Korean J. Math. **34** (2026), No. 2, pp. 217–223
https://dx.doi.org/10.11568/kjm.2026.34.2.217

STRUCTURE OF RINGS WITH INSERTION-OF-NILPOTENT-FACTORS-PROPERTY

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Abstract. In this article, we study the structure of rings satisfying a certain insertion-of-factors property. In particular, we examine how this property interacts with various classes of rings that arise in noncommutative ring theory. These results refine and extend several known properties of IFP rings and unit-IFP rings to the setting of INFP rings.

1. Introduction

Every ring considered in this article is associative and, unless otherwise stated, is assumed to have identity. Let R be a ring. The group of units in R is written by $U(R)$. The Wedderburn radical (i.e., sum of all nilpotent ideals), the upper nilradical (i.e., the sum of all nil ideals), the lower nilradical (i.e., the intersection of all prime ideals), the Jacobson radical, and the set of all nilpotent elements in R are denoted by $N_0(R), N^*(R)$, $N_*(R)$, $J(R)$, and $N(R)$, respectively. It is well-known that $N_0(R) \subseteq N_*(R) \subseteq N^*(R) \subseteq N(R)$ and $N^*(R) \subseteq J(R)$.

The polynomial ring with an indeterminate x over R is denoted by $R[x]$ and $C_{f(x)}$ denotes the set of all coefficients of $f(x)$ for $f(x) \in R[x]$. Denote the n by n full (resp., upper triangular) matrix ring over R by $Mat_n(R)$ (resp., $T_n(R)$). $D_n(R)$ denotes the subring $\{(a_{ij}) \in T_n(R) \mid a_{11} = \cdots = a_{nn}\}$ of $T_n(R)$. Use E_{ij} for the matrix with (i,j)-entry 1 and elsewhere 0. Let $\mathbb{Z}$ (resp., $\mathbb{Z}_n$) denote the ring of integers (modulo n). $C(R)$ denotes the center of R, and write $N_k(R) = \{a \in R \mid a^k = 0\}$ for $k \geq 2$, and $\mathbb{H}$ denotes the Hamilton quaternions over the real number field. We denote $\oplus$ and $\prod$ by the direct sum and the direct product of rings, respectively.

A ring is usually called *reduced* if it has no nonzero nilpotent elements. Following Bell [4], a right (or left) ideal I of a ring R is said to have the *Insertion-of-Factors-Property* (simply, *IFP)* if $ab \in I$ implies $aRb \subseteq I$ for $a, b \in R$. So a ring R is said to be *IFP* if the zero ideal of R has the IFP. Shin [21] used the term *SI* for the IFP, while Narbonne [20] called IFP rings *semicommutative.* A ring is called *Abelian* if every idempotent is central. It is easily checked that commutative rings and reduced

Received February 21, 2026. Revised April 11, 2026. Accepted April 13, 2026.
2010 Mathematics Subject Classification: 16N40, 16U80.
Key words and phrases: INFP ring, nilpotent element, IFP ring, Abelian ring, reduced ring.
This research was supported by Kyungdong University Research Fund, 2026.
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rings are IFP and IFP rings are Abelian. For an IFP ring, it is also easily shown that $N(R) = N_*(R)$ and RaR is nilpotent for all $a \in N(R)$. Following Han et al. [8], a ring R is called *insertion of nilpotent factors property* (shortly, *INFP*) if $ab = 0$ for $a, b \in R$ implies $aN(R)b = 0$. Recently, a ring R is said to satisfy *Insertion-of-Factors-Property on units* (simply, be a *unit-IFP* ring) [15] if $ab = 0$ for $a, b \in R$ implies $aU(R)b = 0$. Notice that IFP rings are clearly unit-IFP but not conversely by [15, Example 1.1]. Every unit-IFP ring is clearly INFP, but this direction is irreversible by [15, Lemma 1.2(1) and Example 2.5].

2. Structure of INFP rings

In this section we study the structure of various kinds of rings in relation to INFP rings.

The following lemma presents basic properties of INFP rings that will be used throughout this article.

LEMMA 2.1. (1) *A ring R is INFP if and only if* $a_1N(R)a_2N(R)a_3 \cdots a_{n-1}N(R)a_n = 0$ *whenever* $a_1a_2 \cdots a_n = 0$ *for* $a_1, a_2, \ldots, a_n \in R$.

