Korean J. Math. **31** (2023), No. 4, pp. 391–403
https://dx.doi.org/10.11568/kjm.2023.31.4.391

COMMON FIXED POINT RESULTS VIA F-CONTRACTION ON C^*-ALGEBRA VALUED METRIC SPACES

Shivani Kukreti, Gopi Prasad*, and Ramesh Chandra Dimri

Abstract. In this work, we establish common fixed point results by utilizing a variant of F-contraction in the framework of C^*-algebra valued metric spaces. We utilize E.A. and C.L.R. property possessed by the mappings to prove common fixed point results in the same metric settings. To validate the applicability of these common fixed point results, we provide illustrative examples too.

1. Introduction

Fixed point theory is an extensive field of research that has been actively explored for over a century. In the realm of metric fixed point theory, Banach contraction principle [1] (Bcp) demonstrated that every contraction mapping has a unique fixed point in a complete metric space. This principle has seen numerous extensions and generalizations in diverse directions, but keeping in view of requirement of this research work, we precisely refer [2–11].

In 2012, Wardwoski [12] introduced F-contractions as a novel generalization of the Bcp. His work included a comparative analysis of the convergence rates between the F-contraction principle and the Bcp. Subsequently, Dung and Wardwoski [13] proposed a weaker version of the F-contraction and established fixed point results. F-contractions have been studied in various settings, under single-valued and set-valued contractions along with partially ordered and graphic structures see for instance ([14–16])

On an another point of note in 2014 Ma et al. [17] introduced the notion of C^*-algebra valued metric spaces providing a more versatile framework than traditional metric spaces by replacing the range set with a unital C^*-algebra. This research work has sparked a cascade of research efforts, with subsequent studies by Ma et al. [18], Ege and Alaca [19], Mlaike et al. [20] and Maheswari et al. [21] contributing to the growing body of knowledge (see, [22–25]). These works not only establish fundamental results within C^*-algebra valued metric spaces but also demonstrate the practical applications of these spaces in solving problems related to fixed point theory and

Received September 17, 2023. Revised November 8, 2023. Accepted November 9, 2023.
2010 Mathematics Subject Classification: 47H10, 54H25.
Key words and phrases: Common fixed point, F-contraction, weakly compatible mappings.
$*$ Corresponding author.

integral-type operators.

Furthermore, the concept of C^*-algebra valued metric spaces, introduced by Ma et al. [17], expanded the scope of metric spaces. In this context, Qiaoling et al. [26] introduced compatible and weakly compatible mappings and investigated common and coincidence fixed points for two weakly compatible mappings, contributing to the development of this field. As researchers continue to delve into this captivating domain, the potential for new insights and applications in mathematics and related fields remains a compelling driving force behind the ongoing exploration of C^*-algebra valued metric spaces.

2. Preliminaries

Throughout this paper, we consistently represent $\mathcal{A}$ as an unital C^*-algebra, characterized by the presence of a unity element denoted as I, and equipped with a linear involution $*$. In this algebra, for any elements a and b belonging to $\mathcal{A}$, it holds that $(ab)^* = a^*b^*$, and an essential property is that $(a)^{**} = a$. Additionally, we identify positive elements within $\mathcal{A}$ by the notation $0_{\mathcal{A}} \leq a$, with $0_{\mathcal{A}}$ representing the zero element belonging to $\mathcal{A}$. The partial ordering relation on $\mathcal{A}$ can be precisely defined as $a \preccurlyeq b$ if and only if $0_{\mathcal{A}} \preccurlyeq b - a$.

A Banach $*$-algebra is called C^*-algebra when it holds the assumption $||a * a|| = ||a||^2$ for all $a \in \mathcal{A}$. In this context, $\mathcal{A}^+$ denotes the set $a \in \mathcal{A} : a \succeq 0_{\mathcal{A}}$, signifying those elements of $\mathcal{A}$ that are greater than or equal to $0_{\mathcal{A}}$. This notation follows the conventions established by Ma et al. in their prior work.

Definition 2.1. [17] Let E be a non empty set. A mapping $\phi : E \times E \to \mathcal{A}$ is called a C^*-algebra valued metric on E if it satisfies the following for all $a, b, c \in E$:
(i) $\phi(a, b) \preccurlyeq 0_{\mathcal{A}}$ and $\phi(a, b) = 0_{\mathcal{A}}$ if and only if $a = b$
(ii) $\phi(a, b) = \phi(b, a)$
(iii) $\phi(a, c) \preccurlyeq \phi(a, b) + \phi(b, c)$.
The triplet $(E, \mathcal{A}, \phi)$ is called C^*-algebra valued metric space.

Definition 2.2. [17] If for any $\epsilon > 0$, a sequence $\{a_n\}$ in $(E, \mathcal{A}, \phi)$ satisfy for all $n, m > k$, $\|\phi(a_n, a_m)\| < \epsilon$ is called Cauchy with respect to $\mathcal{A}$.

Definition 2.3. [17] If for any $\epsilon > 0$, a sequence $\{a_n\}$ in $(E, \mathcal{A}, \phi)$ satisfy for all $n > k$, $\|\phi(a_n, a)\| < \epsilon$ is said to be convergent with respect to $\mathcal{A}$.

Definition 2.4. [17] Convergence of every Cauchy sequence in $\mathcal{A}$ implies a complete C^*-algebra valued metric space.

E.A. property by Aamri and El Moutawakil [27] was introduced in metric space to relax the assumption of completeness with the closeness of the space.

Definition 2.5. [27] The pair of mappings (g, h) satisfies E.A. property if there exists a sequence $\{a_n\}$ in E such that $\lim_{n\to\infty} ga_n = \lim_{n\to\infty} ha_n = g\delta$ for some $\delta \in E$.

Sintunavarat and Kumam introduced the CLR property in the context of metric spaces as a means to address the compactness or closeness of the space.

DEFINITION 2.6. [28] The pair of self mappings, (g, h) satisfies CLR property if there exists a sequence $\{a_n\}$ in E such that

$$\lim_{n\to\infty} ga_n = \lim_{n\to\infty} ha_n = g\delta$$

for some $\delta \in E$.

DEFINITION 2.7. [29,30] Let g and h be self-mappings defined on a non-empty set $\mathcal{X}$. Then
(a) $a \in E$ is a coincidence point of g and h if $ga = ha$
(b) if $b \in E$ is any point so that $b = ga = ha$, then b is a point of coincidence of the mappings g and h.
(c) the pair (g, h) is weakly compatible if g and h commute at their coincidence points, that is, $h(ga) = g(ha)$, whenever $ha = ga$.

3. Main Results

In this section, we present existence and uniqueness of common fixed point results for a pair of weakly contractive self-mappings. Finally we demonstrated findings of main results by some illustrative examples.

