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$\bar{T}$-CURVATURE AND $\bar{C}$-BOCHNER CURVATURE ON LP-SASAKIAN MANIFOLDS ADMITTING A GENERAL CONNECTION

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Abstract. In the present work, it is proved that LP-Sasakian manifolds which are $\bar{T}$-flat, quasi-$\bar{T}$-flat, ξ-$\bar{T}$-flat, ϕ-$\bar{T}$-flat, $\bar{T}$-semi-symmetric, ϕ-$\bar{T}$-Ricci recurrent, $\bar{C}$-Bochner flat, or satisfy $\bar{T} \cdot \bar{S} = 0$, are generalized η-Einstein manifolds. The $\bar{T}$-curvature and $\bar{C}$-Bochner curvature tensors are defined with respect to a general connection $\bar{\nabla}$ which includes the quarter-symmetric metric, Schouten–van Kampen, Tanaka–Webster, and Zamkovoy connections.

1. Introduction

The idea of contact geometry has developed from the mathematical conceptualization of classical mechanics [6]. One of the most significant classes of contact manifolds is the class of Sasakian manifolds [3]. Sasakian manifolds were introduced by Sasaki [12] in 1960, and they can be described as odd-dimensional counterparts of Kähler manifolds. Para-Sasakian manifolds were defined by T. Adati and K. Matsumoto [1] by considering them as special cases of an almost paracontact manifold introduced by I. Sato [13]. The notion of Lorentzian almost paracontact manifolds was developed by K. Matsumoto [9]. Briefly, Lorentzian para-Sasakian manifolds are called LP-Sasakian manifolds. The investigation of LP-Sasakian manifolds is of great importance due to their admissible applications in the general theory of relativity.

It is well-known that the Levi-Civita connection is the unique torsion-free metric connection on a (pseudo-)Riemannian manifold. Besides the Levi-Civita connection, S. Golab introduced the notion of quarter-symmetric connections in [7]. The Schouten–van Kampen connection was introduced in [14] for the study of nonholonomic manifolds. The Tanaka–Webster connection is the canonical affine connection defined on a non-degenerate pseudo-Hermitian CR-manifold [16]. S. Tanno defined the Tanaka–Webster connection on contact metric manifolds [17]. Zamkovoy

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defined a connection on a paracontact metric manifold so that its torsion is the obstruction for the paracontact manifold to be para-Sasakian. It is now known as the Zamkovoy connection and seems to be the paracontact analogue of the (generalized) Tanaka–Webster connection [20].

In 2019, Biswas and Baishya [2] introduced a new connection, the so-called general connection, on an n-dimensional Sasakian manifold M as follows:

$$\bar{\nabla}_U V = \nabla_U V + a[(\nabla_U \eta)(V)\xi - \eta(V)\nabla_U \xi] + b\eta(U)\phi V \tag{1.1}$$

for all $U, V \in \chi(M)$, where $a, b \in \mathbb{R}$, η is a 1-form, ξ is a vector field, ϕ is a tensor field of type $(1,1)$, $\chi(M)$ is the set of all vector fields on M, and ∇ is the Levi-Civita connection.

The connection $\bar{\nabla}$ includes all four connections mentioned above. In fact:

(i) if $a = 0$ and $b = -1$, then (1.1) reduces to the quarter-symmetric metric connection;
(ii) if $a = 1$ and $b = 0$, then (1.1) reduces to the Schouten–van Kampen connection;
(iii) if $a = 1$ and $b = -1$, then (1.1) reduces to the Tanaka–Webster connection;
(iv) if $a = 1$ and $b = 1$, then (1.1) reduces to the Zamkovoy connection.

In a recent paper, the general connection $\bar{\nabla}$ was studied on LP-Sasakian manifolds by Kumar *et al.* [8]. In that paper, the authors computed the projective curvature tensor, conharmonic curvature tensor, and quasi-conharmonic curvature tensor with respect to this connection. They also provided an application in the context of the general theory of relativity.

It is well established that, besides the Riemannian curvature tensor, there exist several types of curvature tensors. The Riemannian geometry of manifolds can be explored by employing curvature tensors characterized by both algebraic and differential properties. The (quasi-)conformal, conharmonic, concircular, (pseudo-)projective, M-projective, and W_i $(i = 1, \ldots, 9)$ curvature tensors are some examples of such curvature tensors. Taking into account this concept, Tripathi and Gupta [18] introduced a new curvature tensor T to unify all these curvature tensors as follows:

$$\begin{aligned} T(U,V)W = {} & a_0 R(U,V)W + a_1 S(V,W)U + a_2 S(U,W)V + a_3 S(U,V)W \\ & + a_4 g(V,W)QU + a_5 g(U,W)QV + a_6 g(U,V)QW \\ & + a_7 r(g(V,W)U - g(U,W)V), \end{aligned} \tag{1.2}$$

for any vector fields U, V, W on M, where R, Q, S, and r are the Riemannian curvature tensor, the Ricci operator, the Ricci tensor, and the scalar curvature, respectively, and $a_i \in \mathbb{R}$ $(i = 0, \ldots, 7)$. Denoting $T(U,V,W,X) = g(T(U,V)W,X)$, we express the curvature tensor T given by (1.2) in its $(0,4)$-tensor form as follows:

$$\begin{aligned} T(U,V,W,X) = {} & a_0 R(U,V,W,X) + a_1 S(V,W)g(U,X) \\ & + a_2 S(U,W)g(V,X) + a_3 S(U,V)g(W,X) \\ & + a_4 g(V,W)S(U,X) + a_5 g(U,W)S(V,X) \\ & + a_6 g(U,V)S(W,X) + a_7 r\big(g(V,W)g(U,X) - g(U,W)g(V,X)\big), \end{aligned} \tag{1.3}$$

where $R(U,V,W,X) = g(R(U,V)W,X)$. From (1.2), we have

$$\begin{aligned} S_T(U,V) = {} & \big(a_0 + (n-1)a_1 + a_2 + a_3 + a_5 + a_6\big)S(U,V) \\ & + \big(a_4 + (n-2)a_7\big) r g(U,V), \end{aligned} \tag{1.4}$$

where $S_T(U,V)$ is the T-Ricci tensor. Equation (1.2) implies that

$$Q_TU = \big(a_0 + (n-1)a_1 + a_2 + a_3 + a_5 + a_6\big)QU + \big(a_4 + (n-2)a_7\big)rU, \tag{1.5}$$

where Q_TU is the T-Ricci operator (see [11, 15, 19]).

