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A BRIEF REVIEW OF PREDATOR-PREY MODELS FOR AN ECOLOGICAL SYSTEM WITH A DIFFERENT TYPE OF BEHAVIORS

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Abstract. The logistic growth model was developed with a single population in mind. We now analyze the growth of two interdependent populations, moving beyond the one-dimensional model. Interdependence between two species of animals can arise when one (the ”prey”) acts as a food supply for the other (the ”predator”). Predator-prey models are the name given to models of this type. While social scientists are mostly concerned in human communities (where dependency hopefully takes various forms), predator-prey models are interesting for a variety of reasons. Some variations of this model produce limit cycles, an interesting sort of equilibrium that can be found in dynamical systems with two (or more) dimensions. In terms of substance, predator-prey models have a number of beneficial social science applications when the state variables are reinterpreted. This paper provides a quick overview of numerous predator–prey models with various types of behaviours that can be applied to ecological systems, based on a survey of various types of research publications published in the last ten years. The primary source for learning about predator–prey models used in ecological systems is historical research undertaken in various circumstances by various researchers. The review aids in the search for literature that investigates the impact of various parameters on ecological systems. There are also comparisons with traditional models, and the results are double-checked. It can be seen that several older predator–prey models, such as the Beddington–DeAngelis predator–prey model, the stage-structured predator–prey model, and the Lotka–Volterra predator–prey model, are stable and popular among academics. For each of these scenarios, the results are thoroughly checked.

1. Introduction

Modeling predator-prey interactions is an essential topic in mathematical biology since one of the primary aspects of ecological systems is the link between different species and their living environment. In conventional predator-prey models, the growth function of prey species in the absence of predators, the predator's mortality function in the absence of prey, and the predator's functional reaction function to the

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prey are all key components. The interplay between animals and their natural environment defines ecological systems [1]. This type of interaction can take place on a variety of spatial and temporal domains [2,3]. Population ecology is nearly as ancient as the study of complicated population dynamics. Volterra and Lotka independently devised a normal model of interacting species in the 1920s, which is now known by their joint names. The predator-prey version of the model showed neutrally stable cycles, despite the fact that it was a simplistic model [4]. Because of its worldwide existence and importance, the dynamic interplay between predators and their prey has long been and will continue to be a dominating issue in both ecology and mathematical ecology [5]- [6]. The conservation of mass principle states that predators can only grow in proportion to what they eat [7]. Predator-prey models are based on two broad principles: the first is that population dynamics can be decomposed into birth and death processes, and the second is that population dynamics can be decomposed into birth and death processes.

The spatial and temporal behaviour of interacting species in ecosystems has piqued people's curiosity in recent years. Due of its universal occurrence and relevance, predator-prey dynamical behaviour has long been and will continue to be one of the most prevalent themes in ecosystems [8]- [13]. Unfortunately, the majority of studies on the spatiotemporal predator-prey system with functional response focus on the bifurcation events caused by changing the control parameter(s), with little attention paid to the Turing pattern selection. In authentic ecological settings, our research provides valuable insights into the dynamics of predator-prey interactions, with a specific focus on the influence of cooperative hunting on pattern dynamics within a diffusive predator-prey model. The significance of hunting cooperation becomes pronounced in ecological systems where prey demonstrate herd behavior, as discussed in previous studies [14].

In this paper, we look at the numerous studies that have been conducted during the last ten years, from 2011 to 2020. Throughout this decade, we have attempted to cover practically every model that has been developed using certain conditions to analyse the influence on the environment by scientists and researchers all around the world. It's essentially review work using the most basic methods of literature survey. Various research articles are available on prey-predator model, which we download.

2. Literature Review

2.1. Spatial Prey-Predator Models. Yuan et al. [15] discussed a spatial predator-prey model with quadratic mortality in predator population and herd behavior in prey population. Under the linear stability investigation, authors get the condition for stationary pattern. With logistic growth in the prey and a square root response function [16], the basic predator-prey model is given by

$$(1)\qquad \begin{cases} \dfrac{dU}{dt} = pU\big(1-\dfrac{U}{Q}\big) - \dfrac{\beta\sqrt{U}V}{1+t_a\beta\sqrt{U}}, \\ \dfrac{dV}{dt} = -mV + \dfrac{b\beta\sqrt{U}V}{1+t_a\beta\sqrt{U}} \end{cases}$$

Where $U(t)$ and $V(t)$ are the prey and hunter densities at time t, respectively. The parameters p represent the prey's growth rate, Q its carrying capacity, and m

the predator's death rate in the absence of prey. The search efficiency of V for U is parameter β, the conversion or consumption rate of prey to predator is parameter b, and the average handling time is parameter t_a.

This research presents the Turing pattern selection in a spatial hunter-prey scenario. The authors begin by obtaining the Turing space and constructing the amplitude equations for the energised modes. The authors then use numerical simulations to outline each of the three classifications: stripes, spots, and spots-stripes combinations of Turing patterns around the onset of Turing bifurcation, demonstrating that the model dynamics exhibit intricate design replica. It's worth noting that the spatial hunter-prey model can't produce Turing structures if the hunter mortality is represented by a linear form. Quadratic mortality is thus the driving force behind the Turing pattern. If the model's parameter m changes little, the model's qualitative dynamics change. The hunter's mortality rate is represented by m. According to the biological perspective, obtained outcomes show that the mortality rate of hunter might has an important job in the spatial hunter-prey model. We can acquire three different patterns by adjusting the value of the hunter mortality m: stripe pattern, spot pattern, and spot-stripe pattern. From the perspective on populace dynamics, anyone can observe that there exists the spot design facsimile-the prey u was the segregated zone with low density, and the rest of was with high density, this implies the prey might break out nearby. The prey in this space is protected.

The above paragraph briefly touches upon the biological perspective of the model outcomes. Expanding on this aspect, future work could involve a more detailed ecological interpretation of the results. How do these patterns impact the predator-prey dynamics in a real-world context? Are there implications for population persistence or ecosystem stability? Such insights could have broader ecological relevance.

Braza [16] discussed about a hunter-prey model with herd behavior in the prey, therefore hunter attack on the prey along the outside passage of the group of prey. This type of model is examined by Ajraldi et al. [67]. With the help of mathematical outcome of the herd behavior, author has considered hunter-prey systems and competitions models in which collaboration terms utilize the square root of the prey populace as opposed to just the prey populace.

The dynamics of the square root system are compared to those of predator–prey systems with a conventional Lotka–Volterra interaction term. To begin, Holling Type-II response functions are obtained by employing time-budget arguments in the models.

$$(2)\qquad \begin{cases} \dfrac{dU}{dt} = gU(1-\dfrac{U}{K}) - \dfrac{\beta UV}{1+t_a\beta U} \\ \dfrac{dV}{dt} = -d_1V + \dfrac{r\beta UV}{1+t_a\beta U} \end{cases}$$

Here $U(t)$ is stands for prey and $V(t)$ is for predator. Growth rate of the prey is the parameter g, carrying capacity is denoted by K, and in the absence of prey, d_1 is the death rate of the predator. The parameter β is the search efficiency of V for U, r is consumption rate or biomass conversion, and t_a is Y's average handling time of U.

If the hunter death rate parameter (d_1) is very large, greater than the consumption rate r of prey, then the hunter normally vanishes, leaving the prey to keep up with itself after some time at the carrying capacity. For hunter death rates d_1 less than yet not very a long way from the rate r it use prey, the hunter and prey exist together in a stable equilibrium. Because d_1 is just moderate in size, the hunter can adequately support itself yet not develop excessively to clear out the prey. If the hunter mortality rate gets more smaller, then the coexistence essentially becomes unstable. When d_1 diminishes, at a Hopf bifurcation the stable steady state becomes unstable. The populaces then, become stably periodic, however the amplitude increases quickly as d_1 diminishes less then the Hopf value. At a specific value of $d_1 = d_1^*$ less than the Hopf value, the branch of stable periodic solution will be ends, beneath which the zero populace steady state is the only stable solution. Fundamentally when the hunter's mortality rate d_1 is very little, its fecundity take it to the end of the prey and afterward itself.