(2) *The class of INFP rings is closed under subrings.*

(3) ([8, Proposition 1.5(1)]) *Every INFP ring is Abelian.*

(4) ([8, Proposition 2.1.(1)]) *$N(R)$ is a subring of an INFP ring R.*

Proof. (1) Let R be INFP and $abc = 0$ for $a, b, c \in R$. Then $aN(R)bc = 0$ by definition, and this yields $aN(R)bN(R)c = 0$ also by definition. Thus we can inductively obtain the proof.

The converse is obvious.

(2) This is clear. □

REMARK 2.2. (1) For any ring R and $n \geq 2$, $T_n(R)$ and $Mat_n(R)$ are not INFP rings by Lemma 2.1(3), since they are not Abelian. We will use these facts without reference. Therefore we see that the concept of an INFP ring is not Morita invariant as well as that the class of INFP rings is not closed under homomorphic images. For example, let R be the ring of quaternions with integer coefficients. Then R is a domain (and so an INFP ring). But the ring R/qR for any odd prime integer q is isomorphic to $Mat_2(\mathbb{Z}_q)$ by the argument in [7, Exercise 2A]. Thus R/qR is not INFP.

(2) The converse of Lemma 2.1(3) does not hold: The ring $D_n(R)$ over an Abelian ring R for $n \geq 4$ is Abelian by [10, Lemma 2], but $E_{12}E_{34} = 0$ for $E_{12}, E_{34} \in D_n(R)$ and $0 \neq E_{14} = E_{12}E_{23}E_{34} \in E_{12}N(D_n(R))E_{34}$. So $D_n(R)$ is not INFP for $n \geq 4$.

(3) Related to Lemma 2.1(4), there exists a unit-IFP ring (and so an INFP ring) R such that $N(R)$ is not an ideal of R in [15, Example 1.1]. Moreover, the ring $T_2(R)$ over a reduced ring R is not INFP. But $N(T_2(R)) = \begin{pmatrix} 0 & R \\ 0 & 0 \end{pmatrix}$ forms a subring of $T_2(R)$.

(4) Köthe's conjecture (i.e., the sum of two nil left ideals is nil) holds for INFP rings: Let R be an INFP ring. By Lemma 2.1(4), we have $a + b \in N(R)$ for $a \in I$ and $b \in J$ where I and J are nil left ideals of R.

Note that there exists a non-INFP ring $T_2(D)$ where D is a division ring , but Köthe's conjecture holds for $T_2(D)$ by [12, Proposition 2.7 and Proposition 4.1].

Moreover, all of the non-trivial factor rings of $T_2(D)$ are $T_2(D)/J(R) \cong D \oplus D$, $T_2(D)/I \cong D$, and $T_2(D)/K \cong D$ and they are clearly IFP (and hence INFP) where $J(R) = \begin{pmatrix} 0 & D \\ 0 & 0 \end{pmatrix}, I = \begin{pmatrix} D & D \\ 0 & 0 \end{pmatrix}, K = \begin{pmatrix} 0 & D \\ 0 & D \end{pmatrix}$. This shows that there exists a non-INFP ring whose factor rings are INFP

Recall that R is called *local* if $R/J(R)$ is a division ring, R is called *semilocal* if $R/J(R)$ is semisimple Artinian, and R is called *semiperfect* if R is semilocal and idempotents can be lifted modulo $J(R)$. Local rings are Abelian and semilocal.

PROPOSITION 2.3. (1) *Let R_λ ($\lambda \in \Lambda$) be rings. Then R_λ is INFP for each $\lambda \in \Lambda$ if and only if $\prod_{\lambda\in\Lambda} R_\lambda$ is INFP if and only if the subring of $\prod_{\lambda\in\Lambda} R_\lambda$ generated by $\oplus_{\lambda\in\Lambda} R_\lambda$ and $1_{\prod_{\lambda\in\Lambda} R_\lambda}$ is INFP.*

(2) *A ring R is INFP if and only if both eR and $(1-e)R$ are INFP for any central idempotent $e \in R$.*

(3) *A ring R is INFP and semiperfect if and only if R is a finite direct sum of local INFP rings.*

Proof. (1) This is a standard closure argument; we include a short proof for completeness. It is enough to show that $\prod_{\lambda\in\Lambda} R_\lambda$ is INFP when R_λ is INFP for each $\lambda \in \Lambda$ by Lemma 2.1(1). Suppose that R_λ is INFP for each $\lambda \in \Lambda$. Let $ab = 0$ for $a = (a_\lambda), b = (b_\lambda) \in \prod_{\lambda\in\Lambda} R_\lambda$. By hypothesis, $a_\lambda N(R_\lambda) b_\lambda = 0$ for each $\lambda \in \Lambda$. This implies that $a \left[\prod_{\lambda\in\Lambda} N(R_\lambda)\right] b = 0$, and so $aN(\prod_{\lambda\in\Lambda} R_\lambda)b = 0$ since $N(\prod_{\lambda\in\Lambda} R_\lambda) \subseteq \prod_{\lambda\in\Lambda} N(R_\lambda)$. Thus $\prod_{\lambda\in\Lambda} R_\lambda$ is INFP.