On the other hand, in [4], Bochner introduced a Kähler analogue of the Weyl conformal curvature tensor by purely formal considerations, which is now recognized as the Bochner curvature tensor. Later, Matsumoto and Chuman [10] constructed the C-Bochner curvature tensor from the Bochner curvature tensor by using the Boothby–Wang fibration [5].

The C-Bochner curvature tensor is given by

$$\begin{aligned}B(U,V)W = R(U,V)W &+ \frac{1}{n+3}\big[S(U,W)V - S(V,W)U + g(U,W)QV - g(V,W)QU \\ &+ S(\phi U,W)\phi V - S(\phi V,W)\phi U + g(\phi U,W)Q\phi V - g(\phi V,W)Q\phi U \\ &+ 2S(\phi U,V)\phi W + 2g(\phi U,V)Q\phi W - S(U,W)\eta(V)\xi + S(V,W)\eta(U)\xi \\ &- \eta(U)\eta(W)QV + \eta(V)\eta(W)QU\big] - \frac{\tau+n-1}{n+3}\big[g(\phi U,W)\phi V \\ &- g(\phi V,W)\phi U + 2g(\phi U,V)\phi W\big] - \frac{\tau-4}{n+3}\big[g(U,W)V - g(V,W)U\big] \\ &+ \frac{\tau}{n+3}\big[g(U,W)\eta(V)\xi - g(V,W)\eta(U)\xi + \eta(U)\eta(W)V - \eta(V)\eta(W)U\big],\end{aligned} \tag{1.6}$$

where S is the Ricci tensor, Q is the Ricci operator, $\tau = \frac{n+r-1}{n+1}$, and r is the scalar curvature of M.

In this paper, we study the $\bar{T}$-curvature and $\bar{C}$-Bochner curvature tensors on LP-Sasakian manifolds with respect to the general connection defined by (1.1). The paper is organized as follows: After the introduction in Section 1, we provide a brief account of LP-Sasakian manifolds in Section 2. In Section 3, we present our main results, showing that LP-Sasakian manifolds which are $\bar{T}$-flat, quasi-$\bar{T}$-flat, ξ-$\bar{T}$-flat, ϕ-$\bar{T}$-flat, $\bar{T}$-semi-symmetric, ϕ-$\bar{T}$-Ricci recurrent, $\bar{C}$-Bochner flat, or satisfy $\bar{T}\cdot\bar{S} = 0$, are generalized η-Einstein manifolds. Finally, concluding remarks are given in Section 4.

2. Preliminaries

An n-dimensional differentiable manifold M is said to be an LP-Sasakian manifold if it admits a 1-form η, a vector field ξ, and a tensor field ϕ of type $(1,1)$ which satisfy the following conditions:

$$\begin{gathered}\eta(\xi) = -1, \quad \eta\circ\phi = 0, \quad \phi\xi = 0, \quad \phi^2U = U + \eta(U)\xi, \\ g(\phi U,\phi V) = g(U,V) + \eta(U)\eta(V), \quad \eta(U) = g(U,\xi), \quad g(\phi U,V) = g(U,\phi V), \\ \nabla_U\xi = \phi U, \quad (\nabla_U\phi)(V) = \eta(V)U + g(U,V)\xi + 2\eta(U)\eta(V)\xi,\end{gathered} \tag{2.1}$$

where ∇ denotes the covariant differentiation with respect to the Lorentzian metric g.

Moreover, in an n-dimensional LP-Sasakian manifold M, the following relations hold:

$$
\begin{aligned}
&\eta(R(U,V)W) = g(V,W)\eta(U) - g(U,W)\eta(V),\\
&R(\xi,U)V = g(U,V)\xi - \eta(V)U,\ R(\xi,V,W,\xi) = -g(\phi V,\phi W),\\
&R(U,V)\xi = \eta(V)U - \eta(U)V,\ R(\xi,U)\xi = U + \eta(U)\xi,\ S(U,\xi) = (n-1)\eta(U),\\
&Q\xi = (n-1)\xi,\ Q\phi = \phi Q,\ S(U,V) = g(QU,V),\ S^2(U,V) = S(QU,V),\\
&S(\phi U,\phi V) = S(U,V) + (n-1)\eta(U)\eta(V)
\end{aligned}
\tag{2.2}
$$

for all $U,V,W \in \chi(M)$, where R, Q, and S are the Riemannian curvature tensor, the Ricci operator, and the Ricci tensor, respectively, with respect to the Levi-Civita connection ∇.

Definition 2.1. An LP-Sasakian manifold (M,ϕ,ξ,η,g) is said to be a generalized η-Einstein manifold if its Ricci tensor S is of the form

$$S(U,V) = f_1 g(U,V) + f_2 g(U,\phi V) + f_3\eta(U)\eta(V),$$

where f_1, f_2, and f_3 are smooth functions on M. In particular, if $f_2 = 0$, the manifold is an η-Einstein manifold; if $f_2 = f_3 = 0$, it is an Einstein manifold; and if $f_3 = 0$, it is a special type of generalized η-Einstein manifold.

Definition 2.2. An LP-Sasakian manifold (M,ϕ,ξ,η,g) is called:

(i) T-flat if $T(U,V)W = 0$,
(ii) quasi-T-flat if $g(T(U,V)W,\phi X) = 0$,
(iii) ξ-T-flat if $T(U,V)\xi = 0$,
(iv) ϕ-T-flat if $g(T(\phi U,\phi V)\phi W,\phi X) = 0$,

for any vector fields U,V,W, and X on M.

Definition 2.3. An LP-Sasakian manifold (M,ϕ,ξ,η,g) is called T-semi-symmetric if it satisfies the condition $R(U,V)\cdot T = 0$ for any vector fields U and V on M.

Definition 2.4. An LP-Sasakian manifold (M,ϕ,ξ,η,g) is called ϕ-T-Ricci recurrent if its T-Ricci operator Q_T satisfies the condition $\phi^2((\nabla_U Q_T)V) = B(U)Q_T V$, where B is a non-zero 1-form.