2.1.1. *Ratio-dependent predator-prey system.* Wang et al. [17] examined the formation of a ratio-dependent predator-prey system with Michaelis-Menten functional response and reaction diffusion. In a spatial domain, the conditions of Turing, wave, and Hopf bifurcation are determined. Furthermore, the authors present a hypothetical analysis of developmental processes that incorporates the distribution of living creatures and their interaction with local diffusion in a spatially distributed population. The outcomes of numerical simulations show that the development of segregated groupings, such as spotted or stripelike or a combination of the two, is the normal dynamics of population density variation. This research indicates that the spatially extended model incorporates spiral waves and chaos in addition to more complicated dynamic patterns in space. Exploring how the insights gained from this research could be applied to real-world scenarios, such as ecological conservation or pest management, could have practical implications. Developing strategies based on the observed patterns to optimize resource allocation or control invasive species could be a relevant avenue for future research.

Hunter-prey models satisfy two general principles: the first is that population dynamics can be divided into birth and death processes, and the second is the conservation of mass principle, which states that predators can only grow in proportion to what they have eaten. The following concepts can be used to design a canonical form of a hunter-prey system:

$$\begin{cases} \dot{M}(t) = Mg(M) - f(M,Q)Q - \mu_M(M)M, \\ \dot{Q}(t) = \gamma f(M,Q)Q - \mu_Q(Q)Q, \end{cases} \tag{3}$$

where $M(t)$, $Q(t)$ are the prey and predator densities, respectively, and t is the time, $g(M)$ is the per capita prey growth rate in the absence of predators, μM and μQ are natural prey and predator mortality, respectively, and $f(M,Q)$ is the functional response. The numerical response, often known as $\gamma f(M,Q)$, is the per capita generation of predators due to predation. In general, one assumes that the primary cause of mortality for the prey is eating.

Here writer has predominantly target on the ratio-dependent hunter-prey system with Michaelis-Menten-type functional response:

$$\begin{cases} \dfrac{\partial M}{\partial t} = m\big(1 - \dfrac{M}{C}\big)M - \dfrac{\beta M}{Q + \beta t_h M}Q + D_1\nabla^2 M, \\ \dfrac{\partial Q}{\partial t} = f\dfrac{\beta M}{Q + \beta t_h M}Q - mQ + D_2\nabla^2 Q, \end{cases} \tag{4}$$

$\forall (M,Q) \in [0,\infty]^2 \setminus (0,0)$, where M, Q denote the prey and predator density, respectively. The diffusion coefficients of prey and predator are D_1, D_2, respectively.

$\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$ is the Laplacian operator in 2-D space. All of the parameters are positive constants: m represents the prey's maximum growth rate, f represents the conversion efficiency, m represents the predator's mortality rate, C represents the carrying capacity, β represents the capture rate, and t_h indicates the handling time.

2.1.2. *Spatial dynamics of the Beddington-DeAngelis hunter-prey model.* Zhang et al. [18] have systematically concentrated on the spatial dynamics of the Beddington-DeAngelis hunter-prey model. Authors investigate the linear stability and acquire the Turing instability condition for the model. Amplitude equations are obtained by the authors and then drive the stability of various designs. It is observed that the model has coexistence of stripe patterns and H_0 hexagon patterns, H_π hexagon designs, and H_0 hexagon designs. To all the more likely depict the real ecosystem like an open system and consider the environmental noise. It is found that noise can diminish the quantity of the patterns and make the patterns highly regular. In addition, noise can produce two types of normal pattern infections. First one is from the H_π hexagon patterns to the standard stripe designs, and the second one is from the coexistence of stripe patterns and H_0 hexagon patterns to the ordinary stripe designs.

Authors created a hunter-prey model using the Beddington-DeAngelis or density-dependent functional response and logistic prey growth, as shown below:

$$\begin{cases} \frac{dP}{dt} = gP(1 - \frac{P}{C}) - \frac{mPQ}{s + P + fQ} \\ \frac{dQ}{dt} = \frac{rmPQ}{s + P + fQ} - dQ, \end{cases} \tag{5}$$

where $P(t)$ and $Q(t)$ are the prey and predator densities at time t, g is the prey's intrinsic growth rate, C is the prey's carrying capacity, r is the prey's conversion rate to predator, and d is the predator's mortality rate. The Beddington-DeAngelis functional response is defined as $\frac{mP}{s+P+fQ}$. The value m represents the maximum number of prey a predator may devour in a given amount of time, whereas the parameter s represents the saturation constant. The term fQ measures the mutual interference between predators, while the parameter f scales the impact of predator interference.

2.2. Prey-Predator Models with Amplitude Equation. Dutt [19] has established an amplitude equation for the system of a weakly nonlinear hypothesis for a model glycolytic diffusion-reaction system. The resulting amplitude equation's linear stability analysis explains the stability and structural changes of many forms of Turing structures. This amplitude equation also takes into account the fact that time-invariant amplitudes in Turing structures are independent of the activator species' complexing reaction, even though complexing reaction has a significant impact on Hopf-wave bifurcation.

Jana et el. [20] describes a prey–hunter model including prey refuge. Accordingly, authors studied about various biomass density of the prey populaces in two areas. Authors include movements for the prey populaces between two areas. Additionally, its noticed that the hunter populace is softly stronger in nature thus authors assume their density dependent death rate. The reason for seeing the hunter population as the stronger is since there is a location that is totally open for the prey population, hence the hunter population's fight for survival should be stronger. They may die if they are substantially weaker, but because of their comparative stronger nature, they are capable of surviving in the current biological structure. The system's dynamical behaviour is investigated. The presence of refuges clearly has an impact on the coexistence of prey and hunter populations, as evidenced by the results. The presence of a prey refuge causes the density of the prey population to increase, while the density of the hunter population decreases, according to biological criteria. It has been shown that the effects of refuges can help to stabilise the system. The density-dependent death rate for hunters is found to play a substantial impact in the dynamics of the proposed model system. The obtained results show that the density-dependent death rate for hunters can cause a stable equilibrium to become unstable, and interestingly, if the density-dependent death rate for hunters reaches its critical value, a fundamental Hopf bifurcation is obtained. Additionally, if the density-dependent death rate for hunters is high, the prey and hunter populaces will converge to their equilibrium levels after a period of repeated oscillations around the equilibrium. Though, as the density-dependent death rate of hunters falls, oscillations rise, the positive steady state vanishes, and the customer population perishes. On average, the dynamics of delay differential equations are substantially more complex than those of conventional differential equations. The authors also looked at the effects of discrete time-delay on the dynamics of the prey hunter system. Time delay has been found to have a significant impact on the dynamics of the prey predator system. The results show that a time delay can make a stable equilibrium unstable, and that when the time delay reaches a critical threshold, a fundamental Hopf bifurcation occurs. The stability of bifurcating periodic solutions and the behaviour of the Hopf bifurcation are investigated using the Center Manifold Theorem and the Normal Form Method. The authors develop the requirement for the system to be globally asymptotically stable in the presence of delay using a decent Lyapunov function at the interior equilibrium point.

Tewa et al. [21] analytically studied the impact of a SIS infectiuos disease in a Predator–Prey model influencing both Predators and Preys or Preys. The response function utilized in this study is Holling function type II. Authors have seen that when the infection shows up in Prey populace, just the Preys can vanish; just the Predator can vanish; the uninfected Preys and Predators can vanish and then, only diseased preys persist. The illness can vanish or persevere locally. The biological explanation of Hopf bifurcation is that the hunter exists together with the sensitive Prey and the infected Prey, showing oscillatory balance behavior. To prevent the vanishing of the species, one should look cautiously a few parameters, in particular, the infection rate, the growth rate of the sensitive populace and the mortality rate of the infective population. Without sickness, the systems are unsteady about the origin. Consequently, there is no chance of extinction of the populaces. This isn't true in the case of a diseased system. A subset of these outcomes are concerned with the [63]- [66] outcomes. However, because the authors of this study used mass action incidence, they had no problems with behaviour near the source. The behaviour surrounding the origin was investigated, and the dynamics near the origin are complicated due to standard incidence, with the populace arriving at the origin either along the axis or in a spiral pattern.