THEOREM 3.1. *Let $(E, \mathcal{A}, \phi)$ be a C^*-algebra valued metric space and two pairs (M, g) and (N, h) of weakly compatible self mappings with $M(E) \subseteq h(E)$ and $N(E) \subseteq h(E)$, either the pair (M, g) or (N, h) satisfies E.A property and there exist $F : \mathcal{A}^+ \to \mathcal{A}$ be a continuous and strictly non- decreasing mapping satisfying for every $\alpha, \beta \in E$, $a_F, b_F, c_F, d_F, e_F \in \mathcal{A}$ with $\|a_F\|, \|b_F\|, \|c_F\|, \|d_F\|, \|e_F\| < 1$ and $\tau > 0$ such that*

$$\begin{aligned} \tau + F(\phi(M\xi, N\gamma)) \preccurlyeq\ & F(max\{a_F\phi(g\xi, M\xi), b_F\phi(h\gamma, N\gamma), c_F\phi(g\xi, h\gamma), \\ & d_F\phi(g\xi, N\gamma), e_F\phi(h\gamma, M\xi)\}) \end{aligned} \tag{3.1}$$

then the self mapping $M, N, g,$ and h have a unique common fixed point in E if either range subspace of $g(E)$ or $h(E)$ is a closed subspace in E.

Proof. Let the pair (N, h) satisfies E.A. property, then there exists a sequence $\{\xi_n\}$ in E such that

$$\lim_{n\to\infty} N(\xi_n) = \lim_{n\to\infty} h(\xi_n) = \delta$$

for some $\delta \in E$.
Further, $N(E) \subseteq g(E)$. Therefore, there exist a sequence $\{\gamma_n\}$ in E such that $N(\xi_n) = g(\gamma_n)$. Thus,

$$\lim_{n\to\infty} g(\gamma_n) = \delta = \lim_{n\to\infty} N(\xi_n)$$

We claim that $\lim_{n\to\infty} M(\gamma_n) = \delta$. On contrary, suppose that $\lim_{n\to\infty} M(\xi_n) = \delta_1 \neq \delta$. Substituting $\xi = \gamma_n$ and $\gamma = \xi_n$ in (3.1) we have,

$$\begin{aligned} \tau + F(\phi(M\gamma_n, N\xi_n)) \preccurlyeq\ & F(max\{a_F\phi(g\gamma_n, M\gamma_n), b_F\phi(h\xi_n, N\xi_n), c_F\phi(g\gamma_n, h\xi_n), \\ & d_F\phi(g\gamma_n, N\xi_n), e_F\phi(h\xi_n, M\gamma_n)\}). \end{aligned} \tag{3.2}$$

Taking limit as $n \to \infty$ in (3.2), we have

$$\tau + F(\phi(\delta_1, \delta)) \preccurlyeq F(max\{a_F\phi(\delta, \delta_1), b_F\phi(\delta, \delta), c_F\phi(\delta, \delta), d_F\phi(\delta, \delta), e_F\phi(\delta, \delta_1)\})$$

We have either $\phi(\delta_1, \delta) \preccurlyeq \{a_F\phi(\delta, \delta_1)$ or $\phi(\delta_1, \delta) \preccurlyeq e_F\phi(\delta, \delta_1)$, which is a contradiction. Thus $\delta_1 = \delta$, that is,

$$\lim_{n\to\infty} M(\gamma_n) = \lim_{n\to\infty} N(\xi_n) = \delta.$$

Let the range space of $f(E)$ forms a closed subspace within E, and there exists a $\nu \in E$ such that $g\nu = \delta$, we can then deduce the following:

$$\lim_{n\to\infty} M(\gamma_n) = \lim_{n\to\infty} N(\xi_n) = \lim_{n\to\infty} h(\xi_n) = \lim_{n\to\infty} g(\gamma_n) = \delta = g\nu.$$

We claim that $M\nu = g\nu$. Substituting $\xi = \nu$ and $\gamma = \xi_n$ in (3.1), we have

$$\begin{aligned}\tau + F(\phi(M\nu, M\xi_n)) \preccurlyeq\ & F(max\{a_F\phi(g\nu, M\nu), b_F\phi(h\xi_n, N\xi_n), c_F\phi(g\nu, h\xi_n), \\ & d_F\phi(g\nu, N\xi_n), e_F\phi(h\xi_n, M\nu)\}).\end{aligned} \tag{3.3}$$

Taking limit as $n \to \infty$ in (3.3), we have

$$\tau + F(\phi(M\nu, \delta)) \preccurlyeq F(max\{a_F\phi(\delta, M\nu), b_F\phi(\delta, \delta), c_F\phi(\delta, \delta), d_F\phi(\delta, \delta), e_F\phi(\delta, M\nu)\}),$$

that is,

$$\tau + F(\phi(M\nu, \delta)) \preccurlyeq F(max\{a_F\phi(\delta, M\nu), 0_{\mathcal{A}}, e_F\phi(\delta, M\nu)\}).$$

So, we have either $\phi(M\nu, \delta) \preccurlyeq a_F\phi(\delta, M\nu)$ or $\phi(M\nu, \delta) \preccurlyeq e_F\phi(\delta, M\nu)$, which is a contradiction. Thus $M\nu = \delta = g\nu$, that is, the coincidence point of the pair (M, g). Now, the pair (M, g) is weakly compatible, that is, $Mg\ \nu = gM\ \nu$ or $M\delta = g\delta$. Since $M(E) \subseteq h(E)$. Therefore, there exists $\Gamma \in E$ such that $M\nu = h\Gamma = g\nu = \delta$.
We have to prove that Γ is a coincidence point of pair (N, h), that is, $N\Gamma = h\Gamma = \delta$. Substituting $\xi = \nu$ and $\gamma = \Gamma$ in (3.1), we have

$$\begin{aligned}\tau + F(\phi(M\nu, N\Gamma)) &\preccurlyeq F(max\{a_F\phi(g\nu, M\nu), b_F\phi(h\Gamma, N\Gamma), c_F\phi(g\nu, h\Gamma), \\ &\qquad d_F\phi(g\nu, N\Gamma), e_F\phi(h\Gamma, M\nu)\}) \\ &= F\left(max\{a_F\phi(\delta, \delta), b_F\phi(\delta, N\Gamma), c_F\phi(\delta, \delta), d_F\phi(\delta, N\Gamma), e_F\phi(\delta, \delta)\}\right) \\ &= F(max\{0_{\mathcal{A}},\ b_F\phi(\delta, N\Gamma),\ d_F\phi(\delta, N\Gamma)\}).\end{aligned}$$

So, have either $\phi(N\Gamma, \delta) \preccurlyeq b_F\phi(\delta, N\Gamma)$ or $\phi(N\Gamma, \delta) \preccurlyeq d_F\phi(\delta, N\Gamma)$, which is a contradiction. Thus $N\Gamma = h\Gamma = \delta$ and Γ is coincidence point of N and h.
Furthermore, the pair (N, h) exhibits weak compatibility implies that $Nh\ \Gamma = hN\ \Gamma$ or equivalently $N\delta = h\delta$. As a result, the element δ serves as a common coincidence point for the mappings M, N, g, and h.
Now, we have to prove that δ is common fixed point of M, N, g and h. Substituting $\xi = \nu$ and $\gamma = \delta$ in (3.1), we have

$$\begin{aligned}\tau + F(\phi(M\nu, N\delta)) &\preccurlyeq F(max\{a_F\phi(g\nu, M\nu), b_F\phi(h\delta, N\delta), c_F\phi(g\nu, h\delta), \\ &\qquad d_F\phi(g\nu, N\delta), e_F\phi(h\delta, M\nu)\}),\end{aligned}$$

that is,

$$\begin{aligned}\tau + F(\phi(\delta, M\delta)) &\preccurlyeq F(max\{\{a_F\phi(\delta, \delta), b_F\phi(\delta, N\delta), c_F\phi(\delta, \delta), d_F\phi(\delta, N\delta), e_F\phi(\delta, \delta)\}), \\ &= F(max\{0_{\mathcal{A}},\ b_F\phi(\delta, N\delta),\ d_F\phi(\delta, N\delta)\}).\end{aligned}$$