Furthermore, if $\{e_1,\dots,e_{n-1},\xi\}$ is a local orthonormal basis on an n-dimensional LP-Sasakian manifold M, then $\{\phi e_1,\dots,\phi e_{n-1},\xi\}$ is also a local orthonormal basis. Using (2.2), we have [15]

$$
\begin{aligned}
&\sum_{i=1}^{n-1} g(e_i,e_i) = \sum_{i=1}^{n-1} g(\phi e_i,\phi e_i) = n-1,\\
&\sum_{i=1}^{n-1} g(e_i,W)S(V,e_i) = \sum_{i=1}^{n-1} g(\phi e_i,W)S(V,\phi e_i) = S(V,W) + (n-1)\eta(V)\eta(W),\\
&\sum_{i=1}^{n-1} S(e_i,e_i) = \sum_{i=1}^{n-1} S(\phi e_i,\phi e_i) = r-n+1,\\
&\sum_{i=1}^{n-1} R(e_i,V,W,e_i) = \sum_{i=1}^{n-1} R(\phi e_i,V,W,\phi e_i) = S(V,W) - g(\phi V,\phi W),
\end{aligned}
\tag{2.3}
$$

$$\sum_{i=1}^{n-1} g(\phi e_i, \phi W) S(\phi V, \phi e_i) = S(\phi V, \phi W),$$
$$\sum_{i=1}^{n-1} g(R(\phi e_i, \phi W)\phi V, \phi e_i) = S(\phi V, \phi W) - g(\phi V, \phi W).$$

3. Curvature Tensors with respect to the General Connection $\bar{\nabla}$ in LP-Sasakian Manifolds

Let $\bar{R}, \bar{S}, \bar{Q}$, and $\bar{r}$ be the Riemannian curvature tensor, Ricci tensor, Ricci operator, and scalar curvature, respectively, with respect to the general connection $\bar{\nabla}$. We recall the following explicit relations from [8]:

$$\begin{aligned} \bar{R}(U,V)W = {} & R(U,V)W + (a+ab+b)\big[g(U,W)\eta(V)\xi \\ & - g(V,W)\eta(U)\xi\big] + a(a+2)\big[g(V,\phi W)\phi U - g(U,\phi W)\phi V\big] \\ & + (ab+b-a)\big[\eta(V)\eta(W)U - \eta(U)\eta(W)V\big] \end{aligned} \tag{3.1}$$

$$\begin{aligned} \bar{S}(V,W) = {} & S(V,W) + (ab+b-a^2-a)g(V,W) + a(a+2)\lambda g(V,\phi W) \\ & + [ab+b-a^2-a+(n-1)(ab+b-a)]\eta(V)\eta(W), \end{aligned} \tag{3.2}$$

$$\bar{S}(\xi,W) = (a+1)(1-b)(n-1)\eta(W), \tag{3.3}$$

$$\begin{aligned} \bar{Q}V = {} & QV + [ab+b-a^2-a]V + a(a+2)\lambda\phi V \\ & + [ab+b-a^2-a+(n-1)(ab+b-a)]\eta(V)\xi, \end{aligned} \tag{3.4}$$

$$\bar{Q}\xi = (a+1)(1-b)(n-1)\xi, \tag{3.5}$$

$$\bar{r} = r - a^2(n-1) + a(a+2)\lambda^2, \tag{3.6}$$

$$\bar{R}(U,V)\xi = (a-ab-b+1)\big[\eta(V)U - \eta(U)V\big], \tag{3.7}$$

$$\begin{aligned} \bar{R}(\xi,V)W = {} & (a+ab+b+1)g(V,W)\xi + 2b(a+1)\eta(V)\eta(W)\xi \\ & + (ab+b-a-1)\eta(W)V, \end{aligned} \tag{3.8}$$

for any $U, V, W \in \chi(M)$, where $\lambda = \operatorname{trace}(\phi)$. Note that the Ricci tensor $\bar{S}$ is symmetric and the curvature tensor $\bar{R}$ satisfies the first Bianchi identity.

We now present our main results. In the following theorems, whenever a relation of the form $\alpha S(U,V) = A\,g(U,V) + B\,g(U,\phi V) + C\,\eta(U)\eta(V)$ is obtained, we assume that $\alpha \neq 0$.

Theorem 3.1. *An n-dimensional $\bar{T}$-flat LP-Sasakian manifold M is a generalized η-Einstein manifold.*

Proof. For a $\bar{T}$-flat LP-Sasakian manifold M, we have $\bar{T}(U,V,W,X) = 0$ from Definition 2.2(i), for any vector fields U, V, W, X on M. From (1.3), we write

$$\begin{aligned} 0 = {} & a_0\bar{R}(U,V,W,X) + a_1\bar{S}(V,W)g(U,X) + a_2\bar{S}(U,W)g(V,X) \\ & + a_3\bar{S}(U,V)g(W,X) + a_4 g(V,W)\bar{S}(U,X) + a_5 g(U,W)\bar{S}(V,X) \\ & + a_6 g(U,V)\bar{S}(W,X) + a_7\bar{r}\big(g(V,W)g(U,X) - g(U,W)g(V,X)\big). \end{aligned} \tag{3.9}$$

Using (3.1), (3.2), and (3.6) in (3.9), we obtain

$$0 = a_0\big\{R(U,V,W,X)$$

$$
\begin{aligned}
&+(a+ab+b)[g(U,W)\eta(V)\eta(X)-g(V,W)\eta(U)\eta(X)]\\
&+a(a+2)[g(V,\phi W)g(\phi U,X)-g(U,\phi W)g(\phi V,X)]\\
&+(ab+b-a)[\eta(V)\eta(W)g(U,X)-\eta(U)\eta(W)g(V,X)]\big\}\\
&+a_1\big\{S(V,W)g(U,X)+(ab+b-a^2-a)g(V,W)g(U,X)\\
&\quad+a(a+2)\lambda g(V,\phi W)g(U,X)\\
&\quad+[ab+b-a^2-a+(n-1)(ab+b-a)]\eta(V)\eta(W)g(U,X)\big\}\\
&+a_2\big\{S(U,W)g(V,X)+(ab+b-a^2-a)g(U,W)g(V,X)\\
&\quad+a(a+2)\lambda g(U,\phi W)g(V,X)\\
&\quad+[ab+b-a^2-a+(n-1)(ab+b-a)]\eta(U)\eta(W)g(V,X)\big\}\\
&+a_3\big\{S(U,V)g(W,X)+(ab+b-a^2-a)g(U,V)g(W,X)\\
&\quad+a(a+2)\lambda g(U,\phi V)g(W,X)\\
&\quad+[ab+b-a^2-a+(n-1)(ab+b-a)]\eta(U)\eta(V)g(W,X)\big\}\\
&+a_4 g(V,W)\big\{S(U,X)+(ab+b-a^2-a)g(U,X)\\
&\quad+a(a+2)\lambda g(U,\phi X)\\
&\quad+[ab+b-a^2-a+(n-1)(ab+b-a)]\eta(U)\eta(X)\big\}\\
&+a_5 g(U,W)\big\{S(V,X)+(ab+b-a^2-a)g(V,X)\\
&\quad+a(a+2)\lambda g(V,\phi X)\\
&\quad+[ab+b-a^2-a+(n-1)(ab+b-a)]\eta(V)\eta(X)\big\}\\
&+a_6 g(U,V)\big\{S(W,X)+(ab+b-a^2-a)g(W,X)\\
&\quad+a(a+2)\lambda g(W,\phi X)\\
&\quad+[ab+b-a^2-a+(n-1)(ab+b-a)]\eta(W)\eta(X)\big\}\\
&+a_7\big(r-a^2(n-1)+a(a+2)\lambda^2\big)\big(g(V,W)g(U,X)\\
&\quad-g(U,W)g(V,X)\big).
\end{aligned}
\tag{3.10}
$$