Considering of the availability of prey, Chen and Zhang [22] developed a delayed hunter-prey model with hunter migration. The model is useful to the significant biological control. Their aim is the impacts of the delay and migration rate on the dynamics of the model. This model can have either two or three equilibria. At the point when the migration rate is consider as a bifurcation parameter, one has in backward bifurcation. Then authors examined the linear stability of the equilibria. The stability of the trivial equilibrium and the boundary equilibrium is very simple. Their stability is delay-free. Though, the result may not be legitimate for the positive equilibrium. Authors got sufficient conditions for the stability and instability for the positive equilibrium. In major circumstances, the stability is depend on delay. Besides, the stability of the positive equilibrium can switch by the delay. Hopf bifurcation can exist at the point where the positive equilibrium drops stability. Utilizing the normal form theory and the center manifold method, authors likewise determined the stability and direction of the Hopf bifurcation. Presently, authors display a few remarks on biological control. The superior outcome is to vanish the prey. It can be obtained by discovering natural adversaries which have main elective food to alive and picking the migration rate smaller than the intrinsic growth rate of the hunter. For this situation, the hunting factor has an significant role rather than the migration rate.Though, eliminating the prey is either not possible or very expensive. The coexistence equilibrium is stable for little delay. it is authentic that the hunter won't require a large time period to change the prey organism. Authors can hold the prey through economic injury level (EIL) by changing the value of the migration rate.

Gambino et al. [23] looked studied how the Turing mechanism causes nonlinear cross-diffusion for two linked reaction-diffusion equations in a two-dimensional space. The possibility that the Turing bifurcation occurs due to a degenerate eigenvalue complicates the inquiry technically, but it results in a large number of patterns (Figure1) that adorn the plane and appear as steady state solutions of the reaction-diffusion system. Squares, mixed-mode, and roll designs, of which supersquares and hexagons are examples. Near the onset of instability, the authors received the amplitude equations, which provide a mathematical explanation of the reaction-diffusion system. The study of amplitude equations has uncovered a number of phenomena, including stable subcritical Turing designs and many branches of stable solutions that lead to hysteresis. Rolls bifurcate supercritically from their equivalent fundamental bifurcation point, and hexagonal patterns form by a subcritical bifurcation. Hence, there is a bistable area where the two rolls and hexagons are stable; nevertheless, rolls appear as a transient state due to a spatially modulated cross-roll instability, forcing the solution toward a mixed modes design. We were only partially successful in explaining the instability as a result of mode competition.

The dynamics of a diffusive hunter-prey system with the Beddington–DeAngelis functional response and two delays were examined by Zuo [24]. The presence of Hopf bifurcations and the stability of constant equilibria are investigated by studying the characteristic equations using the two delays as a bifurcation parameter. Using the normal form hypothesis and the centre manifold of partial functional differential equations, the equations determining the stability and direction of bifurcating periodic solutions are also inferred. The upper–lower solutions technique, in particular, provides sufficient requirements for the global stability of the positive equilibrium.

Xu [25] looked on the dynamics of a diffusive prey–hunter model with broad functional response and prey stage structure. First, he considers Hopf bifurcation and asymptotical stability of equilibrium points for the simplified ODE system. For the equivalent reaction-diffusion system, uniform boundedness, the presence of global solutions, and the stability of equilibrium points have also been investigated. Finally, we demonstrate the significance of huge diffusivity by constructing the presence and nonexistence of nonconstant positive stable states of this reaction-diffusion system.

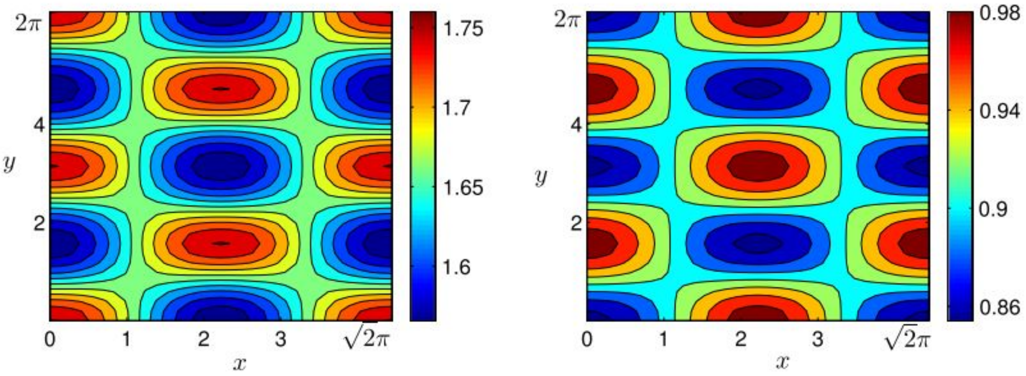


FIGURE 1. Left: the species u. Right: the species v. The parameters are $\mu_1 = 1.2, \mu_2 = 1, \gamma_{11} = 0.5, \gamma_{12} = 0.4, \gamma_{21} = 0.38, \gamma_{22} = 0.4, a_1 = 0.01, a_2 = 0.001, c_1 = 0.1, c_2 = 0.2, \gamma = 28.05, b_2 = 1.1, b = 7.264 > b^c = 7.192$.

The outcomes demonstrate the importance of the hunter species' dispersion rate. The hunter's high diffusion rate will aid in the formation of patterns. However, the absence of spatial patterns can be caused by a high diffusion rate of young prey species or a high diffusion rate of adult prey species.

According to the standpoint of bifurcation evaluation, Aguirre et al. [27] examined a predation model with ratio-dependent functional response and strong Allee effect. By expanding the model vector field over the full first quadrant and modifying the parameters suitably, the authors demonstrated that any viable solution stays both nonnegative and confined. The authors also looked at the existence and local stability of equilibrium sites along the positive horizontal axis. When the size of the prey population equals its carrying capacity, this equilibrium occurs, and it can either be a saddle point or an attractor. The second equilibrium can be either a repeller or a saddle point, with a population size equal to the rescaled Allee parameter $m = \frac{M}{K}$. It was revealed that the origin is frequently a non-hyperbolic attractor. As a result, the origin always has an open two-dimensional bowl of attraction in the first quadrant for every combination of parameter values. As a result, there is long-term extinction when the initial conditions are in this bowl. The sort of bowls of attraction of attractive equilibria on the inner of the first quadrant determines the circumstances for the two species' endurance. They discovered particular types of conditions on parameters that allow a positive equilibrium point to pass through Bogdanov–Takens, saddle–node, and Hopf bifurcations using hypothetical techniques from nonlinear dynamics and bifurcation inquiry. Furthermore, the authors demonstrated that the Bogdanov–Takens point is an organising centre for the probable endurance/extinction situations of the two populations when the hunter growing ability is greater than the rescaled mortality rate, using correct methods for numerical bifurcation examination and the calculation of global invariant manifolds of equilibria, which were carried out in the software package Auto. The features of the bowl of attraction of the attractive equilibrium are illustrated in many conceivable scenarios around the Bogdanov–Takens singularity by the authors. Its bowl boundary appears to be the mathematical Allee threshold that both species must surpass in order to avoid falling into the origin's bowl of attraction, which leads to mutual extinction.

2.3. Time Delay and Hopf Bifurcation Investigation. Khajanchi and Banerjee [28] studied an enhanced quantitative mathematical model of Kuznetsov et al. [29] addressing tumor-immune interaction with discrete time delay. The model's qualitative investigation, which included Hopf bifurcation investigation, was studied. To preserve stability and related criteria, a delay estimation has been acquired. The authors get definite expression for the direction of Hopf bifurcation and the stability of bifurcating periodic solutions utilizing the centre manifold theorem. The logical discoveries have been confirmed through mathematical reenactments. If we fluctuate the time delay and also the system parameters, then the model system shows complex dynamical behavior. The suggested model has an exceptionally rich biological dynamics and shows stable irregular and regular periodic oscillations for various parameter values. The model doesn't consider the biological complexity of tumors but this model target on the generic reaction between the immune system interactions and tumor cells. By Utilizing the model simulations, the eccentric development of tumor cells in vivo and the clinical tests could be clarified or solved. The model indicates for both the low and high tumor load can have oscillatory behaviors. The connection among the immune system and cancer demonstrates more complicated dynamical behaviour such as periodic and non-periodic oscillations, as well as arbitrary behaviours such as chaotic or high periodic behaviours, when the parameters are changed across a huge range. The impact of different system parameters for various amounts of time delay is examined, revealing the system's complex biological dynamics.

Lett et al. [30] did the study of a group of aerial and aquatic hunters assaulting a swarming fish school showed that a multi-explicit gathering of hunters are more productive at irritating a fish school than a solitary-species gathering of predators. Likewise, the irriation is higher when the number of assaults is such that prey have not sufficient time to get back to swarming. These discoveries propose that aerial and aquatic foragers might got common advantages in making mixed foraging gatherings. Even single-species gathering of hunters are more effective at irritating prey by increasing number of assaults.