So, we have either $\phi(M\delta, \delta) \preccurlyeq b_F\phi(\delta, N\delta)$ or $\phi(M\delta, \delta) \preccurlyeq d_F\phi(\delta, N\delta)$ which contradicts. Hence $N\delta = \delta$. Thus, $M\delta = N\delta = g\delta = h\delta = \delta$.
We will arrive at the same result if we make the assumption that the range space of $h(E)$ is closed in E, and the pair (M, g) satisfies the E. A. property.
Now let Ω is another common fixed point of M, N, g and h. Substituting $\xi = \Omega$ and $\gamma = \delta$ in (3.1), we have

$$\begin{aligned}
&\tau + F(\phi(\Omega, \delta))\\
&= \tau + F(\phi(M\Omega, N\delta))\\
&\preccurlyeq F(max\{a_F\phi(g\Omega, M\Omega), b_F\phi(h\delta, N\delta), c_F\phi(g\Omega, h\delta), d_F\phi(g\Omega, N\delta), e_F\phi(h\delta, M\Omega)\})\\
&= F(max\{a_F\phi(\Omega, \Omega), b_F\phi(\delta, \delta), c_F\phi(\Omega, \delta), d_F\phi(\Omega, \delta), e_F\phi(\delta, \Omega)\})\\
&= F(max\{0_{\mathcal{A}}, c_F\phi(\Omega, \delta), d_F\phi(\Omega, \delta), e_F\phi(\delta, \Omega)\}).
\end{aligned}$$

We have either $\phi(\Omega, \delta) \preccurlyeq c_F\phi(\Omega, \delta)$ or $\phi(\Omega, \delta) \preccurlyeq d_F\phi(\Omega, \delta)$ or $\phi(\Omega, \delta) \preccurlyeq e_F\phi(\Omega, \delta)$ which contradicts. Thus $\Omega = \delta$. Hence δ is unique common fixed point of M, N, g and h. □

THEOREM 3.2. *Let $(E, \mathcal{A}, \phi)$ be a C^*-algebra valued metric space and two pairs (M, g) and (N, h) of weakly compatible self mappings with $M(E) \subseteq h(E)$ and $N(E) \subseteq h(E)$, either of the pairs (M, g) or (N, h) satisfies CLR property and there exist $F : \mathcal{A}^+ \to \mathcal{A}$ be a continuous and strictly non- decreasing mapping satisfying for every $\alpha, \beta \in E, a_F, b_F, c_F, d_F, e_F \in \mathcal{A}$ with $\|a_F\|, \|b_F\|, \|c_F\|, \|d_F\|, \|e_F\| < 1$ and $\tau > 0$,*

$$\begin{aligned}
\tau + F(\phi(M\xi, N\gamma)) \preccurlyeq\ & F(max\{a_F\phi(g\xi, M\xi), b_F\phi(h\gamma, N\gamma), c_F\phi(g\xi, h\gamma),\\
& d_F\phi(g\xi, N\gamma), e_F\phi(h\gamma, M\xi)\}).
\end{aligned} \tag{3.4}$$

Then the self mapping M, N, g, and h have a unique common fixed point in E.

Proof. Let the pair (N, h) satisfies CLR_B property, then there exists a sequence $\{\xi_n\}$ in E such that

$$\lim_{n\to\infty} N(\xi_n) = \lim_{n\to\infty} h(\xi_n) = \delta$$

for some $\delta \in E$.
Since, $N(E) \subseteq g(E)$. Therefore, there exists $\nu \in E$, such that $N\xi = g\nu$. Claiming that $M\nu = g\nu = \delta$. Substituting $\xi = \nu$ and $\gamma = \xi_n$ in (3.4), we have

$$\begin{aligned}
\tau + F(\phi(M\nu, N\xi_n)) \preccurlyeq\ & F(max\{a_F\phi(g\nu, M\nu), b_F\phi(h\xi_n, N\xi_n), c_F\phi(g\nu, h\xi_n),\\
& d_F\phi(g\nu, N\xi_n), e_F\phi(h\xi_n, M\nu)\}).
\end{aligned} \tag{3.5}$$

Taking limit as $n \to \infty$ in the above inequality, we have

$$\begin{aligned}
\tau + F(\phi(M\nu, N\xi)) &\preccurlyeq F(max\{a_F\phi(N\xi, M\nu), b_F\phi(N\xi, N\xi), c_F\phi(N\xi, N\xi),\\
&\qquad d_F\phi(N\xi, N\xi), e_F\phi(N\xi, M\nu)\})\\
&= F(max\{a_F\phi(N\xi, M\nu), 0_{\mathcal{A}}, e_F\phi(N\xi, M\nu)\}).
\end{aligned}$$

So, we have either $\phi(M\nu, N\xi_n) \preccurlyeq a_F\phi(N\xi, M\nu)$ or $\phi(M\nu, N\xi) \preccurlyeq e_F\phi(N\xi, M\nu)$, which is contradiction. Thus, $M\nu = g\nu = N\xi = \delta$. Since $M(E) \subseteq h(E)$. Therefore, there exists $\Gamma \in E$, such that $h\Gamma = M\nu = g\nu = \delta$.

Now we shall show that $h\Gamma = N\Gamma = \delta$, that is, Γ is the coincidence point of the pair (N, h). Substituting $\xi = \nu$ and $\gamma = \Gamma$ in (3.4), we have

$$\begin{aligned}\tau + F(\phi(M\xi, N\Gamma)) \preccurlyeq F(max\{&a_F\phi(g\nu, M\nu), b_F\phi(h\Gamma, N\Gamma), c_F\phi(g\nu, h\Gamma),\\ &d_F\phi(g\nu, N\Gamma), e_F\phi(h\Gamma, M\nu)\}),\end{aligned}$$

that is,

$$\begin{aligned}\tau + F(\phi(\delta, N\Gamma)) &\preccurlyeq F\left(max\{a_F\phi(\delta, \delta), b_F\phi(\delta, N\delta), c_F\phi(\delta, \delta), d_F\phi(\delta, N\Gamma), e_F\phi(\delta, \delta\}\right)\\ &= F(max\{0_{\mathcal{A}},\ b_F\phi(\delta, N\Gamma), d_F\phi(\delta, N\Gamma)\}).\end{aligned}$$