Putting $U=X=\xi$ in (3.10) and using (2.1) and (2.2), we obtain

$$
\begin{aligned}
a_1S(V,W)=&\big\{-a_0(a+ab+b+1)-a_1(ab+b-a^2-a)-a_4(a+1)(1-b)(n-1)\\
&-a_7\big(r-a^2(n-1)+a(a+2)\lambda^2\big)\big\}g(V,W)-a_1a(a+2)\lambda g(V,\phi W)\\
&+\big\{-a_0(a+ab+b+1)-a_1\big[ab+b-a^2-a+(n-1)(ab+b-a)\big]\\
&+(a_2+a_3+a_5+a_6)(a+1)(1-b)(n-1)\\
&-a_7\big(r-a^2(n-1)+a(a+2)\lambda^2\big)\big\}\eta(V)\eta(W).
\end{aligned}
$$

Thus, $S(V,W)=f_1g(V,W)+f_2g(V,\phi W)+f_3\eta(V)\eta(W)$. Hence, M is a generalized η-Einstein manifold. □

THEOREM 3.2. *An n-dimensional quasi-$\bar{T}$-flat LP-Sasakian manifold M is a generalized η-Einstein manifold.*

Proof. For a quasi-$\bar{T}$-flat LP-Sasakian manifold M, we have $g(\bar{T}(U,V)W,\phi X)=0$ from Definition 2.2(ii). From (1.3), replacing U with ϕU, we get

$$
0=a_0\bar{R}(\phi U,V,W,\phi X)+a_1\bar{S}(V,W)g(\phi U,\phi X)
$$

$$
\begin{aligned}
&+ a_2\bar{S}(\phi U, W)g(V,\phi X) + a_3\bar{S}(\phi U, V)g(W,\phi X) \\
&+ a_4 g(V,W)\bar{S}(\phi U,\phi X) + a_5 g(\phi U, W)\bar{S}(V,\phi X) + a_6 g(\phi U, V)\bar{S}(W,\phi X) \\
&+ a_7\bar{r}\big(g(V,W)g(\phi U,\phi X) - g(\phi U, W)g(V,\phi X)\big).
\end{aligned} \tag{3.11}
$$

We recall that if $\{e_1,\dots,e_{n-1},\xi\}$ is a local orthonormal basis of vector fields in M, then $\{\phi e_1,\dots,\phi e_{n-1},\xi\}$ is also a local orthonormal basis. Putting $U = X = e_i$ in (3.11) and taking the sum over $i = 1$ to $n-1$, we evaluate the trace:

$$
\begin{aligned}
0 = {} & a_0\sum_{i=1}^{n-1}\bar{R}(\phi e_i, V, W, \phi e_i) + a_1\sum_{i=1}^{n-1}\bar{S}(V,W)g(\phi e_i,\phi e_i) \\
&+ a_2\sum_{i=1}^{n-1}\bar{S}(\phi e_i, W)g(V,\phi e_i) + a_3\sum_{i=1}^{n-1}\bar{S}(\phi e_i, V)g(W,\phi e_i) \\
&+ a_4\sum_{i=1}^{n-1}g(V,W)\bar{S}(\phi e_i,\phi e_i) + a_5\sum_{i=1}^{n-1}g(\phi e_i, W)\bar{S}(V,\phi e_i) \\
&+ a_6\sum_{i=1}^{n-1}g(\phi e_i, V)\bar{S}(W,\phi e_i) \\
&+ a_7\sum_{i=1}^{n-1}\bar{r}\big(g(V,W)g(\phi e_i,\phi e_i) - g(\phi e_i, W)g(V,\phi e_i)\big).
\end{aligned} \tag{3.12}
$$

Using (3.1), (3.2), (3.6), and the trace identities in (2.1), (2.2), and (2.3), we obtain

$$
\begin{aligned}
0 = {} & \big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big)\bar{S}(V,W) \\
&+ \big\{a_0(a+ab+b+1) + a_4\big(\bar{r} - (a+1)(1-b)(n-1)\big) \\
&\quad + a_7\bar{r}(n-2)\big\}g(V,W) \\
&+ \big\{a_0(a+ab+b+1) + (a_2+a_3+a_5+a_6)(a+1)(1-b)(n-1) \\
&\quad - a_7\bar{r}\big\}\eta(V)\eta(W).
\end{aligned} \tag{3.13}
$$

Using (3.2), we get

$$
\begin{aligned}
0 = {} & \big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big)S(V,W) \\
&+ \big\{\big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big)(ab + b - a^2 - a) \\
&\quad + a_0(a+ab+b+1) + a_4\big(\bar{r} - (a+1)(1-b)(n-1)\big) \\
&\quad + a_7\bar{r}(n-2)\big\}g(V,W) \\
&+ \big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big)a(a+2)\lambda g(V,\phi W) \\
&+ \big\{\big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big) \\
&\quad \times \big(ab + b - a^2 - a + (n-1)(ab+b-a)\big) \\
&\quad + a_0(a+ab+b+1) + (a_2+a_3+a_5+a_6)(a+1)(1-b)(n-1) \\
&\quad - a_7\bar{r}\big\}\eta(V)\eta(W).
\end{aligned}
$$