(2) It follows from Lemma 2.1(1) and (1) because $R \cong eR \oplus (1-e)R$ for any central idempotent $e \in R$.

(3) The ring-theoretic decomposition is classical for semiperfect rings; the point here is that the INFP property passes to the local components. Suppose that R is INFP and semiperfect. Since R is semiperfect, R has a finite orthogonal set $\{e_1, e_2, \ldots, e_n\}$ of local idempotents whose sum is 1 by [19, Proposition 3.7.2], say $R = \sum_{i=1}^n e_i R$ such that each $e_i R e_i$ is a local ring. Since R is Abelian by Lemma 2.1(3), each $e_i R$ is an ideal of R with $e_i R = e_i R e_i$. But each $e_i R$ is also an INFP ring as a subring by Lemma 2.1(2).

Conversely assume that R is a finite direct sum of local IFP rings. Then R is semiperfect since local rings are semiperfect, and moreover R is INFP by (1). □

By [9, Lemma 2.2(2)], $N_0(R) = N_*(R) = N^*(R) = N(R)$ for an IFP ring R. But there exists a unit-IFP ring (and so an INFP ring) R such that $N(R) \nsubseteq N_0(R)$ and $N(R) \nsubseteq J(R)$ in [15, Example 1.1].

THEOREM 2.4. *For an INFP ring R, we have the following.*

(1) *$N_0(R) = N_*(R) = N^*(R)$.*

(2) *$J(R[x]) = N_0(R[x]) = N_*(R[x]) = N^*(R[x]) = N_0(R)[x] = N_*(R)[x] = N^*(R)[x] = N_0(R)[x] \subseteq J(R)[x]$. Moreover, $J(R[x]) = J(R)[x]$ when $J(R)$ is nil.*

Proof. We in part quote the proof of [15, Theorem 1.3].

(1) Let $a \in N^*(R)$. Then $a^n = 0$ for some $n \geq 1$ and $RaR \subseteq N^*(R) \subseteq N(R)$. Since R is INFP,

$$a(RaR)a(RaR)a \cdots a(RaR)a(RaR)a = 0,$$

and hence $(RaR)^{2n-1} = 0$ and so $a \in N_0(R)$. Thus we have $N_0(R) = N_*(R) = N^*(R)$.

(2) By help of [1, Theorem 1] and [5, Corollary 4], we have $J(R[x]) \subseteq N^*(R)[x]$ and $N_0(R)[x] = N_0(R[x])$, respectively. By (1) and the facts that $N_*(R[x]) \subseteq N^*(R[x])$, $N^*(R[x]) \subseteq J(R[x])$, and $N^*(R)[x] \subseteq J(R)[x]$. Thus we get

$$J(R[x]) \subseteq N^*(R)[x] = N_*(R)[x] = N_0(R)[x] = N_0(R[x]) \subseteq N_*(R[x]) \subseteq N^*(R[x]) \subseteq J(R[x]),$$

and so

$$J(R[x]) = N^*(R)[x] = N_*(R[x]) = N_0(R[x]) = N^*(R[x]) = N_*(R)[x] = N_0(R)[x] \subseteq J(R)[x].$$

Moreover, $J(R[x]) = J(R)[x]$ since $J(R) = N^*(R)$ when $J(R)$ is nil. $\square$

Theorem 2.4(1) extends the corresponding identity known for IFP rings to the INFP setting. Notice that $J(R[x])$ is always nil in any INFP ring R by Theorem 2.4(2).

3. Extensions over INFP rings

In this section we study the structure of INFP ring in various situations related to ordinary ring extensions which have roles in ring theory.

Following Cohn [6], a ring R is called *reversible* if $ab = 0$ implies $ba = 0$ for $a, b \in R$. It can be easily verified that reduced rings are reversible and reversible rings are IFP and hence INFP. It is shown that $D_2(R)$ over a reduced ring R is reversible by [16, Proposition 1.6] and so it is INFP. But there exists a non-reduced ring A such that $D_2(A)$ is reversible (and so INFP) in [15, Example 2.2].

