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ON ORTHOGONAL REVERSE DERIVATIONS OF SEMIPRIME SEMIRINGS

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Abstract. In this paper, we introduce the notion of orthogonal reserve derivation on semiprime semirings. Some characterizations of semiprime semirimgs are obtained by means of orthogonal reverse derivations. We also investigate conditions for two reverse derivations on semiring to be orthogonal.

1. Introduction

The notion of semiring was first introduced in 1934 by H. S. Vandiver [7]. A semiring is an algebraic structure , consisting of a nonempty set S on which we have defined two associative binary operations addition and multiplication such that the multiplication is distributive over addition. The notion of rings with derivations is quite old and plays a significant role in the integration of analysis, algebraic geometry, and algebra. The study of derivations in rings though initiated long back, but got interested only after Posner [4] who 1957 established two very striking results on derivations in prime rings. The reverse derivations on semiprime rings has been studied by Samman and Alyamani [5]. Here the authors obtain some results of semiprime rings by reverse derivations. In this paper, we introduce the notion of orthogonal reserve derivation on semirings. Some characterizations of semiprime semirings are obtained by means of orthogonal reverse derivations. We also investigate conditions for two reverse derivations on semiring to be orthogonal.

2. Semirings

A *semiring* $(S, +, \cdot)$ is an algebraic system with a non-empty set S together with two binary operations "+" and " · " such that

(S1) $(S, +)$ is a semigroup,
(S2) $(S, \cdot)$ is a semigroup,
(S3) $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ for all $a, b \in S$.

Definition 2.1. ([2]) A semiring $(S, +, \cdot)$ is said to be *additively commutative* if $(S, +)$ is a commutative semigroup. It is said to be *multiplicatively commutative* if

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$(S,\cdot)$ is a commutative semigroup. It is said to be *commutative* if both $(S,+)$ and $(S,\cdot)$ are commutative.

Definition 2.2. ([2]) Let S be a semiring. An element a in S is said to be *additively left cancellative* if $a+b=a+c$ implies $b=c$, for every $b,c\in S$. It is said to be *additively right cancellative* if $b+a=c+a$ implies $b=c$. It is said to be *additively cancellative* if it is both left and right cancellative. A semiring S is said to be *additively cancellative* if all elements in S are additively cancellative.

Definition 2.3. ([2]) Let S be a semiring. Then

(i) S is said to be *prime* if $aSb=0$ implies $a=0$ or $b=0$ for all $a,b\in S$.
(ii) S is said to be *semiprime* if $aSa=0$ implies $a=0$, for all $a\in S$.
(iii) S is said to be 2-*torsion free* if $2a=0$ implies $a=0$ for all $a\in S$.

Definition 2.4. ([2]) Let S be a semiring. An additive mapping $d:S\to S$ is called a *derivation* if

$$d(xy)=d(x)y+xd(y)$$

for all $x,y\in S$.

Example 2.5. Let S be a semiring and $M_2(S)=\left\{\begin{pmatrix} a & 0\\ b & c\end{pmatrix}: a,b,c\in S\right\}$. Then $M_2(S)$ is a semiring. Define a map $d:M_2(S)\to M_2(S)$ by

$$d\begin{pmatrix} a & 0\\ b & c\end{pmatrix}=\begin{pmatrix} 0 & 0\\ b & 0\end{pmatrix}$$

Then d is a derivation on S.

3. Orthogonal reverse derivations of semiprime semirings

Throughout this paper, we assume that S is a semiring with additive identity 0 and addition is commutative.

Definition 3.1. Let S be a semiring. An additive mapping $d:S\to S$ is a *reverse derivation* if

$$d(xy)=d(y)x+yd(x)$$

for all $x,y\in S$ and a *Jordan derivation* if $d(x^2)=d(x)x+xd(x)$ for all $x\in S$.

Obviously, if S is commutative semiring, then both reverse derivation and derivation of S are the same. It can be easily seen that the reverse derivation is not a derivation, in general, but it is a Jordan derivation.