Thus,

$$
S(V,W) = f_1 g(V,W) + f_2 g(V,\phi W) + f_3\eta(V)\eta(W).
$$

Hence, M is a generalized η-Einstein manifold. □

THEOREM 3.3. *An n-dimensional ξ-$\bar{T}$-flat LP-Sasakian manifold M is a generalized η-Einstein manifold.*

Proof. For a ξ-$\bar{T}$-flat LP-Sasakian manifold M, we have $\bar{T}(U,V)\xi = 0$. From (1.2), taking the inner product on both sides with W, we deduce

$$\begin{aligned} 0 = a_0\bar{R}(U,V,\xi,W) &+ a_1\bar{S}(V,\xi)g(U,W) + a_2\bar{S}(U,\xi)g(V,W) \\ &+ a_3\bar{S}(U,V)g(\xi,W) + a_4 g(V,\xi)\bar{S}(U,W) + a_5 g(U,\xi)\bar{S}(V,W) \\ &+ a_6 g(U,V)\bar{S}(\xi,W) + a_7\bar{r}\big(g(V,\xi)g(U,W) - g(U,\xi)g(V,W)\big). \end{aligned} \tag{3.14}$$

Substituting $V = \xi$ into (3.14) and evaluating the explicit terms such as $\bar{R}(U,\xi,\xi,W) = (ab+b-a-1)g(U,W) + (a-ab-b+1)\eta(U)\eta(W)$, we obtain

$$\begin{aligned} 0 = a_0&\big[(ab+b-a-1)g(U,W) + (a-ab-b+1)\eta(U)\eta(W)\big] \\ &- a_1(a+1)(1-b)(n-1)g(U,W) + a_2\bar{S}(U,\xi)\eta(W) \\ &+ a_3\bar{S}(U,\xi)\eta(W) - a_4\bar{S}(U,W) + a_5\eta(U)\bar{S}(\xi,W) \\ &+ a_6 g(U,\xi)\bar{S}(\xi,W) + a_7\bar{r}\big(-g(U,W) - \eta(U)\eta(W)\big). \end{aligned} \tag{3.15}$$

Hence,

$$\begin{aligned} a_4\bar{S}(U,W) = &\big[a_0(ab+b-a-1) - a_1(a+1)(1-b)(n-1) - a_7\bar{r}\big]g(U,W) \\ &+ \big[a_0(a-ab-b+1) + (a_2+a_3+a_5+a_6)(a+1)(1-b)(n-1) \\ &\quad - a_7\bar{r}\big]\eta(U)\eta(W). \end{aligned}$$

Using (3.2), we get

$$\begin{aligned} a_4 S(U,W) = &\big\{a_0(ab+b-a-1) - a_1(a+1)(1-b)(n-1) - a_4(ab+b-a^2-a) \\ &- a_7\big(r - a^2(n-1) + a(a+2)\lambda^2\big)\big\}g(U,W) - a_4 a(a+2)\lambda g(U,\phi W) \\ &+ \big\{a_0(a-ab-b+1) + (a_2+a_3+a_5+a_6)(a+1)(1-b)(n-1) \\ &- a_4\big(ab+b-a^2-a+(n-1)(ab+b-a)\big) \\ &- a_7\big(r - a^2(n-1) + a(a+2)\lambda^2\big)\big\}\eta(U)\eta(W). \end{aligned} \tag{3.16}$$

Thus,

$$S(U,W) = f_1 g(U,W) + f_2 g(U,\phi W) + f_3\eta(U)\eta(W).$$

Hence, M is a generalized η-Einstein manifold. □

THEOREM 3.4. *An n-dimensional ϕ-$\bar{T}$-flat LP-Sasakian manifold M is a generalized η-Einstein manifold.*

Proof. For a ϕ-$\bar{T}$-flat LP-Sasakian manifold M, we have $g(\bar{T}(\phi U,\phi V)\phi W,\phi X) = 0$. From (1.3), we have

$$\begin{aligned} 0 = a_0\bar{R}(\phi U,\phi V,\phi W,\phi X) &+ a_1\bar{S}(\phi V,\phi W)g(\phi U,\phi X) \\ &+ a_2\bar{S}(\phi U,\phi W)g(\phi V,\phi X) + a_3\bar{S}(\phi U,\phi V)g(\phi W,\phi X) \\ &+ a_4 g(\phi V,\phi W)\bar{S}(\phi U,\phi X) + a_5 g(\phi U,\phi W)\bar{S}(\phi V,\phi X) \\ &+ a_6 g(\phi U,\phi V)\bar{S}(\phi W,\phi X) \\ &+ a_7\bar{r}\big(g(\phi V,\phi W)g(\phi U,\phi X) - g(\phi U,\phi W)g(\phi V,\phi X)\big). \end{aligned} \tag{3.17}$$

Setting $U = X = e_i$ and evaluating the sum from $i = 1$ to $n-1$, we get

$$
\begin{aligned}
0 = {} & a_0 \sum_{i=1}^{n-1} \bar{R}(\phi e_i, \phi V, \phi W, \phi e_i) + a_1 \sum_{i=1}^{n-1} \bar{S}(\phi V, \phi W) g(\phi e_i, \phi e_i) \\
& + a_2 \sum_{i=1}^{n-1} \bar{S}(\phi e_i, \phi W) g(\phi V, \phi e_i) + a_3 \sum_{i=1}^{n-1} \bar{S}(\phi e_i, \phi V) g(\phi W, \phi e_i) \\
& + a_4 \sum_{i=1}^{n-1} g(\phi V, \phi W) \bar{S}(\phi e_i, \phi e_i) + a_5 \sum_{i=1}^{n-1} g(\phi e_i, \phi W) \bar{S}(\phi V, \phi e_i) \\
& + a_6 \sum_{i=1}^{n-1} g(\phi e_i, \phi V) \bar{S}(\phi W, \phi e_i) \\
& + a_7 \bar{r} \sum_{i=1}^{n-1} \big(g(\phi V, \phi W) g(\phi e_i, \phi e_i) - g(\phi e_i, \phi W) g(\phi V, \phi e_i) \big).
\end{aligned} \tag{3.18}
$$