So we have either $\phi(\delta, N\Gamma) \preccurlyeq b_F\phi(\delta, N\Gamma)$ or $\phi(\delta, N\Gamma) \preccurlyeq d_F\phi(\delta, N\Gamma)$, which is a contradiction. Hence $N\Gamma = \delta$, that is, $N\Gamma = h\Gamma = \delta$ and Γ is the coincidence point of N and h. Owing to weak compatibility of the pair, we have $Nh\ \Gamma = hN\ \Gamma$, or $N\delta = h\delta$. Hence δ is a common coincidence point of M, N, g and h.
To prove that δ is common fixed point of M, N, g and h, substitute $\xi = \nu$ and $\gamma = \delta$ in (4), we have

$$\begin{aligned}\tau + F(\phi(M\nu, N\delta)) \preccurlyeq F(max\{&a_F\phi(g\nu, M\nu), b_F\phi(h\delta, N\delta), c_F\phi(g\nu, h\delta),\\ &d_F\phi(g\nu, N\delta), e_F\phi(h\delta, M\nu)\}),\end{aligned}$$

that is,

$$\begin{aligned}\tau + F(\phi(\delta, N\gamma)) &\preccurlyeq F(max\{a_F\phi(\delta, \delta), b_F\phi(N\delta, N\delta), c_F\phi(\delta, N\delta), d_F\phi(\delta, N\delta), e_F\phi(N\delta, \delta)\})\\ &= F(max\{0_{\mathcal{A}}, c_F\phi(\delta, N\delta), d_F\phi(\delta, N\delta), e_F\phi(N\delta, \delta)\}).\end{aligned}$$

So we have either $\phi(\delta, N\delta) \preccurlyeq c_F\phi(\delta, N\delta)$ or $\phi(\delta, N\delta) \preccurlyeq d_F\phi(\delta, N\delta)$, or $\phi(\delta, N\delta) \preccurlyeq e_F\phi(\delta, N\delta)$ which is a contradiction. Hence $N\delta = \delta$. Thus, $M\delta = N\delta = g\delta = h\delta = \delta$, that is, δ a common fixed point of M, N, g and h.
Now, let Ω is another common fixed point of M, N, g and h. By substituting $\xi = \Omega$ and $\gamma = \delta$ in (3.4), we have

$$\begin{aligned}\tau + F(\phi(M\Omega, N\delta)) &\preccurlyeq F(max\{a_F\phi(g\Omega, M\Omega), b_F\phi(h\delta, N\delta), c_F\phi(g\Omega, h\delta),\\ &\qquad d_F\phi(g\Omega, N\delta), e_F\phi(h\delta, M\Omega)\}),\\ &= F(max\{a_F\phi(\Omega, \Omega), b_F\phi(\delta, \delta), c_F\phi(\Omega, \delta), d_F\phi(\Omega, \delta), e_F\phi(\delta, \Omega)\}),\\ &= F(max\{0_{\mathcal{A}}, c_F\phi(\Omega, \delta), d_F\phi(\Omega, \delta), e_F\phi(\delta, \Omega)\}).\end{aligned}$$

We either have, $\phi(\Omega, \delta) \preccurlyeq c_F\phi(\Omega, \delta)$, $\phi(\Omega, \delta) \preccurlyeq d_F\phi(\Omega, \delta)$ or $\phi(\Omega, \delta) \preccurlyeq e_F\phi(\Omega, \delta)$, which is a contradiction. This implies that $\Omega = \delta$. Hence δ is a unique common fixed point of M, N, g and h. □

THEOREM 3.3. *Let $(E, \mathcal{A}, \phi)$ be a C^*-algebra valued metric space and two pairs (M, g) and (N, h) of weakly compatible self mappings with $M(E) \subseteq h(E)$ and $N(E) \subseteq h(E)$, either the pair (M, g) or (N, h) satisfies E.A. property and there exist $F : \mathcal{A}^+ \to \mathcal{A}$ be a continuous and strictly non- decreasing mapping satisfying for every $\alpha, \beta \in E$, $a_F, b_F, c_F, d_F, e_F \in \mathcal{A}$ with $\|a_F\| + \|b_F\| + \|c_F\| + \|d_F\| + \|e_F\| < 1$ and $\tau > 0$ such that*

$$\begin{aligned}\tau + F(\phi(M\xi, N\gamma)) \preccurlyeq\ \ F(max\{&a_F\phi(g\xi, M\xi), b_F\phi(h\gamma, N\gamma), c_F\phi(g\xi, h\gamma),\\ &d_F\phi(g\xi, N\gamma), e_F\phi(h\gamma, M\xi)\})\end{aligned} \tag{3.6}$$

then the self mappings M, N, g, and h have a unique common fixed point in E if either range subspace of $g(E)$ or $h(E)$ is a closed subspace in E.

Proof. Let the pair (N, h) satisfies E.A. property, then there exists a sequence $\{\xi_n\}$ in E such that

$$\lim_{n\to\infty} N(\xi_n) = \lim_{n\to\infty} h(\xi_n) = \delta,$$

for some $\delta \in E$.
Further, $N(E) \subseteq g(E)$. Therefore, there exist a sequence $\{\gamma_n\}$ in E such that $N(\xi_n) = g(\gamma_n)$. Thus,

$$\lim_{n\to\infty} g(\gamma_n) = \delta = \lim_{n\to\infty} N(\xi_n)$$

We claim that $M(\gamma_n) = \delta$ and on contrary, suppose that $\lim_{n\to\infty} M(\xi_n) = \delta_1 \neq \delta$. Substituting $\xi = \gamma_n$ and $\gamma = \xi_n$ in (3.6) we have

$$\begin{aligned} \tau + F(\phi(M\gamma_n, N\xi_n)) \preccurlyeq \;& F(max\{a_F\phi(g\gamma_n, M\gamma_n) + b_F\phi(h\xi_n + N\xi_n) + c_F\phi(g\gamma_n, h\xi_n) \\ & + d_F\phi(g\gamma_n, N\xi_n) + e_F\phi(h\xi_n, M\gamma_n)\}) \end{aligned} \tag{3.7}$$

Taking limit as $n \to \infty$, we have

$$\begin{aligned} \tau + F(\phi(\delta_1, \delta)) & \preccurlyeq F(max\{a_F\phi(\delta, \delta_1) + b_F\phi(\delta, \delta) + c_F\phi(\delta, \delta) \\ & \quad + d_F\phi(\delta, \delta) + e_F\phi(\delta, \delta_1)\}). \end{aligned}$$

We have $\phi(\delta_1, \delta) \preccurlyeq (a_F + e_F)\phi(\delta, \delta_1)$, which is a contradiction. Thus, $\delta_1 = \delta$, that is,

$$\lim_{n\to\infty} M(\gamma_n) = \lim_{n\to\infty} N(\xi_n) = \delta.$$

Now, let $g(E)$ be the closed subspace of E and $g\nu = \delta$ for some $\nu \in E$. Subsequently, we obtain

$$\lim_{n\to\infty} M(\gamma_n) = \lim_{n\to\infty} N(\xi_n) = \lim_{n\to\infty} h(\xi_n) = \lim_{n\to\infty} g(\gamma_n) = \delta = g\nu.$$