Example 3.2. Let S be a semiring and $M_3(S)=\left\{\begin{pmatrix} a & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & b\end{pmatrix}: a,b\in S\}\right\}$. Then $M_3(S)$ is a semiring. Define a map $d:M_3(S)\to M_3(S)$ by

$$d\begin{pmatrix} a & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & b\end{pmatrix}=\begin{pmatrix} 0 & a & 0\\ 0 & 0 & 0\\ 0 & b & 0\end{pmatrix}$$

Then d is not a reverse derivation on S but derivation.

Example 3.3. Let R be the real field and let $d : R \to R$ be a reverse derivation. Consider $M = M_{1\times 2}(R)$, where $M = M_{1\times 2}(R)$ is a matrix. Then it is clear that M is a semiring. Let $N = \{(x, x)|x \in R\} \subset M$. Then N is a subsemiring of M. Define a self-map $D : N \to N$ by $D((x, x)) = (d(x), d(x))$. Let $a = (x_1, x_1)$ and $b = (x_2, x_2)$. Then we have

$$\begin{aligned} D(ab) &= D((x_1, x_1)(x_2, x_2)) \\ &= D(x_1x_2, x_1x_2) \\ &= (d(x_2)x_1 + x_2d(x_1), d(x_2)x_1 + x_2d(x_1)) \\ &= (d(x_2)x_1, d(x_2)x_1) + (x_2d(x_1), x_2d(x_1)) \\ &= (d(x_2), d(x_2))(x_1, x_1) + (x_2, x_2)(d(x_1), d(x_1)) \\ &= D((x_2, x_2))a + bD((x_1, x_1)) \\ &= D(b)a + bD(a). \end{aligned}$$

Hence D is a reverse derivation on S.

Proposition 3.4. *Let d be a reverse derivation of S. Then the following conditions hold:*

(i) *If S is a semiring with characteristic 2, then d^2 is a derivation of S.*
(ii) *Let S is additively cancellative. If e is an idempotent element of S, then $ed(e)e = 0$.*

Proof. (i) Let d be a reverse derivation of S. Then we have

$$\begin{aligned} d^2(xy) &= d(d(xy)) = d(d(y)x + yd(x)) \\ &= d(x)d(y) + xd^2(y) + d^2(x)y + d(x)d(y) \\ &= d^2(x)y + xd^2(y), \end{aligned}$$

(ii) Let e is an idempotent element of S. Then we have $d(e) = d(ee) = d(e)e + ed(e)$. Multiplying by e in equation on left, we obtain $ed(e) = ed(e)e + eed(e) = ed(e)e + ed(e)e$. Since S is additively cancellative, we have $ed(e)e = 0$. □

Definition 3.5. Let S be a semiring. Then a semiring S is said to be *anticommutative* if $ab + ba = 0$, for all $a, b \in S$.

Proposition 3.6. *Let d be a reverse derivation of S. If S is anticommutative, then we have,*

$$d(xy^n) = \begin{cases} y^n d(x) & \text{if } n \text{ is even,} \\ y^{n-1}d(xy) & \text{if } n \text{ is odd.} \end{cases}$$

In particular,

$$d(x^n) = \begin{cases} x^{n-1}d(x) & \text{if } n \text{ is odd,} \\ 0 & \text{if } n \text{ is even.} \end{cases}$$

Proof. First, we consider that n is even. If $n = 2k$, $k \in \mathbb{N}$, then for $k = 1$, we have

$$\begin{aligned} d(xy^2) &= d(y^2)x + y^2d(x) \\ &= (d(y)y + yd(y))x + y^2d(x) \\ &= y^2d(x). \end{aligned}$$

Suppose that it hold for $k = m$. That is, $d(xy^{2m}) = y^{2m}d(x)$. Note that

$$\begin{aligned} d(xy^{2(m+1)}) &= d((xy^{2m})y^2) \\ &= d(y^2)xy^{2m} + y^2 d(xy^{2m}) \\ &= y^2 y^{2m} d(x) \\ &= y^{2(m+1)} d(x). \end{aligned}$$