However, we have the following which is compared with Remark 2.2(1,2).

Proposition 3.1. *For a ring R the following conditions are equivalent:*
(1) *R is a reduced ring;*
(2) *$D_3(R)$ is an IFP ring;*
(3) *$D_3(R)$ is a unit-IFP ring;*
(4) *$D_3(R)$ is a INFP ring;*
(5) *$AN(D_3(R))B = 0$ whenever $AB = 0$ for $A, B \in D_3(R)$.*

Proof. The equivalent relations (1) $\Rightarrow$ (2) $\Rightarrow$ (3) and (1) $\Rightarrow$ (2) $\Rightarrow$ (4) come from [15, Theorem 1.9] and [8, Proposition 1.6], respectively.

The implication (4) $\Rightarrow$ (5) is clear by definition.

(5) $\Rightarrow$ (1): Assume that (5) holds. We apply the proof of [13, Proposition 2.8]. Assume on the contrary that there exists $0 \neq a \in R$ with $a^2 = 0$. Consider two matrices

$$A = \begin{pmatrix} a & a & -1 \\ 0 & a & -1 \\ 0 & 0 & a \end{pmatrix} \text{ and } B = \begin{pmatrix} a & 0 & a \\ 0 & a & 1 \\ 0 & 0 & a \end{pmatrix}$$

in $D_3(R)$. Then $AB = 0$, but $AE_{12}B = aE_{13} \neq 0$ for $E_{12} \in N(D_3(R))$, which contradicts to (5). Thus R is reduced. $\square$

The condition "R is a reduced ring" in Proposition 3.1 cannot be weaken by the condition "R is a reversible ring by next example.

Example 3.2. We recall the ring R in [16, Example 2.1]. Let

$$A = \mathbb{Z}_2\langle a_0, a_1, a_2, b_0, b_1, b_2, c\rangle$$

be the free algebra generated by noncommuting indeterminates $a_0, a_1, a_2, b_0, b_1, b_2, c$ over $\mathbb{Z}_2$. Next, let I be the ideal of A generated by

$$\begin{aligned}&a_0b_0, a_0b_1 + a_1b_0, a_0b_2 + a_1b_1 + a_2b_0, a_1b_2 + a_2b_1, a_2b_2, a_0rb_0, a_2rb_2,\\ &b_0a_0, b_0a_1 + b_1a_0, b_0a_2 + b_1a_1 + b_2a_0, b_1a_2 + b_2a_1, b_2a_2, b_0ra_0, b_2ra_2,\\ &(a_0+a_1 + a_2)r(b_0 + b_1 + b_2), (b_0 + b_1 + b_2)r(a_0 + a_1 + a_2), \text{and } r_1r_2r_3r_4,\end{aligned}$$

where the constant terms of $r, r_1, r_2, r_3, r_4 \in A$ are zero. Now set $R = A/I$. We identify

$$a_0, a_1, a_2, b_0, b_1, b_2, c$$

with their images in R for simplicity. Then R is reversible by [16, Example 2.1] but not reduced clearly.

Now, we show that $D_2(R)$ is not INFP. Consider

$$A = \begin{pmatrix} a_0 & a_1 \\ 0 & a_0 \end{pmatrix}, B = \begin{pmatrix} b_0 & b_1 \\ 0 & b_0 \end{pmatrix} \in D_2(R).$$

Then $AB = 0$, but

$$ACB = \begin{pmatrix} a_0 & a_1 \\ 0 & a_0 \end{pmatrix}\begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix}\begin{pmatrix} b_0 & b_1 \\ 0 & b_0 \end{pmatrix} = \begin{pmatrix} 0 & a_0cb_1 + a_1cb_0 \\ 0 & b_0 \end{pmatrix} \neq 0 \text{ for } C = \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix} \in N(D_2(R)),$$

since $a_0cb_1 + a_1cb_0 \notin I$ by the construction of I. Thus $D_2(R)$ is not INFP.

Consequently, $D_n(R)$ is not INFP for $n \geq 2$ by Lemma 2.1(4).

For a ring R and $n \geq 2$, let $V_n(R)$ be the ring of all matrices (a_{ij}) in $D_n(R)$ such that $a_{st} = a_{(s+1)(t+1)}$ for $s = 1, \ldots, n-2$ and $t = 2, \ldots, n-1$. Note that $V_n(R) \cong \frac{R[x]}{x^nR[x]}$.