We claim that $M\nu = g\nu$. Substituting $\xi = \nu$ and $\gamma = \xi_n$ in (3.6), we have

$$\begin{aligned} \tau + F(\phi(M\nu, N\xi_n)) \preccurlyeq \;& F(max\{a_F\phi(g\nu, M\nu) + b_F\phi(h\xi_n + N\xi_n) + c_F\phi(g\nu, h\xi_n) \\ & + d_F\phi(g\nu, N\xi_n) + e_F\phi(h\xi_n, M\nu)\}). \end{aligned} \tag{3.8}$$

Taking limit as $n \to \infty$, we have

$$\begin{aligned} \tau + F(\phi(M\nu, \delta)) & \preccurlyeq F(a_F\phi(\delta, M\nu) + b_F\phi(\delta, \delta) + c_F\phi(\delta, \delta) \\ & \quad + d_F\phi(\delta, \delta) + e_F\phi(\delta, M\nu)), \end{aligned}$$

that is,

$$\tau + F(\phi(M\nu, \delta)) \preccurlyeq F(a_F\phi(\delta, M\nu) + e_F\phi(\delta, M\nu)).$$

So we have $\phi(M\nu, \delta) \preccurlyeq (a_F + e_F)\phi(\delta, M\nu)$ which is a contradiction. Thus, $M\nu = \delta = g\nu$, that is, the coincidence point of the pair (M, g).
Now the pair (M, g) are weakly compatible, that is, $Mg\nu = gM\nu$ or $M\delta = g\delta$. Since $M(E) \subseteq h(E)$. Therefore, there exists $\Gamma \in E$ such that $M\nu = h\nu = g\nu = \delta$.
We have to prove that Γ is a coincidence point of pair (N, h), that is $N\Gamma = h\Gamma = \delta$. Substituting $\xi = \nu$ and $\gamma = \Gamma$ in (3.6), we have

$$\begin{aligned} \tau + F(\phi(M\nu, N\Gamma)) & \preccurlyeq F(\{a_F\phi(g\nu, M\nu) + b_F\phi(h\Gamma + N\Gamma) + c_F\phi(g\nu, h\Gamma) \\ & \quad + d_F\phi(g\nu, N\Gamma) + e_F\phi(h\Gamma, M\nu)\}), \end{aligned}$$

that is,

$$\begin{aligned}\tau + F(\phi(\delta, N\Gamma)) &\preccurlyeq F(\{a_F\phi(\delta,\delta) + b_F\phi(\delta + N\Gamma) + c_F\phi(\delta,\delta)\\ &\qquad\qquad + d_F\phi(\delta, N\Gamma) + e_F\phi(\delta,\delta)\})\\ &= F(b_F\phi(\delta + N\Gamma) + d_F\phi(\delta, N\Gamma)).\end{aligned}$$

So, we have $\phi(\delta, N\Gamma) \preccurlyeq (b_F + d_F)\phi(\delta, N\delta)$, which is a contradiction. Hence, $N\Gamma = \delta$. Thus $N\Gamma = h\Gamma = \delta$ and Γ is coincidence point of N and h.
Further, the pair (N, h) are weak compatible, that is $Nh\ \Gamma = hN\ \Gamma$, or $N\delta = h\delta$. Therefore, δ is a common coincidence point of M, N, g and h.
Now we shall show that δ is a common fixed point of M, N, g and h. Substituting $\xi = \nu$ and $\gamma = \delta$ in (3.6), we have

$$\begin{aligned}\tau + F(\phi(M\nu, N\delta)) \preccurlyeq &F(\{a_F\phi(g\nu, M\nu) + b_F\phi(h\delta + N\delta) + c_F\phi(g\nu, h\delta)\\ &\qquad\qquad + d_F\phi(g\nu, N\delta) + e_F\phi(h\delta, M\nu)\}),\end{aligned}$$

that is,

$$\begin{aligned}\tau + F(\phi(\delta, N\gamma)) &\preccurlyeq F(\{a_F\phi(\delta,\delta) + b_F\phi(N\delta, N\gamma) + c_F\phi(\delta,\delta)\\ &\qquad\qquad + d_F\phi(\delta, N\gamma) + e_F\phi(\delta,\delta)\})\\ &= F(b_F\phi(\delta + N\delta) + e_F\phi(N\delta, \delta)).\end{aligned}$$

We have shown that $\phi(N\delta, \delta) \preccurlyeq (d_F + e_F)\phi(\delta, N\delta)$, which is a contradiction. Therefore, we conclude that $N\delta = \delta$. Consequently, we have $M\delta = N\delta = g\delta = h\delta = \delta$.
The same conclusion can be obtained when assuming that the range space of $h(E)$ is closed in E, and the pair (M, g) will satisfy the E.A. property.
Now, let Ω be another common fixed point of M, N, g, and h. By substituting $\xi = \Omega$ and $\gamma = \delta$ into equation (3.6), we obtain

$$\begin{aligned}\tau + F(\phi(\Omega, \delta)) &= \tau + F(\phi(M\Omega, N\delta))\\ &\preccurlyeq F(\{a_F\phi(g\Omega, M\Omega) + b_F\phi(h\delta, N\delta) + c_F\phi(g\Omega, h\delta)\\ &\qquad\qquad + d_F\phi(g\Omega, N\delta) + e_F\phi(h\delta, M\Omega)\}),\\ &= F(d_F\phi(\Omega,\Omega) + b_F\phi(N\delta,\delta) + c_F(\Omega,\delta) + d_F(\Omega,\delta) + e_F(\delta,\Omega)),\\ &= F(c_F(\Omega,\delta) + d_F(\Omega,\delta) + e_F(\delta,\Omega)).\end{aligned}$$

From this substitution, we derive the expression $\phi(\Omega, \delta) \preccurlyeq (c_F + d_F + e_F)\phi(\Omega, \delta)$, which is a contradiction. Therefore, we conclude that $\Omega = \delta$, that is, δ is a unique common fixed point of M, N, g, and h. □

If we set $d_F = e_F = 0$ in the Theorem 3.3, we obtain the following result

Corollary 3.4. *Let $(E, \mathcal{A}, \phi)$ be a C^*-algebra valued metric space and two pairs (M, g) and (N, h) of weakly compatible self mappings with $M(E) \subseteq h(E)$ and $N(E) \subseteq h(E)$, either the pair (M, g) or (N, h) satisfies E.A property and there exists $F : \mathcal{A}^+ \to \mathcal{A}$ be a continuous and strictly non- decreasing mapping satisfying for every $\alpha, \beta \in E, a_F, b_F, c_F, d_F, e_F \in \mathcal{A}$ with $\|a_F\| + \|b_F\| + \|c_F\| < 1$ and $\tau > 0$ such that*

$$\tau + F(\phi(M\xi, N\gamma)) \preccurlyeq F(a_F\phi(g\xi, M\xi), b_F\phi(h\gamma, N\gamma), c_F\phi(g\xi, h\gamma)).$$

Then M, N, g, and h have a unique common fixed point in E if either range subspace of $g(E)$ or $h(E)$ is a closed subspace in E.