Sharma and Samanta [31] have built an eco-epidemiological hunter prey model where just the prey populace is infected by an infectious infection. Thus, the prey populace is classified into two sub categories: infected and susceptible. Additionally, authors have included a susceptible prey endurance limit for Allee impact. Then, at that point authors have talked about the dynamical nature of the system at different equilibrium points and the stability of equilibrium points for both weak and strong Allee effect. The system's stability flips around the interior equilibrium, and a Hopf bifurcation exists around the interior equilibrium, using the susceptible prey survival limit for Allee effect as the bifurcation parameter. All significant analytical discoveries are confirmed mathematically by utilizing MATLAB. For both weak and strong Allee effects, mathematical investigations display the dynamical nature of the system at various equilibrium points for various sets of parameters. These data show a good contrast between the behavioural changes in the model's dynamics for weak and strong Allee effects, which is undoubtedly very helpful in understanding the model's dynamics and agrees with our in-depth analyses. It can be seen that the sensitive prey survival limit for the Allee effect assumes a vital role in maintaining populace stability. Our mathematical investigations show how the stability alters the interior equilibrium when a is used as the bifurcation parameter. At last, the authors find that the eco-epidemic hunter-prey model for Allee effect with susceptible prey survival limit displays fascinating dynamics. However, due to the assumptions made and the problems in calculating the model parameters, the mathematical model introduced in this study should be approached with caution. Just sensitive prey species are thought to grow logically, while infected prey species are unable to reproduce since they die before reaching reproductive age. In this way, the authors may fine-tune the model by assuming the infected prey populace's logistic development. The infection in the hunter populace can likewise be incorporated, which might give us an exceptionally rich elements.

Zhang et al. [32] used the diffusion to investigate pattern development in a spatial hunter-prey model with hyperbolic mortality rate. The Turing space was initially partitioned, with the Hopf bifurcation curve and Turing instability curves surrounding it. After that, the authors examined the stability of certain basic bifurcations, such as squares, stripes, and spots, using the amplitude equations. Finally, if the control parameter is not in the Turing space, we analytically investigated the starting value-controlled design simulations. Furthermore, the authors discovered the dynamical distinction between the various death rates. For example, the occurrence of the positive equilibrium, which exists only if certain requirements are met for quadratic and linear [33] death rates [15]; however, it frequently exists for hyperbolic death rates, which are used when modelling plankton population dynamics [34]. According to an environmental perspective, in general homogeneous cases with quadratic and linear mortalities, both prey and hunter populaces will end ultimately due to the non-presence of positive equilibrium. In any event, if it exists under some condition, linear death rate in the spatial model can not generate Turing instability however fascinating grid, and winding designs have been found [33]. For this situation, the positive equilibrium is either consistently stable or unstable, but none of the forms can generate Turing instability. In ecology, there are two related phenomena: (1) hunter and prey populations coexist in the relevant 2-D region, and (2) hunter and prey densities oscillate, which is related to irregular designs. Although quadratic mortality cannot generate squares [15], it can cause Turing instability. Positive equilibrium exists in the absence of hyperbolic mortality, and when it is stable, the hunter and prey exist together in general. Stable Turing designs structure, which not just suggests both hunter and prey persevere in space, but in addition some environmental implications: spots are expected as a hunter guard function, and stripes are connected to the hunter guard and social communication [35]. The environmental implications of spirals and squares are not satisfactory right now.

The spatial patterns formation of hunter prey systems is a significant issue. Huang et al. [36] have address the problem by constructing a time-discrete and new space hunter-prey model with a Beddington-DeAngelis functional response to characterise the predation relationship. The discrete model is based on a coupled map lattice with a nonlinear interaction between the hunter-prey reaction stage and the dispersal stage. The parametric requirements for design production are derived by investigating Hopf instability and Turing instability for the discrete model. Mathematical simulations display an interesting collection of spatiotemporal patterns, containing irregular and regular patterns of labyrinth, stripes, spots, spirals, mosaics, gaps, circles, and some inermediate patterns in-between. The designs obtained cover the maximum of hunter prey design types documented in the literature. Moreover, the discrete model forecast the existence of spatiotemporal choas, which are liable for the production of uneven patterns. This study exhibits that the discrete model's nonlinear mechanisms grab the comlexity of design production in hunter-prey systems better.

Ma et al. [37] studied and examined a stage-structured hunter prey model with square root functional responses and pulses. With the help of simulation, authors display the rich dynamic characteristics of the related system. The global attractivity of the hunter-eradication periodic solution of system is initially shown to be dependent on the harvesting effect and mortality. Not just the worldwide attractivity furthermore, the lastingness are represented adequately in this part, yet in addition the complicated bifurcation charts are provided. In paper [38], Jiao and Meng inferred that abundance collecting could cause the annihilation of hunter populace, and sensible reaping could guarantee the supportable improvement of natural assets, and the conduct of rash loading on prey assumed a significant part for the changelessness of the framework they examined. Our outcomes in this paper are steady with those in [38]. Nonetheless, we research seriously fascinating dynamical practices of system that Meng and Jiao had not broke down. The got outcomes suggest that, if the mean length of adolescent period tau is bigger than a specific worth, the adult hunter will be too alarm to create, henceforth, we should control gathering in a less level to guarantee the feasible improvement because of the long adolescent period tau of juvenile hunter. Moreover, bifurcation outlines of imprudent period T show that the system displays stable, multiplying occasional, mayhem, cycles, course, etc. We can be certain that time delay and drive assume a significant part in the administration of agrarian assets, which makes the biological system shows more intricate and flighty practices.

Mbava et al. [39] examined the dynamics of a hunter-prey model with super hunter infection. The hunter is confronting extinction due to huge competition from the super hunter. The infection is assumed to be biological control, allowing the hunter population to recapture from a low population. The outcomes spotlight that without extra death rate on hunter by super hunter, the hunter populace survives eradication. At present levels of illness incidence, the super hunter populace is vanished out by the sickness. Though, the super hunter populace survives by eradication if the illness occurrence rate is low. Perseverance of all populaces is probable on account of low infection occurrence rate and no extra mortality granted on hunter. A subsystem with two species, hunter and prey, is also assumed as an extraordinary example to find the impact of removing the super hunter from the system on the hunter's survival. This is regarded as a unique feature of the smaller parks. The outcomes demonstrates that the hunter populace flourishes good in the absence of its primary competitor, with its populace ascending to at least double its starting value.

Wang et al. [40] studied a spatial Lesile type hunter prey system with time delay and Holling type III functional response. To know the effect of diffusion and delay on the instability, authors have made hypothetical investigation and mathematical simulations. Because of the equilibrium can't be displayed in a applicable closed form, we can't talk about its qualitative properties in ordinary daily practice. Initially, we talk about the qualitative characteristics and number of +ve equilibrium through the actual parameters. Besides, there are two kinds of instability that authors have discussed here: delay promoted instability and diffusion induced instability. At last, mathematical simulations are carry out to display the hypothetical discoveries (Figure2). Both the hypothetical and mathematical outcomes display that the interaction between time delay and diffusion can bring about stationary patterns. Though, it

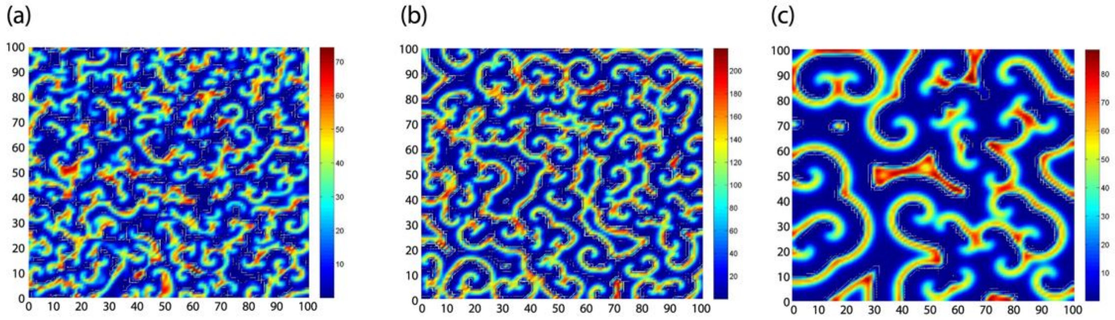


Figure 2. Three types of spirals induced by delay. The parameters are (a) $\eta = 1, \tau = 1.2, D_1 = 0.002, D_2 = 0.1, u^* = 0.3, v^* = 19.13333333, \epsilon = 80, \gamma = 0.01567944251$. (b) $\eta = 0.8, \tau = 1.5, D_1 = 0.002, D_2 = 0.08, u^* = 0.3, v^* = 52.73333334, \epsilon = 240, \gamma = 0.005689001264$. (c) $\eta = 0.85, \tau = 1.5, D_1 = 0.01, D_2 = 0.38, u^* = 0.3, v^* = 19.13333333, \epsilon = 80, \gamma = 0.01567944251$.

must be noticed that the technique used in this study is just appropriate for little time delays. If the delay is huge, different methods should be applied to discover the Turing instability condition.