Since

$$
\sum_{i=1}^{n-1} \bar{R}(\phi e_i, \phi V, \phi W, \phi e_i) = \bar{S}(\phi V, \phi W) + (a + ab + b + 1) g(\phi V, \phi W),
$$

we obtain

$$
\begin{aligned}
0 = {} & \big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big) \bar{S}(\phi V, \phi W) + \big\{a_0(a + ab + b + 1) \\
& + a_4\big(\bar{r} - (a+1)(1-b)(n-1)\big) + a_7 \bar{r}(n-2)\big\} g(\phi V, \phi W).
\end{aligned} \tag{3.19}
$$

Substituting $\bar{S}(\phi V, \phi W) = \bar{S}(V, W) + (a+1)(1-b)(n-1)\eta(V)\eta(W)$ and $g(\phi V, \phi W) = g(V, W) + \eta(V)\eta(W)$ into (3.19), we obtain

$$
\begin{aligned}
0 = {} & \big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big) S(V, W) \\
& + \big\{ \big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big)(ab + b - a^2 - a) \\
& \quad + a_0(a + ab + b + 1) + a_4\big(\bar{r} - (a+1)(1-b)(n-1)\big) + a_7 \bar{r}(n-2) \big\} g(V, W) \\
& + \big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big) a(a+2) \lambda g(V, \phi W) \\
& + \big\{ \big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big) \\
& \quad \times \big(ab + b - a^2 - a + (n-1)(ab + b - a) + (a+1)(1-b)(n-1)\big) \\
& \quad + a_0(a + ab + b + 1) + a_4\big(\bar{r} - (a+1)(1-b)(n-1)\big) + a_7 \bar{r}(n-2) \big\} \eta(V)\eta(W).
\end{aligned}
$$

Thus,

$$
S(V, W) = f_1 g(V, W) + f_2 g(V, \phi W) + f_3 \eta(V)\eta(W).
$$

Therefore, M is a generalized η-Einstein manifold. □

THEOREM 3.5. *An n-dimensional $\bar{T}$-semi-symmetric LP-Sasakian manifold M is a generalized η-Einstein manifold.*

Proof. From Definition 2.3, if an LP-Sasakian manifold M is $\bar{T}$-semi-symmetric, it satisfies $\bar{R}(U, V) \cdot \bar{T} = 0$. Using the identity of derivations for the curvature tensor, this expands to [15]:

$$
\begin{aligned}
0 = {} & g(\bar{T}(U, V)W, U) + g(U, U)\eta(\bar{T}(\xi, V)W) + g(U, V)\eta(\bar{T}(U, \xi)W) \\
& - \eta(V)\eta(\bar{T}(U, U)W) + g(U, W)\eta(\bar{T}(U, V)\xi) - \eta(W)\eta(\bar{T}(U, V)U).
\end{aligned} \tag{3.20}
$$

Taking the trace of (3.20) with respect to the local orthonormal frame field $\{e_1, \ldots, e_{n-1}, \xi\}$, we obtain

$$
\begin{aligned}
0 &= \sum_{i=1}^{n-1} g(\bar{T}(e_i, V)W, e_i) + \sum_{i=1}^{n-1} g(e_i, e_i)\eta(\bar{T}(\xi, V)W) \\
&\quad + \sum_{i=1}^{n-1} g(e_i, V)\eta(\bar{T}(e_i, \xi)W) - \sum_{i=1}^{n-1} \eta(V)\eta(\bar{T}(e_i, e_i)W) \\
&\quad + \sum_{i=1}^{n-1} g(e_i, W)\eta(\bar{T}(e_i, V)\xi) - \sum_{i=1}^{n-1} \eta(W)\eta(\bar{T}(e_i, V)e_i).
\end{aligned} \tag{3.21}
$$

Evaluating the individual sums provides

$$
\begin{aligned}
\sum_{i=1}^{n-1} g(\bar{T}(e_i, V)W, e_i) &= \big(a_0 + (n-1)a_1 + a_2 + a_3 + a_5 + a_6\big)\bar{S}(V, W) \\
&\quad + \big[a_0(a + ab + b + 1) + a_4\big(\bar{r} + (a+1)(1-b)(n-1)\big) \\
&\quad + a_7\bar{r}(n-2)\big]g(V, W) + \big[a_0(a + ab + b + 1) - a_7\bar{r} \\
&\quad - (a_2 + a_3 + a_5 + a_6)(a+1)(1-b)(n-1)\big]\eta(V)\eta(W),
\end{aligned}
$$

$$
\begin{aligned}
\sum_{i=1}^{n-1} g(e_i, e_i)\eta(\bar{T}(\xi, V)W) &= (n-1)\big[-a_1\bar{S}(V, W) - \big(a_0(a + ab + b + 1) \\
&\quad + a_4(a+1)(1-b)(n-1) + a_7\bar{r}\big)g(V, W) \\
&\quad + \big((a_2 + a_3 + a_5 + a_6)(a+1)(1-b)(n-1) \\
&\quad - a_0(a + ab + b + 1) - a_7\bar{r}\big)\eta(V)\eta(W)\big],
\end{aligned}
$$

$$
\begin{aligned}
\sum_{i=1}^{n-1} g(e_i, V)\eta(\bar{T}(e_i, \xi)W) &= -a_2\bar{S}(V, W) + \big[-a_0(a - ab - b + 1) \\
&\quad - a_5(a+1)(1-b)(n-1) + a_7\bar{r}\big]g(V, W) \\
&\quad + \big[-a_0(a - ab - b + 1) \\
&\qquad - (a_2 + a_5)(a+1)(1-b)(n-1) + a_7\bar{r}\big]\eta(V)\eta(W),
\end{aligned}
$$

$$
\begin{aligned}
-\sum_{i=1}^{n-1} \eta(V)\eta(\bar{T}(e_i, e_i)W) &= -\big[a_3\big(\bar{r} + (a+1)(1-b)(n-1)\big) \\
&\quad + a_6(n-1)^2(a+1)(1-b)\big]\eta(V)\eta(W),
\end{aligned}
$$

$$
\begin{aligned}
\sum_{i=1}^{n-1} g(e_i, W)\eta(\bar{T}(e_i, V)\xi) &= -a_3\bar{S}(V, W) - a_6(a+1)(1-b)(n-1)g(V, W) \\
&\quad - (a_3 + a_6)(a+1)(1-b)(n-1)\eta(V)\eta(W),
\end{aligned}
$$

$$
-\sum_{i=1}^{n-1} \eta(W)\eta(\bar{T}(e_i, V)e_i) = -\big[-a_0(a - ab - b + 1)(n-1)
$$

$$+ a_2\big(\bar{r} + (a+1)(1-b)(n-1)\big)$$
$$+ a_5(n-1)^2(a+1)(1-b) - a_7\bar{r}(n-1)\big]\eta(V)\eta(W).$$