THEOREM 3.5. *Let $(E, \mathcal{A}, \phi)$ be a C^*-algebra valued metric space and two pairs (M, g) and (N, h) of weakly compatible self mappings with $M(E) \subseteq h(E)$ and $N(E) \subseteq h(E)$, either the pair (M, g) or (N, h) satisfies CLR property and there exist $F : \mathcal{A}^+ \to \mathcal{A}$ be a continuous and strictly non- decreasing mapping satisfying for every $\alpha, \beta \in E, a_F, b_F, c_F, d_F, e_F \in \mathcal{A}$ with $\|a_F\| + \|b_F\| + \|c_F\| + \|d_F\| + \|e_F\| < 1$ and $\tau > 0$ such that*

$$\tau + F(\phi(M\xi, N\gamma)) \preccurlyeq F(a_F\phi(g\xi, M\xi) + b_F\phi(h\gamma, N\gamma) + c_F\phi(g\xi, h\gamma) + d_F\phi(g\xi, N\gamma) + e_F\phi(h\gamma, M\xi)). \tag{3.9}$$

Then the self mapping M, N, g, and h have a unique common fixed point in E.

Proof. Let the pair (N, h) satisfies the CLR_B property, then it implies the existence of a sequence $\{\xi_n\}$ in the space E such that

$$\lim_{n\to\infty} N(\xi_n) = \lim_{n\to\infty} h(\xi_n) = N\xi = \delta$$

for some $\xi \in E$.
Given that $N(E) \subseteq g(E)$, there exists a $\nu \in E$ such that $N\xi = g\nu$. We assert that $M\nu = N\nu = \delta$. To demonstrate this claim, we can substitute $\xi = \nu$ and $\gamma = \xi_n$ into equation (3.9), resulting in

$$\tau + F(\phi(M\nu, N\xi_n)) \preccurlyeq F(a_F\phi(g\nu, M\nu) + b_F\phi(h\xi_n, N\xi_n) + c_F\phi(g\nu, h\xi_n) + d_F\phi(g\nu, N\xi_n) + e_F\phi(h\xi_n, M\nu)). \tag{3.10}$$

Taking the limit as $n \to \infty$, we have

$$\begin{aligned}\tau + F(\phi(M\nu, N\xi)) &\preccurlyeq F(a_F\phi(N\xi, M\nu) + b_F\phi(N\xi, N\xi) + c_F\phi(N\xi, N\xi) \\ &\quad + d_F\phi(g\xi, N\xi) + e_F\phi(N\xi, M\nu)) \\ &= F(a_F\phi(N\nu, M\nu) + e_F\phi(N\xi, M\nu)),\end{aligned}$$

So we have $\phi(M\nu, N\xi_n \preccurlyeq (a_F + e_F)\phi(N\xi, M\nu)$ which contradicts. Hence, $M\nu = g\nu = N\nu = \delta$. Since $M(E) \subseteq h(E)$. Therefore, there exists $\Gamma \in E$ such that $h\nu = M\nu = g\nu = \delta$.
Our aim is to demonstrate that $h\Gamma = N\Gamma = \delta$, thereby confirming that Γ indeed serves as the coincidence point for the pair (N, h). By substituting $\xi = \nu$ and $\gamma = \Gamma$ into equation (3.9), we obtain

$$\begin{aligned}\tau + F(\phi(M\nu, N\Gamma)) \preccurlyeq &F(a_F\phi(g\nu, M\nu) + b_F\phi(h\Gamma, N\Gamma) + c_F\phi(g\nu, h\Gamma) \\ &+ d_F\phi(g\nu, N\Gamma) + e_F\phi(h\Gamma, M\nu)),\end{aligned}$$

that is,

$$\begin{aligned}\tau + F(\phi(M\nu, N\Gamma)) &\preccurlyeq F(a_F\phi(g\nu, M\nu) + b_F\phi(\delta, N\Gamma) + c_F\phi(g\nu, h\Gamma) \\ &\quad + d_F\phi(\delta, N\Gamma) + e_F\phi(\delta, \delta)) \\ &= F(b_F\phi(\delta, N\Gamma) + d_F\phi(\delta, N\Gamma)).\end{aligned}$$

So we have $\phi(\delta, N\Gamma) \preccurlyeq (b_F + d_F)\phi(\delta, N\Gamma)$ which is a contradiction. Hence $N\Gamma = \delta$, that is, $N\ \Gamma = h\Gamma = \delta$ and Γ is a common coincidence point of N, and h. From (3.9), we have $Nh\ \Gamma = hN\ \Gamma$ or $N\delta = h\delta$. Therefore, δ is a common coincidence point of M, N, g and h.

To establish that δ is a common fixed point of M, N, g, and h, we can substitute $\xi = \nu$ and $\gamma = \delta$ into equation (3.9), resulting in

$$\begin{aligned}\tau + F(\phi(M\nu, N\delta)) \preccurlyeq & F(a_F\phi(g\nu, M\nu) + b_F\phi(h\delta, N\delta) + c_F\phi(g\nu, h\delta) \\ & + d_F\phi(g\nu, N\delta) + e_F\phi(h\delta, M\nu)),\end{aligned}$$

that is,

$$\begin{aligned}\tau + F(\phi(\delta, N\delta)) &\preccurlyeq F(a_F\phi(\delta, \delta) + b_F\phi(N\delta, N\delta) + c_F\phi(\delta, N\delta) \\ &\qquad + d_F\phi(\delta, N\delta) + e_F\phi(N\delta, \delta)) \\ &= F(c_F\phi(\delta, N\delta) + d_F\phi(\delta, N\delta) + e_F\phi(N\delta, \delta)).\end{aligned}$$

So we have $\phi(\delta, N\delta) \preccurlyeq (c_F+d_F+e_F)\phi(\delta, N\delta)$, which is a contradiction. Hence $N\delta = \delta$. Thus, $M\delta = N\delta = g\delta = h\delta = \delta$, that is, common fixed point of M, N, g and h is δ. Now, let Ω be another common fixed point of M, N, g, and h. By substituting $\xi = \Omega$ and $\gamma = \delta$ into equation (3.9), we obtain

$$\begin{aligned}\tau + F(\phi(\Omega, \delta)) &= \tau + F(\phi(M\Omega, N\delta)) \\ &\preccurlyeq F(a_F\phi(g\Omega, M\Omega) + b_F\phi(h\delta, N\delta) + c_F\phi(g\Omega, h\delta) \\ &\qquad + d_F\phi(g\Omega, N\delta) + e_F\phi(h\delta, M\Omega)) \\ &= F(a_F\phi(\Omega, \Omega) + b_F\phi(\delta, \delta) + c_F\phi(\Omega, \delta) + d_F\phi(\Omega, \delta) + e_F\phi(\delta, \Omega)) \\ &= F(c_F\phi(\Omega, \delta) + d_F\phi(\Omega, \delta) + e_F\phi(\delta, \Omega)).\end{aligned}$$