Xu et al. [42] investigated a hunter-prey model with prey species defence mechanisms. Initially, the existence of Hopf bifurcation, local stability of equilibriums without trial, and singular dynamics near to the origin equilibrium were investigated. The authors then showed that if the positive equilibrium is locally asymptotically stable or does not exist, periodic orbits do not exist. If the positive equilibrium is unstable, the uniqueness and existence of breaking point cycles are proved. We were able to determine the model's global dynamics by using the outcomes. The dynamics of a hunter-prey model with a protection component are more rich than conventional models and make ecological sense. The rise of oscillation is due to the protection component of the prey species, according to the conventional L-V predation model. Also, traditional models normally estimate the origin equilibrium as a saddle point, implying that the prey species will recover regardless of how little it recovers in comparison to the hunter species. Conversely, solution behaviour around the origin equilibrium is singular for non-dimensional models. When the prey populace is reasonable lesser than the hunter populace, the prey species goes extinct first, followed by the hunter species. This is sensible from an ecological standpoint.

Banerjee et al. [43] have investigated a spatio-temporal prey-hunter model with ratio dependent functional response and cross-diffusion terms. The impacts of cross-diffusion to the model have been studied by using numerical simulations and amplitude equations in nearby area of Turing bifurcation curve. The functional response relies upon the densities of both hunter and prey populace. Though, when the prey just only dependent functional response is considered, then the self-dispersion framework don't give any Turing pattern except if nonlinear or linear cross-dispersion terms are involved in the system. The shifting of the species of a populace is exceptionally affected by the attendance, nonappearance or plenitude of the other. So, cross-dispersion terms are incorporated in the spatio-temporal prey-hunter model, and the obtained patterns are influenced by them. In the spatio-temporal expansion of the supposed model, cross-diffusion plays a meaningful role in pattern resolution and selection. Cross-diffusion variables are included to account for the impact of a single individual's population density on the movement of different species that are separated from arbitrary diffusion of both species.

Chen [44] did an investigation on the diffusive hunter-prey system. A wide range of hunter-prey models was covered by authors which incorporate some famous ones but additionally some less examined ones. The conversion rate is an important component that influences the dynamics of a general hunter-prey model, according to the authors, and there are very few complex patterns for small and large conversion rates. Consequently, this incident can happen ordinarily in hunter-prey models, as shown in [45] for a unique hunter-prey model with a nonlinear development rate. Authors show the nonexistence of the positive steady states in the case of large conversion rate.

2.4. Eco-epidemiological Model, Turing Instability and Turing Patterns. Huang et al. [45] built a delayed eco-epidemiological model. In addition, the model is mathematically investigated in terms of dynamics like positive solutions, equilibrium stability, and their existence. Moreover, utilizing the incubation duration as a bifurcation parameter, it can be seen that if the parameter reaches specific critical values, a Hopf bifurcation might happen around the equilibrium points. The Hopf bifurcation's stability and direction about the inside equilibrium point were also examined. The age-structure, stage-structure, and spatial part of ecological interactions have all been recognized as significant ingredients in how ecological communities are formed, and knowing the importance of these impacts is difficult both hypothetically and observationally [Wang et al. [46]]. The dynamic examination for the epidemiological prey–hunter model is far from complete due to its complexity [Wang et al. [46]; Das [47]]. Under certain boundary conditions, it is necessary to better understand the dynamical behaviours of an epidemiological prey–hunter model with age-structure, reaction-diffusion, or stage-structure impacts.

Tan et al. [48] investigated the dynamic behaviour of a reaction–diffusion hunter–prey model with Beddington–DeAngelis functional response in an experimental case study. The comparative ODE system allows for complex dynamics as well as stable limit cycles and a Hopf bifurcation. They have demonstrated global and local stability, Turing designs, and fixed patterns with dispersion and cross-dispersion in [Jiang et al. [49]; Zhang and Fu [50]]. The Hopf bifurcation and Turing instability are investigated in this paper. Additionally, they demonstrate the existence of Hopf bifurcation, nonconstant steady state solution, and Turing–Hopf bifurcation using numerical simulations, demonstrating that diffusion coefficients and parameters affect spatiotemporal behaviour. More investigations are mandatory to inspect the quantitative and qualitative changes of pattern production.

The stability and dynamics of the hunter-prey system are significantly affected by epidemic transmission, with the transmission rate playing a key role. Zhang et al. [51] employed a probabilistic cellular automaton (PCA) technique to investigate the spatiotemporal dynamics of a hunter-prey system with an infected predator. In a geographical populace, obtaining a conjunction state of prey, vulnerable hunters, and infected hunters is impossible. This is not the same as using the mean-field estimate to investigate a non-spatial populace, when Hopf bifurcation occurs and the inner equilibrium becomes unstable, and a periodic solution appears as the disease rate rises. The outcomes suggest that presenting an infected hunter with a high transmission rate is advantageous to the prey population's stubbornness in space. A low transmission rate, on the other hand, will result in the cohabitation of prey and sensitive hunter populaces. In brief, management approaches can be developed to reduce the infected hunter's transmission rate for the benefit of biological control.

In hypothetical ecology, serious investigations of the mechanisms and scenarios of pattern development, in models of interacting populaces, have consistently been a fascination, as their realization helps to improve the comprehension of real-world natural systems. Singh and Banerjee [52] performed an analysis on a diffusive hunter-prey model with type II functional response in hunters and non-zero initial conditions and zero-flux boundary conditions. The authors conducted thorough research of both spatiotemporal and non-spatial models, as well as an analysis of alternative design production scenarios in the diffusive hunter-prey model with hunter collaboration. We first obtained the condition for diffusive instability and distinguished the relevant region in the space of controlling parameters while focusing on the spatiotemporal model. In our study, the regulating parameters are the hunting cooperation coefficient, the hunter's fundamental reproduction number (C), and the ratio of dispersion coefficient. The authors use extensive mathematical simulations to investigate the system's properties using parameter values from both the non-Turing and Turing domains. The model simulation has been split into two domains: Turing and non-Turing domains. The impact of hunting collaboration in hunters, as well as the fundamental reproduction quantity of hunters, has been highlighted by the authors. The authors confirmed that in the non-Turing domain, with $C > 1$, the hunter population rose with slight hunting cooperation and decreased with an increase in the hunting cooperation coefficient by simulating numerically. Furthermore, for $C < 1$, an increase in hunting cooperation among hunters aids their survival. In the Turing domain, where $C < 1$, hunting cooperation among hunters is critical for coexistence. We acquire many types of diffusive patterns by varying the values of fundamental reproduction number and cooperation coefficient, such as mixed patterns, stripe patterns, and patchy patterns. Anyone can see the pattern development for preys from the perspective of population dynamics, meaning that the prey are spread with low density and the residual region is with high density, implying that the preys have isolated in small gatherings over a large area and are protected. Similarly, spot formation in hunters ensures that, even when hunting cooperatively, the hunters are dispersed and segregated, the hunter lives. Big African hunters like lions (Pathera leo), leopards (Panthera pardus), and cheetahs (Acinonyx jubatus) regularly predate ungulates two times their size, posing a risk of injury or death to the hunter during prey capture. However, cooperative hunting can easily overcome this risk, potentially increasing hunting success rates. [53]

Rao et al. [54] focus on the complicated dynamics of a Monod-Haldane-type hunter-prey cooperation model, which includes: (a) A predetermined time delay in the development of the prey; (b) dispersion in both the hunter and the prey. When the associated ODE model includes either a single inside equilibrium with two inside attractors or two inside equilibria, numerical simulations were run to show the effects of dispersion and delay. Theoretical and numerical results reveal that dissemination can either destabilise or stabilise the system, that a large delay can destabilise the system, and that the combination of dissemination and delay can exacerbate the system's instability (Figure3 - 5). Also, if this ODE system has two interior equilibria, time delay or prey dispersion, or both, could lead to the hunter's extinction. The findings could provide us with useful biological information on population management for prey-hunter cooperative systems.