Hence it holds for $k = m + 1$. Therefore $d(xy^{2k}) = y^{2k}d(x)$ for every positive integer k. Now, we consider that n is odd. If $n = 2k - 1$, $k \in \mathbb{N}$, then for $k = 1$, it is clear. Suppose that it hold for $k = l$. That is, $d(xy^{2l-1}) = y^{2l-2}d(xy)$. Note that

$$\begin{aligned} d(xy^{2l+1}) &= d((xy^{2l-1})y^2) \\ &= d(y^2)xy^{2l-1} + y^2 d(xy^{2l-1}) \\ &= y^2 y^{2l-2} d(xy) \\ &= y^{2l} d(xy). \end{aligned}$$

Hence it holds for $k = l + 1$. Therefore $d(xy^{2k-1}) = y^{2k-2}d(xy)$ for every positive integer l. This completes the proof.

□

Lemma 3.7. *Let S be an additively cancellative semiring and d_1, d_2 are reverse derivations such that $d_1 d_2$ is a derivation of S, then $d_1(x)d_2(y) + d_2(x)d_1(y) = 0$, for every $x, y \in S$.*

Proof. Since d_1, d_2 are reverse derivations, we have, for every $x, y \in S$,

$$\begin{aligned} d_1 d_2(xy) &= d_1(d_2(xy)) = d_1(d_2(y)x + yd_2(x)) \\ &= d_1(x)d_2(y) + xd_1(d_2(y)) + d_1(d_2(x))y + d_2(x)d_1(y). \end{aligned}$$

Also, $d_1 d_2$ is a derivation of S, we have

$$d_1 d_2(xy) = d_1 d_2(x)y + xd_1 d_2(y).$$

Since S is an additively cancellative semiring, we have $d_2(x)d_1(y) + d_1(x)d_2(y) = 0$, for every $x, y \in S$. □

Theorem 3.8. *Let S be a 2-torsion free additively cancellative semiprime semiring and d_1, d_2 are reverse derivations such that the following conditions are valid:*

(i) $d_1(x)d_2(x) = d_2(x)d_1(x)$, *for every* $x \in S$,

(ii) $d_1 d_2$ *is a derivation of* S.

Then at least one of d_1, d_2 is zero.

Proof. Since $d_1 d_2$ is a derivation and S is an additively cancellative semiring, by Lemma 3.7, we have

$$d_2(x)d_1(y) + d_1(x)d_2(y) = 0, \tag{1}$$

for every $x, y \in S$. Replacing y by x, we have

$$d_2(x)d_1(x) + d_1(x)d_2(x).$$

Using the commutativity of d_1 and d_2, we get $d_1(x)d_2(x) + d_1(x)d_2(x) = 0$, and so $2d_1(x)d_2(x) = 0$. Since S is 2-torsion free, we have $d_1(x)d_2(x) = 0$. Let $A = \{x \in S : d_1(x) \neq 0\}$. Then clearly $A \neq \phi$. Then it is easy to see that $d_2(x) = 0$ for every $x \in A$.

That is, multiplying by $d_2(x)$ on the left side of equation $d_1(x)d_2(x) = 0$, we have $d_2(x)d_1(x)d_2(x) = 0$. This implies $d_2(x) = 0$ since S is semiprime. Now, we have to prove that $d_2(x) = 0$, for every $x \in S/A$. Let $x \in S/A$. If $d_2(y) = 0$ in equation (1), we have $d_2(x)d_1(y) = 0$. If $y \in A$, then we obtain $d_1(y) \neq 0$. Multiplying by $d_2(x)$ on the right side of the equation $d_2(x)d_1(y) = 0$, we have $d_2(x)d_1(y)d_2(x) = 0$. Since S is semiprime, we get $d_2(x) = 0$, for every $x \in S/A$. Thus $d_2(x) = 0$, for every $x \in S/A$ and $x \in A$. This implies that $d_2(x) = 0$, for every $x \in S$. Hence $d_2 = 0$. This completes the proof.