So we have either $\phi(\Omega, \delta) \preccurlyeq (c_F + d_F + e_F)\phi(\Omega, \delta)$, which contradicts. Thus $\Omega = \delta$. Hence δ is a unique common fixed point of M, N, g, and h. □

If we choose $d_F = e_F = 0$ in Theorem 3.5 we have the following Corollary :

Corollary 3.6. *Let $(E, \mathcal{A}, \phi)$ be a C^*-algebra valued metric space and two pairs (M, g) and (N, h) of weakly compatible self mappings with $M(E) \subseteq h(E)$ and $N(E) \subseteq h(E)$, either the pair (M, g) or (N, h) satisfies CLR property and there exist $F : \mathcal{A}^+ \to \mathcal{A}$ be a continuous and strictly non-decreasing mapping satisfying for every $\alpha, \beta \in E, a_F, b_F, c_F \in \mathcal{A}$ with $\|a_F\| + \|b_F\| + \|c_F\| < 1$ and $\tau > 0$,*

$$\tau + F(\phi(M\xi, N\gamma)) \preccurlyeq F(a_F\phi(g\xi, M\xi), b_F\phi(h\gamma, N\gamma), c_F\phi(g\xi, h\gamma)).$$

Then the mapping M, N, g and h have a unique common fixed point in E.

Example 3.7. Let $E = [0, 2], \mathcal{A} = \mathcal{C}$. Defining $\phi : E \times E \to \mathcal{A}$ as $\phi(\xi, \gamma) = |\xi - \gamma|$ and a function $F : \mathcal{A}^+ \to \mathcal{A}$ by $F(t) = \ln(t) + t$. We can easily prove $(E, \mathcal{A}, \phi)$ to be complete C^*-algebra valued metric space.
Let the four self maps M, N, g and h be defined on E as

$$M(\xi) = \begin{cases} \xi & \text{if } \xi \in [0, 1] \\ 1 & \text{if } \xi \in (1, 2] \end{cases}, \; N(\xi) = \begin{cases} \xi & \text{if } \xi \in [0, 1] \\ 2 & \text{if } \xi \in (1, 2] \end{cases}, \; g(\xi) = 4\xi, \; h(\xi) = 8\xi.$$

Firstly, To establish that the pair (M, g) satisfies the E.A. property, we begin by considering a sequence $\{\xi_n\} \subset E$ defined as $\{\xi_n\} = \left\{\frac{1}{\sqrt{2n^2+2}}\right\}$. With this sequence in mind, we observe that there exists a sequence $\{\xi_n\}$ in E for which the following limits hold:

$$\lim_{n\to\infty} M\xi_n = \lim_{n\to\infty} g\xi_n = 0.$$

As a result, we can conclude that the pair (M, g) indeed satisfies the E.A. property. The following cases arise :
Case (i): Let $\xi, \gamma \in [0,1]$, clearly $ME \subset hE$ and $NE \subset gE$.

$$\begin{aligned}
&\phi(M\xi, N\gamma) = 0,\ \phi(g\xi, M\xi) = 7\xi,\\
&\phi(g\xi, N\gamma) = |8\xi - \gamma|,\ \phi(h\xi, N\gamma) = 3\xi,\\
&\phi(g\xi, h\gamma) = |8\xi - 4\gamma|,\ \phi(h\gamma, M\xi) = |4\gamma - \xi|.
\end{aligned}$$

Therefore, $a_F, b_F, c_F \in \mathcal{A}^+$ with $a_F = \frac{1}{4}$, and $b_F = c_F = \frac{1}{4}$

$$\begin{aligned}
&F(a_F\phi(g\xi, M\xi) + b_F\phi(h\gamma, N\gamma) + c_F\phi(g\xi, h\gamma))\\
&\quad = F(7a_F\xi + 3b_F\gamma + c_F|8\xi - 4\gamma|)\\
&\quad = \ln(7a_F\xi + 3b_F\gamma + c_F|8\xi - 4\gamma|)\\
&\quad + (7a_F\xi + 3b_F\gamma + c_F|8\xi - 4\gamma)\\
&\quad \succcurlyeq \tau + F(\phi(M\xi, N\gamma)).
\end{aligned}$$

Thus, $\tau + F(\phi(\xi,\gamma)) \preccurlyeq F(a_F\phi(g\xi, M\gamma) + b_F(h\gamma, N\gamma) + c_F(g\xi, h\gamma))$ for all $\xi, \gamma \in [0,1]$.

Similarly, we can show for $a_F, b_F, c_F, d_F, e_F \in \mathcal{A}^+$, with $_F = \frac{1}{5} = b_F = c_F = d_F$ and $e_F = \frac{1}{6}$,

$$\tau + F(\phi(\xi,\gamma)) \preccurlyeq F(a_F\phi(g\xi, M\gamma) + b_F(h\gamma, N\gamma) + c_F(g\xi, h\gamma) + d_F\phi(g\xi, N\gamma) + e_F\phi(h\gamma, M\xi))$$

for all $\xi, \gamma \in [0,1]$.
Case (ii): Let $\xi, \gamma \in (1,2]$, clearly $ME \subset hE$ and $NE \subset gE$.

$$\begin{aligned}
&\phi(M\xi, N\gamma) = 1,\ \phi(g\xi, M\xi) = 8\xi - 1,\\
&\phi(g\xi, N\gamma) = 8\xi - 2,\ \phi(h\xi, N\gamma) = 4\gamma - 2,\\
&\phi(g\xi, h\gamma) = 8\xi - 4\gamma,\ \phi(h\gamma, M\xi) = 4\gamma - 1.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&F(a_F(g\xi, M\xi) + b_F(h\gamma, N\gamma) + c_F(g\xi, h\gamma))\\
&\quad = F(a_F(8\xi - 1) + b_F(4\gamma - 2) + c_F(8\xi - 4\gamma)\\
&\quad = \ln(a_F(8\xi - 1) + b_F(4\gamma - 2) + c_F(8\xi - 4\gamma)\\
&\quad + (a_F(8\xi - 1) + b_F(4\gamma - 2) + c_F(8\xi - 4\gamma))\\
&\quad \succcurlyeq \tau + F(\phi(M\xi, N\gamma)).
\end{aligned}$$

Thus, $\tau + F(\phi(\xi,\gamma)) \preccurlyeq F(a_F\phi(g\xi, M\xi) + b_F(h\gamma, N\gamma) + c_F(g\xi, h\gamma))$ for all $\xi, \gamma \in (1,2]$.

Similarly, we can show for $a_F, b_F, c_F, d_F, e_F \in \mathcal{A}^+$, with $a_F = \frac{1}{5} = b_F = c_F = d_F$ and $e_F = \frac{1}{6}$,

$$\tau + F(\phi(\xi,\gamma)) \preccurlyeq F(a_F\phi(g\xi, M\gamma) + b_F(h\gamma, N\gamma) + c_F(g\xi, h\gamma) + d_F\phi(g\xi, N\gamma) + e_F\phi(h\gamma, M\xi))$$

for all $\xi, \gamma \in (1,2]$.
Also, the range space of $g(E)$ is a closed in E and the pairs (M, g) and (N, h) are weakly compatible. Hence, by Theorem (3.3) the mappings have a unique common fixed point and indeed, 0 is unique common fixed point.