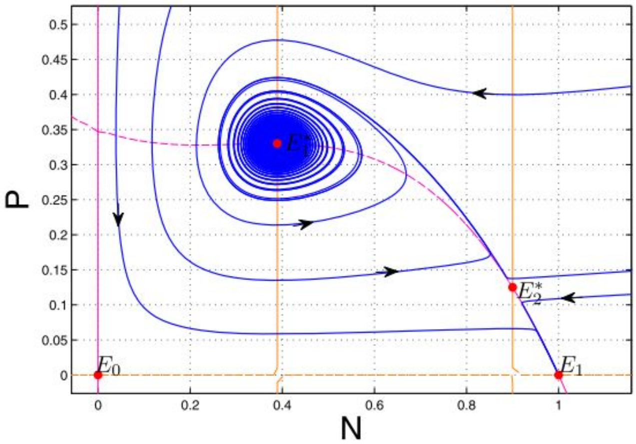


FIGURE 3. $r = 1.0$, $a = 0.35$, $b = 0.1$, $d = 0.72$: $E_1^* = (0.3889, 0.3301)$ is locally asymptotically stable.

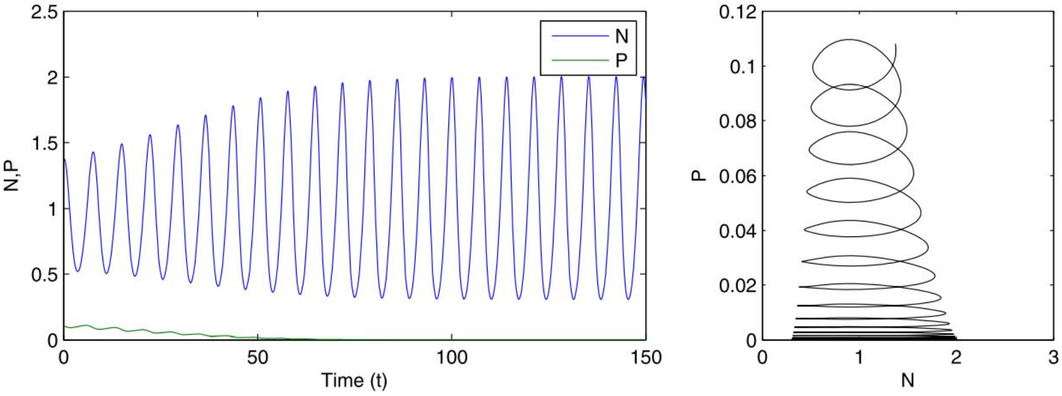


FIGURE 4. $\tau = 1.716 > \tau_0 = 0.676$: Large delay causes large oscillations that lead to the extinction of predator.

Lin et al. [55] studied a new homogeneous diffusive hunter-prey system with herd behaviour extending to the square root of the prey population. The authors are able to demonstrate the existence of both spatially homogeneous and inhomogeneous periodic solutions bifurcating from the positive constant steady state solutions using the classical Hopf bifurcation theorem pertinent to the generic reaction-difusion equations due to [56]. The equations for determining the instability and stability of homogeneous periodic solutions are obtained by the authors (Figure 6 - 7). There are three main points of focus: (i) Instead of considering the scenario where the saturation rate is equal to zero, the authors explore the more general case when the saturation rate is greater than zero. (ii) Show that the Hopf bifurcation points of the system are significantly more complex than those in [56]; (iii) Use the first component of the positive equilibrium as the bifurcation parameter, rather than the mortality rate of the hunter population and the diffusion coefficient. The findings aim to provide a clearer

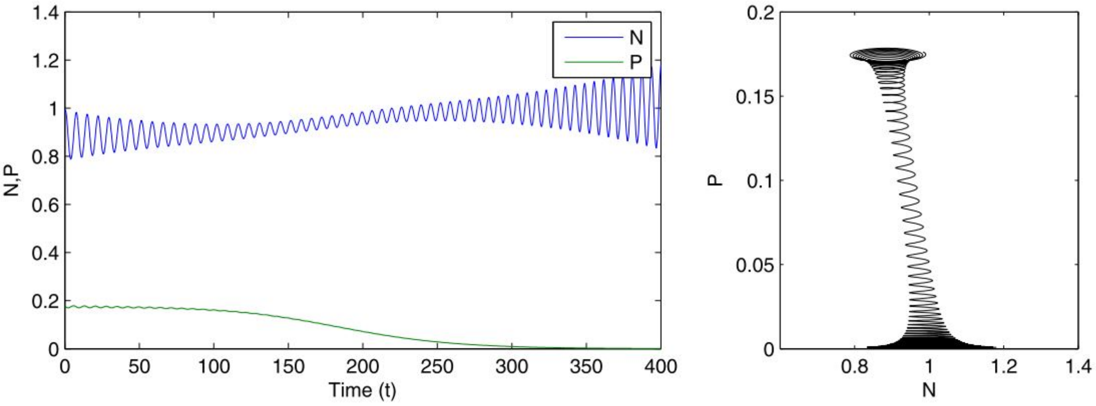


FIGURE 5. $\tau = 1.615$: Large delay leads to the extinction of predator.

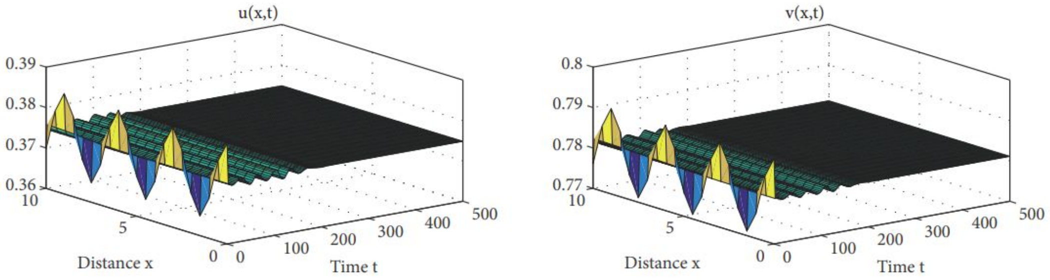


FIGURE 6. Numerical simulations of a system with parameters in a stable homogenous equilibrium solution. The left: component u; the right: component v.

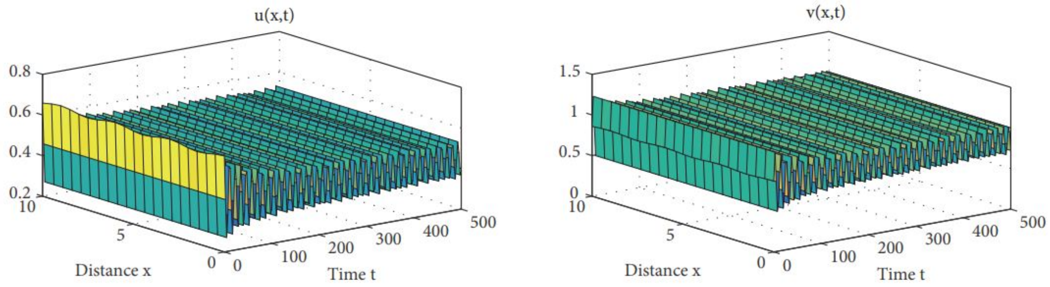
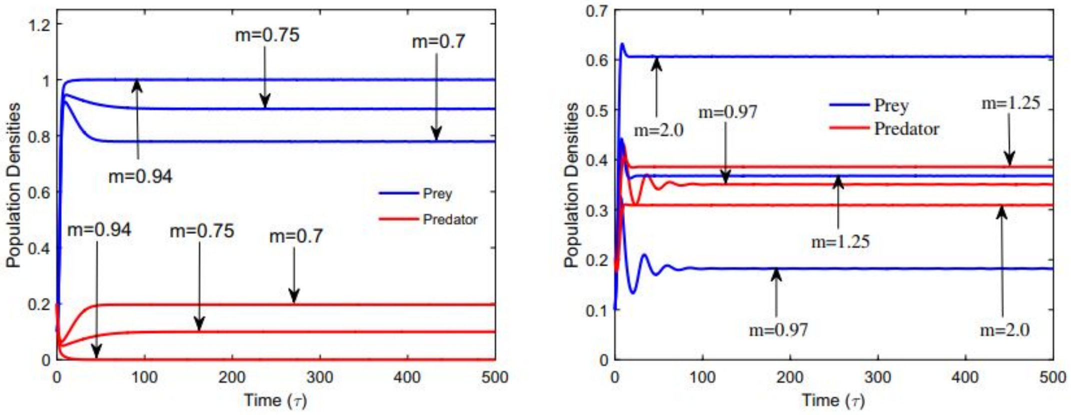


FIGURE 7. Simulations of the Hopf bifurcating spatially homogeneous periodic solutions of system using numerical methods with parameters.

explanation of the mechanism underlying the creation of spatiotemporal patterns in hunter-prey interactions in ecology.