□

PROPOSITION 3.9. *Let d be a reverse derivation of a prime semiring S and let $a \in S$. If $ad(x) = 0$, for every $x \in S$, then $a = 0$ or d is zero.*

Proof. Let $ad(x) = 0$, for every $x \in S$. Then replacing x by xy, we have

$$0 = ad(xy) = a(d(y)x + yd(x)) = ad(y)x + ayd(x) = ayd(x),$$

for every $x, y \in S$. Since S is a prime semiring, if $d(x) \neq 0$, for some $x \in S$, then $a = 0$. □

Now, we give the derivation of orthogonality of two reverse derivations.

DEFINITION 3.10. Let d and g be two reverse derivations on S. Then d and g are said to be *orthogonal* if $d(x)g(y) = 0 = g(y)d(x)$, for all $x, y \in S$.

Consider $S = S_1 \times S_2$, where S_1 and S_2 are semirings. The addition and multiplication on S are defined as follows,

$$(a, b) + (c, d) = (a + c, b + d) \text{ and } \quad (a, b)(c, d) = (ac, bd),$$

for all $a, c \in S_1$ and $b, d \in S_2$. Under these operations, S is a semiring.

EXAMPLE 3.11. Let $K = \{0, a, b, c\}$ be a set in which "+" and "·" are defined by

$+$	0	a	b	c
0	0	a	b	c
a	a	a	c	c
b	b	c	c	c
c	c	c	c	c

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	c	c
c	0	c	c	c

Then it is easy to check that $(S, +, \cdot)$ is a semiring. Define a self-map $d : S \to S$ by

$$d(x) = \begin{cases} 0 & \text{if } x = 0 \\ c & \text{if } x = a, b, c \end{cases}$$

and define a self-map $g : S \to S$ by

$$g(x) = \begin{cases} 0 & \text{if } x = 0 \\ b & \text{if } x = a \\ c & \text{if } x = b, c \end{cases}$$

Then d and g are reverse derivations on the semiring S.

Let $S_1 = S \times S$. Define a self-map $d_1 : S_1 \to S_1$ by $d_1(x, y) = (d(x), 0)$ and $g_1 : S_1 \to S_1$ by $g_1(x, y) = (0, g(y))$. Since

$$\begin{aligned} d_1((x,y)(u,v)) &= d_1(xu, uv) = (d(xu), 0) \\ &= (d(u)x + ud(x), 0 + 0) \\ &= (d(u)x, 0y) + (ud(x), v0) \\ &= (d(u), 0)(x, y) + (u, v)(d(x), 0) \\ &= d_1(u, v)(x, y) + (u, v)d_1(x, y) \end{aligned}$$

then d_1 is a reverse derivation on S_1. Similarly, g_1 is a reverse derivation on S_1. Furthermore,

$$\begin{aligned} d_1(x,y)g_1(u,v) &= (d(x), 0)(0, g(v)) \\ &= (0, 0) \\ &= (0, g(v))(d(x), 0) \\ &= g_1(u, v)d_1(x, y) \end{aligned}$$

Then we know that d_1 and g_1 are orthogonal reverse derivations on S_1.

Lemma 3.12. *Let S be a 2-torsion free semiprime semiring and $a, b \in S$. Then the following conditions are equivalent:*

(i) $asb = 0$,
(ii) $bsa = 0$,
(iii) $asb + bsa = 0$, *for every* $s \in S$.

If one of these conditions is fulfilled, then $ab = ba = 0$.

Proof. (i) ⇒ (ii): Let $asb = 0$, for every $a, b, s \in S$. Multiplying by bs on left side and multiplying by sa on right side, we have $(bsa)s(bsa) = 0$. Since S is semiprime, we have $bsa = 0$.

(ii) ⇒ (iii): Let $bsa = 0$. Multiplying by as on left side and multiplying by sb, on right side we have $(asb)s(asb) = 0$. Since S is semiprime, we have $asb = 0$, which implies $asb + bsa = 0$.