Conclusion

In this paper, we pointed out the fact that several diversified physical problems modeled in the framework of C^*-algebra valued metric spaces have been very useful for future research directions. We established common fixed point results by utilizing a variant of F-contraction on the same metric settings. We used some weaker notions such as E.A. and C.L.R. properties possessed by the involved self-mappings to prove fixed point results. Further, we presented non-trivial examples to vindicate that the claims are novel and original. Thus, these findings supply yet another view on common fixed point results with some new core theoretical results and examples.

Authors' contributions : All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

Aknowledgement : The referees have reviewed the paper very carefully. The authors express their deep thanks for the comments.

Compliance with ethical standards

Competing interests : The author declares that there is no conflict of interests.

References

[1] S. Banach, *Sur les operations dans les ensembles abstraits et leur application aux equations integrals*, Fund. Math. **3** (1922), 133–181.
[2] S.G. Matthews, *Partial metric topology*, Ann. N. Y. Acad. Sci. **728** (1994), 183–197.
[3] S. Czerwik, *Contraction mappings in b-metric spaces*, Acta Math. Univ. Ostrav. **1** (1993), 5–11.
[4] A. Branciari, *A fixed point theorem of Banach-Caccioppoli type on a class of generalized metric spaces*, Publ. Math. **57** (2000), 31–37.
[5] M. Abbas, G. Jungck, *Common fixed point results for noncommuting mappings without continuity in cone metric spaces*, J. Math. Anal. Appl. **341** (2008), 416–420.
[6] J. Jachymski, *The contraction principle for mappings on a metric space with a graph*, Proc. Am. Math. Soc. **136** (4), (2008), 1359–1373.
[7] Y. J. Cho, R. Saadati, S. Wang, *Common fixed point theorems on generalized distance in ordered cone metric spaces*, Comput. Math. Appl., **61** (2011), 1254–1260.
[8] R.C. Dimri, G. Prasad, *Coincidence theorems for comparable generalized non-linear contractions in ordered partial metric spaces*, Commun. Korean Math. Soc. **32** (2) (2017), 375–387.
[9] G. Prasad, *Fixed point theorems via w-distance in relational metric spaces with an application*, Filomat **34**(6) (2020), 1889–1898.
[10] G. Prasad, R.C. Dimri, A. Bartwal, *Fixed points of Suzuki contractive mappings in relational metric spaces*, Rend. Circ. Mat. Palermo, Ser II **69** (2020), 1347–1358. https://doi.org/10.1007/s12215-019-00475-4
[11] G. Prasad, D. Khantwal, *Fixed points of JS-contractive mappings with applications*, J. Anal. **31** (4) (2023), 2687–2701. https://doi.org/10.1007/s41478-023-00598-z
[12] D. Wardowski, *Fixed points of a new type of contractive mappings in complete metric spaces*, Fixed Point Theory Appl. **2012**,94, (2012).
[13] D. Wardwoski, N. V. Dung, *Fixed points of F-weak contraction on complete metric spaces*, Demonstr. Math. **47** (2014).
[14] D. Klim, D. Wardowski, *Fixed points of dynamic processes of set-valued F contractions and application to functional equations*, Fixed Point Theory Appl. **2015**, 22, (2015).

[15] I. Altun, G. Minak, H. Dag, *Multivalued F-contractions on complete metric spaces*, J. Nonlinear Convex Anal. **16** (2015), 659–666.

[16] M. Nazam, C. Park, A. Hussain, M. Arshad, J. R. Lee, *Fixed point theorems for F-contractions on closed ball in partial metric spaces*, J. Comput. Anal. Appl. **27** (2019), 759–769.

[17] Z.H. Ma, L.N. Jiang, H.K. Sun, *C^*-algebra valued metric spaces and related fixed point theorems*, Fixed Point Theory Appl. **2014**, 206, (2014).

[18] Z.H. Ma, L.N. Jiang, *C^*-algebra valued b-metric spaces and related fixed point theorems*, Fixed Point Theory Appl. **2015**, 222, (2015). https://doi.org/10.1186/s13663-015-0471-6

[19] M.E. Ege, C. C. Alaca, *C^*-algebra-valued S-metric spaces*, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat. **67** (2) (2018), 165–177.

[20] N. Mlaike, M. Asim, M. Imdad, *C^*-algebra valued partial b-metric spaces and fixed point results with an application*, Mathematics **8** (8), (2020). https://doi.org/10.3390/math8081381

[21] J.U. Maheswari, A. Anbarasan, M. Gunaseelan, V. Parvaneh, H. Bonab, *Solving an integral equation via C^*-algebra-valued partial b-metrics*, Fixed Point Theory Algorithms Sci Eng **2022**, 18 (2022).

[22] W. Tapanyo, W. Ratiphaphongthon, A. Arunchai, *Completion of C^*-algebra-valued metric spaces*, Thai J. Math., **20** (3), (2022), 1119–1136.

[23] G. Mani, A.J. Gnanaprakasam, O. Ege, A. Aloqaily, N. Mlaiki, *Fixed point results in C^*-algebra-valued partial b-metric spaces with related application*, Mathematics **11** (5), 1158, (2023) 1–9.

[24] M. Paul, K. Sarkar, K. Tiwary, *Some unique common fixed point results on $C*$-algebra valued metric space using C_*-class function*, Ann. Math. Comput. Sci. **16**, (2023), 1–20.

[25] S. Narzary, D. Das, Y.M. Singh, M.S. Khan, S. Sessa, *C^*-algebra-valued partial modular metric spaces and some fixed point results*, Symmetry **15** (6) (2023), 1135.

[26] X. Qiaoling, J. Lining, Z. Ma, *Common fixed point theorems in C^*-algebra valued metric spaces*, J. Nonlinear Sci. Appl. **9** (2016), 4617–4627.

[27] D. El. Moutawakil, M. Aamri, *Some new common fixed point theorems under strict contractive conditions*, J. Math. Anal. Appl. **270** (2002), 181–188.

[28] P. Kumam, W. Sintunavarat, *Common fixed point theorem for a pair of weakly compatible mappings in fuzzy metric space*, J. Appl. Math. **2011** (2011). https://doi.org/10.1155/2011/637958

[29] K. Goebel, *A coincidence theorem*, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. **16** (1968), 733–735.

[30] G. Jungck, B. E. Rhoades, *Fixed points for set valued functions without continuity*, Indian J. Pure Appl. Math. **29** (3), (1998), 227–238.

Shivani Kukreti
Department of Mathematics, H.N.B. Garhwal University,
Srinagar Garhwal, India-246174
E-mail: kukretishivani6@gmail.com

Gopi Prasad
Department of Mathematics,
Dr. Shivanand Nautiyal Goverment Post Graduate College,
Karanprayag, Uttarakhand, India-246444
E-mail: gopiprasad127@gmail.com

Ramesh Chandra Dimri
Department of Mathematics, H.N.B. Garhwal University,
Srinagar Garhwal, India-246174
E-mail: dimrirc@gmail.com