2.4.1. *Investigation of the impact of quadratic and linear mortality on herd behaviour.* Singh and Banerjee [57] investigate the impact of quadratic and linear mortality on herd behaviour in a hunter-prey scenario. According to the nonspatial study, the system with increasing linear mortality is wiped out, whereas the system with quadratic mortality coexists. The hunter-prey system with linear mortality in a spatiotemporal



(a)

(b)

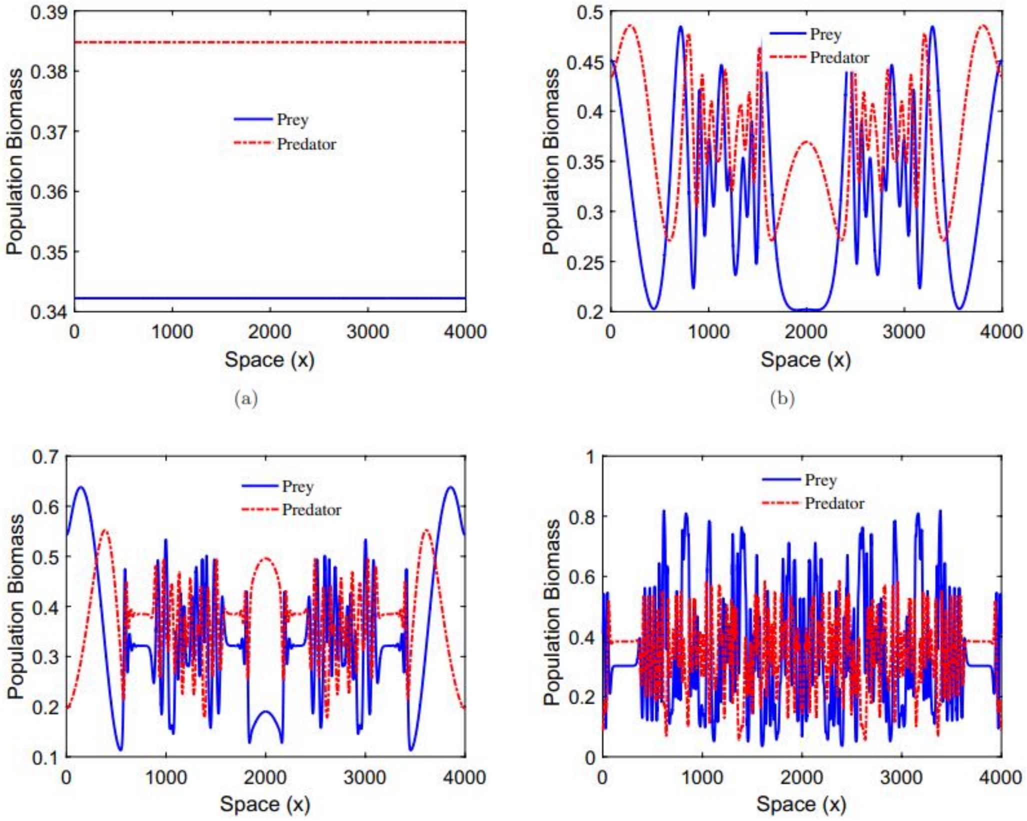
FIGURE 8. (a) Dynamics of preys (u) and predators (v) in the nonspatial domain of the model and (b) dynamics of preys (u) and predators (v) in the nonspatial domain of the model if we select the quadratic death rate for predator population. Other parameter values are $\beta = 0.8$ and $\gamma = 0.01$.

model exhibits four essential dynamics, including stable, smooth oscillatory, intermittent chaos, and chaos encompassing the majority of the space (Figure8) .

The non-Turing design is regular and smooth, and it expands with time, resulting in a chaotic pattern in both the prey and hunter populations (Figure9). This circumstance does not appear to have a Turing design. The system with quadratic mortality, on the other hand, demonstrates diffusive induced instability, resulting in Turing designs. We get several kinds of diffusive patterns when we change the death rate, such as mixed patterns and patchy patterns, implying that there is a spot design replication, where the prey is in a segregated zone with low density and the leftover area has a high density. The spot-stripe pattern transforms into spots as the ratio of diffusion coefficients increases, implying that the prey and hunter populations are converging (Figure10 - 11). Individuals' adaptability for the two species within their surroundings can help the preys and hunters work together more effectively.

Lu et al. [58] investigated an eco-epidemiological model with time delay that addressed the hunter's gestation time. In this scenario, the hunter population is infected with a contagious disease, and infected hunters may recover from the disease and become susceptible once more. The local stability of all feasible equilibria and the occurrence of Hopf bifurcations at without disease and coexistence equilibria are established, respectively, by inspecting reated characteristic equations. Sufficient requirements for the global stability of the coexistence equilibrium, the sickness-free equilibrium, and the hunter-extinct equilibrium of the system are determined using Lyapunov functionals and LaSalle's invariance principle, respectively.

Alidousti [59] investigated the effects of harvesting hunters and scavengers as well as the impact of fractional derivative on a prey-hunter scavenger model. In the interior equilibria, boundedness, positivity of solutions, as well as equilibria stability and Hopf bifurcation are investigated. After a given sum, it is found that fractional derivative promotes chaotic system stability. The results of the inquiry were confirmed by numerical simulation, which showed that harvesting hunter and fractional derivative



(a)

(b)

(c)

(d)

Figure 9. One-dimensional non-Turing dynamic patterns of the model at time moment $\tau = 2500$: solid blue line for prey and dashed red line for predator. (a) $m = 0.47$, (b) $m = 0.46$, (c) $m = 0.45$ and (d) $m = 0.434$. Other parameter values are $\beta = 0.8$, $\gamma = 0.01$ and $D = 1.0$.

cuts became the reason for beefing and consistency in this model. It has been observed that chaos is absent in this ecological model, and it has been established that fractional order plays a critical part in this incidence.

Bezabih et al. [60] established a susceptible-infected prey and hunter of eco-epidemiological model based on physiologically significant and significant assumptions. Every possible equilibrium point has been calculated. The stability of endemic and disease-free equilibrium points is investigated on a global and local scale. In a simulation research, it was discovered that prey species are declining due to viral disease, yet this works in conjunction with the hunter's ability to effectively consume the prey, resulting in a rise in the hunter. When infective prey was completely consumed by hunters, susceptible prey had no chance of being infected by infective prey. As a result of the irresistible disease wiping out the susceptible prey, the prey-hunter system stabilises for a while.

Under zero-flux boundary conditions and non-zero initial conditions, Singh and Dubey [61] built a diffusive hunter-prey model with hunting cooperation in hunters and Holling type IV functional response. The authors investigated both spatiotemporal and non-spatial models in depth, focusing on potential pattern formation scenarios in the diffusive hunter-prey model with hunting cooperation among hunters. The authors initially identified the comparison domain in the space of control parameters