(iii) ⇒ (i): Let $asb + bsa = 0$, for every $s, a, b \in S$. Multiplying by bs on left side, we have $bs(asb) + bs(bsa) = 0$. Also, multiplying by as on left side, we get

$$(asb)s(asb) + (asb)s(bsa) = 0. \tag{2}$$

Multiplying by sa on right side, we have $(asb)sa + (bsa)sa = 0$. Also, multiplying by sb, on right side, we get

$$(asb)s(asb) + (bsa)s(asb) = 0. \tag{3}$$

Adding equation (2) to (3) and using (3), we have $2((asb)s(asb)) = 0$. Since S is 2-torsion free and S is semiprime, we have $asb = 0$, for all $x \in S$.

Let $asb = 0$. Multiplying by b on left side and multiplying by a on right side, we have $(ba)s(ba) = 0$. Since S is semiprime, we have $ba = 0$. Similarly, from $bsa = 0$, we can prove that $ab = 0$. □

Lemma 3.13. *Let S be a 2-torsion free semiprime semiring and let d and g be additive mappings of S into itself, satisfying $d(x)sg(x) = 0$, for any $s \in S$. Then $d(x)sg(y) = 0$, for every $x, y \in S$.*

Proof. Suppose that $d(x)sg(x) = 0$, for any $x, s \in S$. Replacing x by $x+y$, we have

$$\begin{aligned} 0 &= d(x+y)sg(x+y) \\ &= d(x)sg(x) + d(x)sg(y) + d(y)sg(x) + d(y)sg(y) \\ &= d(x)sg(y) + d(y)sg(x). \end{aligned}$$

Hence we have

$$0 = d(x)sg(y) + d(y)sg(x).$$

Multiplying by $d(x)sg(y)$ on the left side of this equation, then we have

$$0 = (d(x)sg(y))s_1(d(x)sg(y)) + (d(x)sg(y))s_1(d(y)sg(x))$$

for any $s_1 \in S$. By Lemma 3.12, we have $(d(x)sg(y))s_1(d(x)sg(y)) = 0$. Since S is semiprime, $d(x)sg(y) = 0$, for every $x, y \in S$. □

Theorem 3.14. *Let S be a 2-torsion free semiprime semiring and let d and g be reverse derivations. Then*

$$d(x)g(y) + g(x)d(y) = 0, \tag{4}$$

for all $x, y \in S$ if and only if d and g are orthogonal.

Proof. Suppose that $d(x)g(y)+g(x)d(y) = 0$, for every $x, y \in S$. Replacing y by xy in (4)

$$\begin{aligned} 0 &= d(x)g(xy) + g(x)d(xy) \\ &= d(x)(g(y)x + yg(x)) + g(x)(d(y)x + yd(x)) \\ &= (d(x)g(y) + g(x)d(y))x + d(x)yg(x) + g(x)yd(x). \end{aligned}$$

By hypothesis, $d(x)yg(x)+g(x)yd(x) = 0$, and so by Lemma 3.12, we have $d(x)yg(x) = 0 = g(x)yd(x)$, for every $x, y \in S$. Hence, by Lemma 3.13, we get $d(x)yg(z) = 0 = g(x)yd(z)$, for any $x, y, z \in S$. This proves that d and g are orthogonal.

Conversely, assume that d and g are othogonal.Then we have $d(x)g(y) = 0 = g(x)d(y)$, which implies that $d(x)g(y) + g(x)d(y) = 0$, for all $x, y \in S$. □

Remark. Suppose that d and g are reverse derivations of a semiring S. Then the following identities are immediate from the definition of reverse derivations.

$$\begin{aligned} (dg)(xy) &= d(g(xy)) = d(g(y)x + yg(x)) \\ &= (dg)(x)y + d(x)g(y) + g(x)d(y) + x(dg)(y), \end{aligned} \tag{5}$$

for any $x, y \in S$.

Similarly, we have

$$\begin{aligned} (gd)(xy) &= g(d(xy)) = g(d(y)x + yd(x)) \\ &= (gd)(x)y + g(x)d(y) + d(x)g(y) + x(gd)(y), \end{aligned} \tag{6}$$

for any $x, y \in S$.

The following theorem gives a few criteria on the orthogonality of reverse derivations.