Substituting these expressions into (3.21), we obtain

$$S(V,W) = f_1 g(V,W) + f_2 g(V,\phi W) + f_3\eta(V)\eta(W).$$

Therefore, M is a generalized η-Einstein manifold. □

THEOREM 3.6. *An n-dimensional ϕ-$\bar{T}$-Ricci recurrent LP-Sasakian manifold M is a generalized η-Einstein manifold.*

Proof. From Definition 2.4, we recall that an LP-Sasakian manifold M is ϕ-$\bar{T}$-Ricci recurrent if $\phi^2((\bar{\nabla}_U\bar{Q}_T)V) = B(U)\bar{Q}_T V$, for any vector fields U, V on M, where B is a non-zero 1-form. This relation yields $g(\bar{\nabla}_U\bar{Q}_T\xi,\phi V) - g(\bar{Q}_T(\bar{\nabla}_U\xi),\phi V) = 0$, [15]. Having in mind this structure and using (3.4), (3.5), and (3.6), we deduce

$$\begin{aligned}(3.22)\quad 0 = &-(a+1)\big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big)\bar{S}(\phi U,\phi V)\\ &+ (a+1)^2\big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big)\times(1-b)(n-1)g(\phi U,\phi V).\end{aligned}$$

By replacing $\bar{S}(\phi U,\phi V) = \bar{S}(U,V) + (a+1)(1-b)(n-1)\eta(U)\eta(V)$ and applying (3.2), equation (3.22) expands to

$$\begin{aligned}0 = &-(a+1)\big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big)S(U,V)\\ &- (a+1)\big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big)\big[(ab + b - a^2 - a)g(U,V) + a(a+2)\lambda g(U,\phi V)\\ &\quad + \big(ab + b - a^2 - a + (n-1)(ab + b - a) + (a+1)(1-b)(n-1)\big)\eta(U)\eta(V)\big]\\ &+ (a+1)^2\big(a_0 + a_1(n-1) + a_2 + a_3 + a_5 + a_6\big)(1-b)(n-1)\big(g(U,V) + \eta(U)\eta(V)\big).\end{aligned}$$

Thus,

$$S(U,V) = f_1 g(U,V) + f_2 g(U,\phi V) + f_3\eta(U)\eta(V).$$

Hence, M is a generalized η-Einstein manifold. □

THEOREM 3.7. *An n-dimensional LP-Sasakian manifold M satisfying $\bar{T}\cdot\bar{S} = 0$ is a generalized η-Einstein manifold.*

Proof. Following [15], suppose that $\bar{S}(\bar{T}(U,V)\xi,\xi) = 0$ for any vector fields U, V on M. By (1.2), we have

$$\begin{aligned}(3.23)\quad 0 = &\, a_0\bar{S}(\bar{R}(U,V)\xi,\xi) + a_1\bar{S}(V,\xi)\eta(U) + a_2\bar{S}(U,\xi)\eta(V) + a_3\bar{S}(U,V)\bar{S}(\xi,\xi)\\ &+ a_4\eta(V)\bar{S}(\bar{Q}U,\xi) + a_5\eta(U)\bar{S}(\bar{Q}V,\xi) + a_6 g(U,V)\bar{S}(\bar{Q}\xi,\xi)\\ &+ a_7\bar{r}\big(\eta(V)\bar{S}(U,\xi) - \eta(U)\bar{S}(V,\xi)\big)\end{aligned}$$

for any vector fields U, V on M. Using the components from (3.3), (3.5), and (3.7), taking $\eta(\bar{Q}U) = (a+1)(1-b)(n-1)\eta(U)$, the substitution yields

$$\begin{aligned}a_3\bar{S}(U,V) = &\big[a_1 + a_2 + (a_4 + a_5)(a+1)(1-b)(n-1)\big]\eta(U)\eta(V)\\ (3.24)\quad &- a_6(a+1)(1-b)(n-1)g(U,V).\end{aligned}$$

Assuming $a_3 \neq 0$ and expanding $\bar{S}(U,V)$ via (3.2),

$$\begin{aligned}a_3 S(U,V) = &-\big[a_3(ab + b - a^2 - a) + a_6(a+1)(1-b)(n-1)\big]g(U,V)\\ &- a_3 a(a+2)\lambda g(U,\phi V) + \big[a_1 + a_2 + (a_4 + a_5)(a+1)(1-b)(n-1)\\ &- a_3\big(ab + b - a^2 - a + (n-1)(ab + b - a)\big)\big]\eta(U)\eta(V).\end{aligned}$$

Thus,

$$S(U,V) = f_1 g(U,V) + f_2 g(U,\phi V) + f_3 \eta(U)\eta(V).$$

Therefore, M is a generalized η-Einstein manifold. □

Theorem 3.8. *An n-dimensional $\bar{C}$-Bochner flat LP-Sasakian manifold M is a generalized η-Einstein manifold.*

Proof. Let M be a $\bar{C}$-Bochner flat LP-Sasakian manifold. Then $\bar{B}(U,V)W = 0$ for any vector fields U, V, W on M. By (1.6),

$$\begin{aligned}
0 = \bar{R}(U,V)W &+ \frac{1}{n+3}\big[\bar{S}(U,W)V - \bar{S}(V,W)U + g(U,W)\bar{Q}V - g(V,W)\bar{Q}U \\
&+ \bar{S}(\phi U,W)\phi V - \bar{S}(\phi V,W)\phi U + g(\phi U,W)\bar{Q}\phi V - g(\phi V,W)\bar{Q}\phi U \\
&+ 2\bar{S}(\phi U,V)\phi W + 2g(\phi U,V)\bar{Q}\phi W - \bar{S}(U,W)\eta(V)\xi + \bar{S}(V,W)\eta(U)\xi \\
&- \eta(U)\eta(W)\bar{Q}V + \eta(V)\eta(W)\bar{Q}U\big] - \frac{\tau+n-1}{n+3}\big[g(\phi U,W)\phi V \\
&- g(\phi V,W)\phi U + 2g(\phi U,V)\phi W\big] - \frac{\tau-4}{n+3}\big[g(U,W)V - g(V,W)U\big] \\
&+ \frac{\tau}{n+3}\big[g(U,W)\eta(V)\xi - g(V,W)\eta(U)\xi + \eta(U)\eta(W)V - \eta(V)\eta(W)U\big].
\end{aligned}$$

