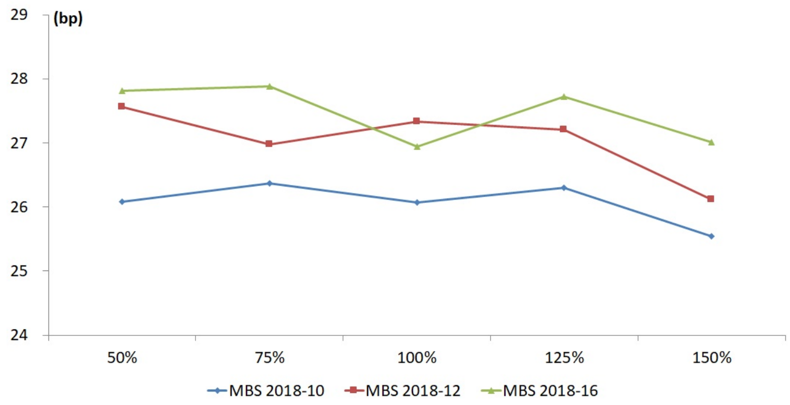

FIGURE 9. Fair Spreads with Respect to the Aging Sensitivity of the Prepayment

The values on the x-axis are the relative values of the aging sensitivity, $\beta_4 = 0.120272$ in Table 3. There is no obvious correlation in Figure 9. This is mostly because the aging motive in the prepayment-rate process is a predictable factor that increases monotonically without falling.

Figure 10 demonstrates the relation between the fair spreads and the long-term average of the baseline prepayment-rate process, $\tilde{\theta}_p$ in Table 3.

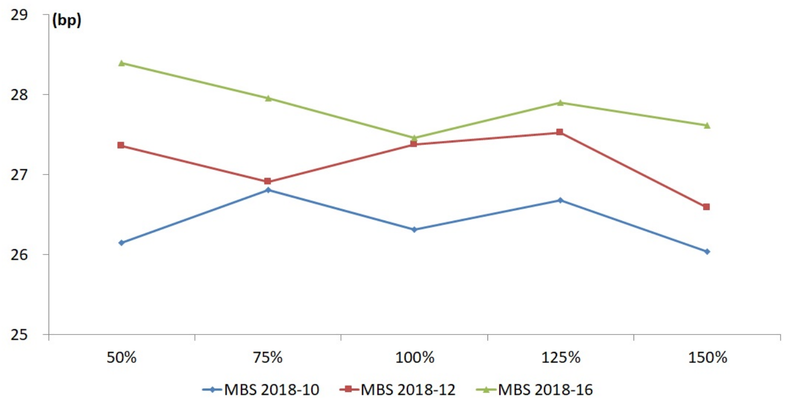

FIGURE 10. Fair Spreads with Respect to the Long-Term Average Prepayment Rates

The values on the x-axis denote the relative values of the long-term average of the baseline prepayment-rate process, $\tilde{\theta}_p = 0.0144944$ in Table 3. No clear correlation is apparent between the fair spreads and $\tilde{\theta}_p$; this is probably because the value itself is a long-term average after eliminating the effects of refinancing and the aging of the prepayment rate, which affects the cash-flow volatility less than the other factors.

Now, Figure 11 represents the changes in fair spreads with respect to the volatility level of the baseline prepayment-rate process, σ_p.

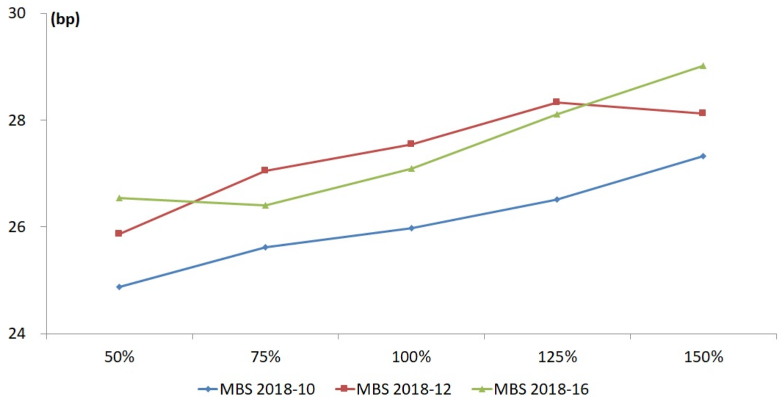

FIGURE 11. Fair Spreads with Respect to the Volatility of the Prepayment-Rate Process

The percent values on the x-axis are the relative percent levels of the volatility of the baseline prepayment-rate process in Table 3. A higher volatility in the prepayment rate exposes the investors to a greater cash-flow uncertainty, which leads to a higher price for the call option embedded in the KCMO MBSs, and thus, a higher fair spread, as shown in Figure 11.

4. Conclusion

In this study, we suggested a prepayment-risk-neutral pricing model for MBSs with a reduced-form prepayment rate model. We believe that this paper contributes, in particular, in the valuation of MBSs since it is hard to find a study investigating the value of a call option embedded in a MBS. We also investigated the implications in the pricing of the two types of MBS, KCMO and KPT MBS, and showed that the volatilities of the interest rate and the prepayment rate were the key factors determining the fair spread between the coupon rates of the two MBSs.

This study has limitations as in the following. The issuance data of MBSs is used for the analysis due to the lack of trading data of MBSs after issuance in Korea. Considering that the issuance prices of most financial products, including MBS, have an issuance premium on the initial issue, we admit that there may be some distortion in the price used in this paper. In addition, since the MBS studied in this paper has a simpler structure with only one tranche, due to the complex pricing of a multiple-tranche MBS, the pricing of multiple-tranche MBSs is definitely a promising further research topic.

We believe that this study can provide insight into the policies on MBSs in emerging countries where long-term bond markets are not well developed as Korea. If they want to securitize long-term mortgage loans by issuing a MBS with those long-term mortgages as its underlying pool, they may need to consider a structure of multiple tranches and call options, similar to the case of Korea.

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