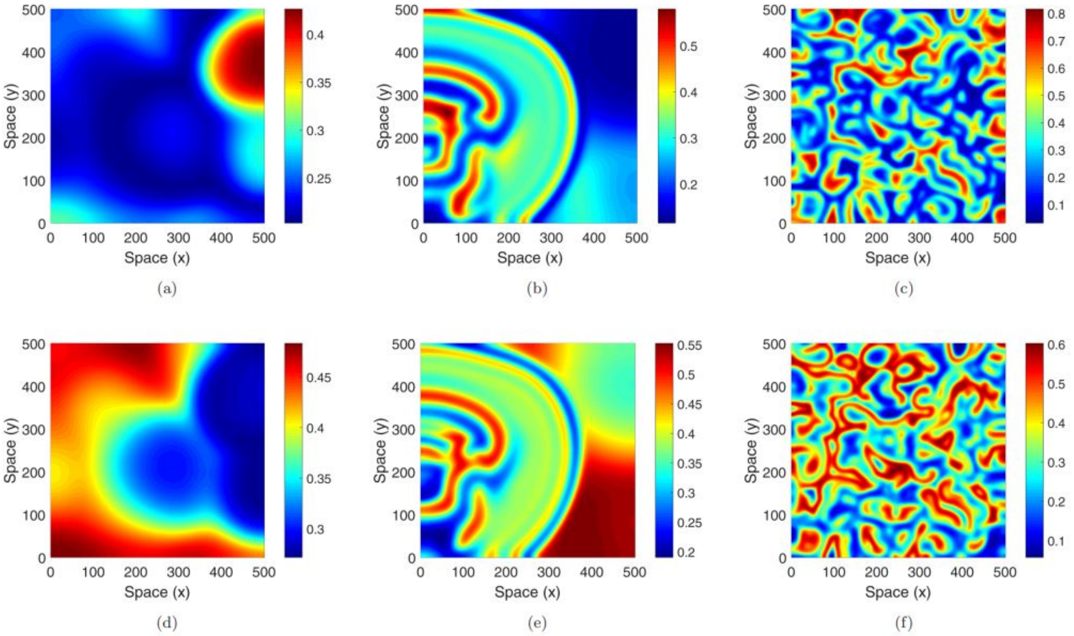
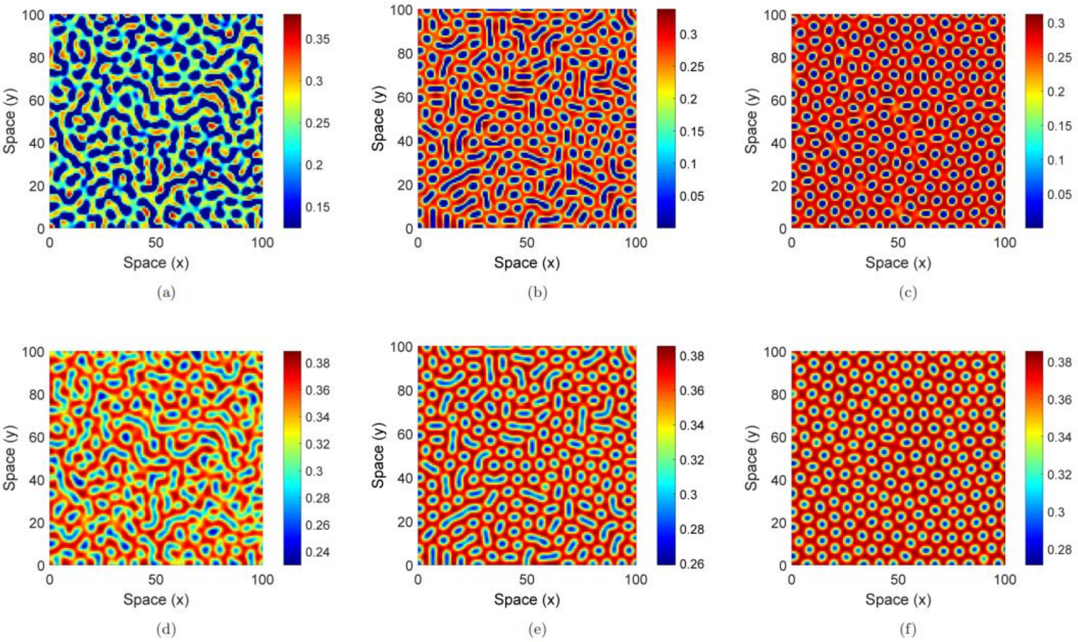


FIGURE 10. Two-dimensional non-Turing dynamic patterns of the model for prey (upper panel) and predator (lower panel) at time moment $\tau = 2500$. (a) and (d) $m = 0.46$, (b) and (e) $m = 0.45$, (c) and (f) $m = 0.434$. Other parameter values are $\beta = 0.8$, $\gamma = 0.01$ and $D = 1.0$.

and obtained the requirement for diffusive instability. The control parameters in this study are the dispersion coefficient ratio and the hunting cooperation coefficient. We investigate the rise and types of spatial patterns of the system using extensive numerical simulations and parameter values from the Turing region. Model simulation has been divided into two categories: spatial and non-spatial domains. The impact of hunting collaboration in hunters, as well as the hunters' carrying capacity, has been highlighted. The increase in hunting cooperation in hunters helps them to survive in non-spatial space, according to numerical simulation. Hunting collaboration among hunters plays a critical role in coexistence in the spatial space. We acquire many forms of diffusive patterns by varying the values of the cooperation coefficient, for example, mixed pattern, stripe pattern, and patchy pattern. According to populace dynamics, there is a pattern production (spot) for preys, implying that the preys are distributed with low density and the remaining area is dense, implying that the preys have isolated themselves in small gatherings over a large area and are protected. Hunters' spot development passes on that with hunting cooperation, the hunters are dispersed and constrained, but they persist. Big African hunters like lions (Panthera leo), cheetahs (Acinonyx jubatus), and leopards (Panthera pardus) consistently predate ungulates two times their mass with the risk of injury or death to the hunter during prey capture, but they can undoubtedly be overwhelmed by cooperative hunting, which could improve hunting success rates [62].



(a) (b) (c)

(d) (e) (f)

Figure 11. Two-dimensional Turing dynamic patterns of the model (if we select the quadratic death rate for predator population) for prey (upper panel) and predator (lower panel) at time moment $\tau = 2500$. (a) and (d) $m = 0.93$, (b) and (e) $m = 0.97$, (c) and (f) $m = 1.0$. Other parameter values are $\beta = 0.8$, $\gamma = 0.01$ and $D = 0.08$.

3. Research gaps and future directions for research

During the last two decades, most of the research in ecology is based on local linear stability, existence, Hopf-Bifurcation, and global Stability of the model. Some researchers used Turing ideas for the temporal prey- predator model. Though Turing's patterns have the scope in biological study especially for prey-predator, for study the behavior of group or individual on land or in oceans. Till now all the previous research and researcher used only the temporal study of the prey-predator system but in future we can study the spatiotemporal prey-predator system with the reaction-diffusion part and the effect of time delay. However, as we look to the future, an exciting avenue emerges for delving into the intricate interplay between space and time. This involves the incorporation of reaction-diffusion mechanisms and the exploration of time-delay effects in spatiotemporal prey-predator systems. By extending our analytical frameworks to encompass both spatial and temporal dimensions, we stand to gain a deeper understanding of the intricate dynamics governing ecological systems. As the boundaries of ecological research continue to expand, embracing these complexities promises to uncover novel insights that will enrich our grasp of the natural world.

4. Conclusion

In conclusion, the study of Prey-Predator systems within the realm of ecology has unveiled a dynamic and intricate interplay between species that has captivated researchers for decades. The past decade has witnessed remarkable progress in understanding these systems, particularly in the context of various behavioral adaptations exhibited by both prey and predators. Through an extensive review of research papers from the last ten years, it is evident that the incorporation of behavioral dynamics has enriched our comprehension of these interactions and shed light on the nuances that shape ecosystems.

Behavioral adaptations in prey, such as defensive mechanisms and foraging strategies, have been shown to significantly influence their survival and reproductive success. These adaptations are not only integral to the survival of prey individuals but also have cascading effects on the structure of entire communities. Meanwhile, predator behaviors encompassing hunting tactics, movement patterns, and decision-making processes have been found to intricately coevolve with those of their prey. The interdependence between these behaviors forms a complex web of interactions, dictating the ebb and flow of populations within ecosystems.

Furthermore, the integration of technological advancements, such as advanced tracking and modeling techniques, has permitted a more nuanced exploration of these behavioral dynamics. These tools have enabled researchers to observe real-time interactions in unprecedented detail and to develop more accurate predictive models, enhancing our ability to anticipate the outcomes of prey-predator dynamics under various scenarios.

However, it is apparent that there remain gaps in our understanding, suggesting exciting avenues for future research. Exploring the impact of anthropogenic factors, climate change, and habitat alteration on these intricate relationships is of paramount importance. Additionally, delving deeper into the molecular and genetic underpinnings of behavioral adaptations could provide insights into the mechanisms driving evolutionary change in these systems.

In summation, the past decade has demonstrated that behavioral adaptations are central to the functioning of Prey-Predator systems in ecology. This review has highlighted the multifaceted nature of these interactions, showcasing the delicate balance that sustains ecosystems. As we continue to unravel the mysteries of the natural world, it is imperative that we recognize the significance of behavioral adaptations in shaping the intricate tapestry of life on Earth. Through continued interdisciplinary collaboration and innovative research, we are poised to uncover even more profound insights into the fascinating dynamics of Prey-Predator interactions in the years to come.

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