Putting $W = \xi$ in the above equation and using (2.1), we obtain

$$\begin{aligned}
0 = \bar{R}(U,V)\xi &+ \frac{1}{n+3}\big[\bar{S}(U,\xi)V - \bar{S}(V,\xi)U + 2\big(\eta(U)\bar{Q}V - \eta(V)\bar{Q}U\big) \\
&- \bar{S}(U,\xi)\eta(V)\xi + \bar{S}(V,\xi)\eta(U)\xi\big] + \frac{2\tau-4}{n+3}\big(\eta(V)U - \eta(U)V\big).
\end{aligned} \tag{3.25}$$

Putting $U = \xi$ in (3.25) and taking the inner product with X, we get

$$\begin{aligned}
0 = \bar{R}(\xi,V,\xi,X) &+ \frac{1}{n+3}\big[\bar{S}(\xi,\xi)g(V,X) - 2\eta(X)\bar{S}(V,\xi) - 2\bar{S}(V,X) \\
&- 2\eta(V)\bar{S}(X,\xi) - \bar{S}(\xi,\xi)\eta(V)\eta(X)\big] + \frac{2\tau-4}{n+3}\big[g(V,X) + \eta(V)\eta(X)\big].
\end{aligned} \tag{3.26}$$

Using (3.2), (3.3), and (3.7), (3.26) reduces to

$$\begin{aligned}
\frac{2}{n+3}\bar{S}(V,X) = &\left(a - ab - b + 1 - \frac{(a+1)(1-b)(n-1)}{n+3} + \frac{2\tau-4}{n+3}\right)\big[g(V,X) + \eta(V)\eta(X)\big] \\
&- \frac{2(a+1)(1-b)(n-1)}{n+3}\eta(V)\eta(X).
\end{aligned}$$

By (3.2), we obtain

$$S(V,X) = f_1 g(V,X) + f_2 g(V,\phi X) + f_3 \eta(V)\eta(X).$$

Thus, M is a generalized η-Einstein manifold. □

4. Conclusion

In this paper, we establish several results on the relationships between the $\bar{T}$-curvature tensor, the $\bar{C}$-Bochner curvature tensor, and the generalized η-Einstein condition on LP-Sasakian manifolds with respect to a general connection $\bar{\nabla}$. Since the $\bar{T}$-curvature tensor provides a unified framework for several well-known curvature tensors and the connection $\bar{\nabla}$ includes four different affine connections as special cases, the results obtained here yield a broad generalization in the setting of LP-Sasakian manifolds. The methods and results presented in this paper may also be useful for further studies on other contact metric manifolds and related problems in mathematical physics.

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References

[1] T. Adati and K. Matsumoto, *On conformally recurrent and conformally symmetric P-Sasakian manifolds*, TRU Math. **13** (1977), 25–32.

[2] A. Biswas and K. K. Baishya, *A general connection on Sasakian manifolds and the case of almost pseudo-symmetric Sasakian manifolds*, Sci. Stud. Res. Ser. Math. Inform. **29**(1) (2019), 59–72.

[3] D. E. Blair, *Contact Manifolds in Riemannian Geometry*, Lecture Notes in Mathematics, Vol. 509, Springer-Verlag, Berlin–New York, 1976.

[4] S. Bochner, *Curvature and Betti numbers II*, Ann. of Math. **50** (1949), 77–93. https://doi.org/10.2307/1969353

[5] W. M. Boothby and H. C. Wang, *On contact manifolds*, Ann. of Math. **68** (1958), 721–734.

[6] H. Geiges, *A brief history of contact geometry and topology*, Expo. Math. **19**(1) (2001), 25–53.

[7] S. Golab, *On semi-symmetric and quarter-symmetric linear connections*, Tensor (N.S.) **29** (1975), 249–254.

[8] R. Kumar, L. Chawngthu, O. Bahadır, and M. A. Khan, *Geometry of LP-Sasakian manifolds admitting a general connection*, Mathematics **13**(6) (2025), 902. https://doi.org/10.3390/math13060902

[9] K. Matsumoto, *On Lorentzian paracontact manifolds*, Bull. Yamagata Univ. Natur. Sci. **12**(2) (1989), 151–156.

[10] M. Matsumoto and G. Chūman, *On the C-Bochner curvature tensor*, TRU Math. **5** (1969), 21–30.

[11] H. G. Nagaraja and G. Somashekhara, *T-curvature tensor in (k, μ)-contact manifolds*, Mathematica Aeterna **2** (2012), 523–532.

[12] S. Sasaki, *On differentiable manifolds with certain structures which are closely related to almost contact structure. I*, Tohoku Math. J. **12** (1960), 459–476. https://doi.org/10.2748/tmj/1178244407

[13] I. Sato, *On a structure similar to the almost contact structure*, Tensor (N.S.) **30** (1976), 219–224.

[14] J. A. Schouten and E. R. van Kampen, *Zur Einbettungs- und Krümmungstheorie nichtholonomer Gebilde*, Math. Ann. **103** (1930), 752–783.

[15] R. J. Shah, *On T-curvature tensor in LP-Sasakian manifolds*, Kathmandu Univ. J. Sci. Eng. Technol. **9**(2) (2013), 69–79. https://doi.org/10.3126/kuset.v9i2.65842

[16] N. Tanaka, *On non-degenerate real hypersurfaces, graded Lie algebras and Cartan connections*, Japan. J. Math. **2**(1) (1976), 131–190.

[17] S. Tanno, *Variational problems on contact Riemannian manifolds*, Trans. Amer. Math. Soc. **314** (1989), 349–379.

[18] M. M. Tripathi and P. Gupta, *T-curvature tensor on a semi-Riemannian manifold*, J. Adv. Math. Stud. **4**(1) (2011), 117–129.

[19] M. M. Tripathi and P. Gupta, *On $(N(k), \xi)$-semi-Riemannian manifolds: pseudosymmetries*, Int. Electron. J. Geom. **5**(2) (2012), 95–167.

[20] S. Zamkovoy, *Canonical connections on paracontact manifolds*, Ann. Global Anal. Geom. **36**(1) (2009), 37–60.
https://doi.org/10.1007/s10455-008-9147-3

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