As a corollary to Lemma 2.1(4) and Proposition 3.1, we have the following but the converse does not hold by Example 3.2.

Corollary 3.3. *If R is a reduced ring, then $V_n(R)$ is INFP for $n \geq 2$.*

Observe that if the polynomial ring $R[x]$ is INFP then so is R by Lemma 2.1(4), but there exists an INFP ring R such that $R[x]$ is not INFP by the following.

Example 3.4. We use the ring and argument in [11, Example 2]. Let $A = \mathbb{Z}_2\langle a_0, a_1, a_2, b_0, b_1, b_2, c\rangle$ be the free algebra with noncommuting indeterminates $a_0, a_1, a_2, b_0, b_1, b_2, c$ over $\mathbb{Z}_2$. Let B be the set of all polynomials with zero constant terms in A. Next, consider the ideal I of A generated by

$$\begin{aligned}&a_0b_0, a_1b_2 + a_2b_1, a_0b_1 + a_1b_0, a_0b_2 + a_1b_1 + a_2b_0, a_2b_2,\\ &\qquad a_0rb_0, a_2rb_2, (a_0 + a_1 + a_2)r(b_0 + b_1 + b_2), r_1r_2r_3r_4\end{aligned}$$

where $r \in A$ and $r_1, r_2, r_3, r_4 \in B$. Set $R = A/I$, and identify $a_0, a_1, a_2, b_0, b_1, b_2, c$ with their images in R for simplicity. Then R is IFP by [11, Example 2] and hence it is INFP.

For $f(x) = a_0 + a_1x + a_2x^2, g(x) = b_0 + b_1x + b_2x^2 \in R[x]$, $f(x)g(x) = 0$ but $f(x)cg(x) \neq 0$ since $a_0cb_1 + a_1cb_0 \notin I$, noting $c \in N(R[x])$. Thus $R[x]$ is not INFP.

For an algebra R over a commutative ring S, the *Dorroh extension* of R by S is the Abelian group $R \oplus S$ with multiplication given by

$$(r_1, s_1)(r_2, s_2) = (r_1r_2 + s_1r_2 + s_2r_1, s_1s_2)$$

for $r_i \in R$ and $s_i \in S$. We have the following theorem.

Theorem 3.5. *Let R be an algebra with identity over a commutative domain S. Then R is INFP if and only if the Dorroh extension $D = R \oplus S$ is INFP.*

Proof. Note that $R = \{r + s \mid (r, s) \in D\}$ and $N(D) = (N(R), 0)$ since S is a commutative domain. It suffices to show the necessity by Lemma 2.1(4).

Assume that R is INFP and let $(r_1, s_1)(r_2, s_2) = 0$ for $(r_i, s_i) \in D$. Then $r_1r_2 + s_1r_2 + s_2r_1 = 0$ and $s_1s_2 = 0$. Since S is a domain, $s_1 = 0$ or $s_2 = 0$. Let $s_1 = 0$, then $r_1r_2+s_2r_1 = 0$ and $r_1(r_2+s_2) = 0$. Since R is INFP, $r_1N(R)(r_2+s_2) = 0$. This implies $(r_1, 0)N(D)(r_2, s_2) = 0$. Similarly, if $s_2 = 0$ then we get $(r_1, s_1)N(D)(r_2, 0) = 0$.

Therefore the Dorroh extension D is INFP. □

An element u of a ring R is *right regular* if $ur = 0$ implies $r = 0$ for $r \in R$. Similarly, *left regular* elements can be defined. An element is *regular* if it is both left and right regular (i.e., not a zero divisor).

Proposition 3.6. *Let M be a multiplicatively closed subset of a ring R consisting of central regular elements. Then R is INFP if and only if $M^{-1}R$ is INFP.*

Proof. It follows from the fact $N(M^{-1}R) = M^{-1}N(R)$. □

Recall the ring of *Laurent polynomials* in x, written by $R[x; x^{-1}]$. Letting $M = \{1, x, x^2, \ldots\}$, M is clearly a multiplicatively closed subset of central regular elements in $R[x]$ such that $R[x; x^{-1}] = M^{-1}R[x]$. So Proposition 3.6 yields the following.

Corollary 3.7. *Let R be a ring. Then $R[x]$ is INFP if and only if $R[x; x^{-1}]$ is INFP.*

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