Theorem 3.15. *Let S be a 2-torsion free semiprime semiring and let d and g be reverse derivations. Then d and g are orthogonal if and only if $dg = 0$.*

Proof. Suppose that $dg = 0$. Then by using the identity (5) above, we obtain

$$d(x)g(y) + g(x)d(y) = 0,$$

for every $x, y \in S$. Therefore, by Theorem 3.14, d and g are orthogonal.

Conversely, since d and g are orthogonal, we have $d(x)yg(z) = 0$ for every $x, y, z \in S$. Hence we get

$$\begin{aligned} 0 &= d(d(x)yg(z)) = d(yg(z))d(x) + yg(z)d(d(x)) \\ &= (dg)(z)yd(x) + g(z)d(y)d(x) + yg(z)d(d(x)) \\ &= (dg)(z)yd(x). \end{aligned}$$

Replacing x by $g(z)$, we have $(dg)(z)y(dg)(z) = 0$, for any $z \in S$. Since S is semiprime, we obtain $(dg)(z) = 0$, for every $z \in S$, that is, $dg = 0$. □

Theorem 3.16. *Let S be a 2-torsion free semiprime semiring and let d and g be reverse derivations. Then d and g are orthogonal if and only if $dg + gd = 0$.*

Proof. Suppose that $dg + gd = 0$. Then we have,

$$\begin{aligned} 0 &= (dg + gd)(xy) \\ &= (dg)(x)y + d(x)g(y) + g(x)d(y) + x(dg)(y) \\ &\quad + (gd)(x)y + g(x)d(x) + d(x)g(y) + x(gd)(y) \\ &= (dg + gd)(x)y + 2d(x)g(y) + 2g(x)d(y) + x((dg)(y) + (gd)(y)), \end{aligned}$$

for every $x, y \in S$. Since S is 2-torsion free, we obtain $d(x)g(y) + g(x)d(y) = 0$, and so by Theorem 3.14, d and g are orthogonal.

Conversely, let d and g be orthogonal reverse derivations. By Theorem 3.15, we have $dg = gd = 0$. Hence $dg + gd = 0$. □

Theorem 3.17. *Let S be a 2-torsion free semiprime semiring and let d and g be reverse derivations. Then d and g are orthogonal if and only if dg is a derivation of S.*

Proof. Suppose that dg is a derivation on S. Then we have

$$(dg)(xy) = (dg)(x)y + x(dg(y)).$$

Comparing this expression with (5), we obtain $d(x)g(y) + g(x)d(y) = 0$, and so by Theorem 3.14, d and g are orthogonal.

Conversely, if d and g are orthogonal, by Theorem 3.15, $dg = 0$. Thus dg is a derivation of S. □

Corollary 3.18. *Let S be a 2-torsion free semiprime semiring and let d and g be orthogonal reverse derivation of S. Then $d = 0$ or $g = 0$.*

Theorem 3.19. *Let S be a 2-torsion free semiprime semiring and let d be a reverse derivation of S. If d^2 is a derivation of S, then $d = 0$.*

Proof. Since d^2 is a derivation of S, we have $d^2(xy) = d^2(x)y + xd^2(y)$ and

$$\begin{aligned} d^2(xy) &= d(d(xy)) = d(d(y)x + yd(x)) \\ &= d(x)d(y) + d(x)d(y) + xd^2(y) + d^2(x)y \\ &= 2d(x)d(y) + d^2(x)y + xd^2(y). \end{aligned}$$

Hence we have $2d(x)d(y) = 0$. Since S is semiprime, we get $d(x)d(y) = 0$, for any $x, y \in S$. Replacing x by sx, we have

$$\begin{aligned} 0 &= d(sx)d(y) = (d(x)s + xd(s))d(y) \\ &= d(x)sd(y). \end{aligned}$$

Replacing y by $x + y$, we have

$$\begin{aligned} 0 &= d(x)sd(x + y) \\ &= d(x)s(d(x) + d(y)) \\ &= d(x)sd(x) + d(x)sd(y) = d(x)sd(x). \end{aligned}$$

Since S is 2-torsion free, we obtain $d(x) = 0$, for any $x \in S$, i.e., $d = 0$. $\square$

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