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KRONECKER FUNCTION RINGS AND PRÜFER-LIKE DOMAINS

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Abstract. Let D be an integral domain, $\bar{D}$ be the integral closure of D, $*$ be a star operation of finite character on D, $*_w$ be the so-called $*_w$-operation on D induced by $*$, X be an indeterminate over D, $N_* = \{f \in D[X] | c(f)^* = D\}$, and $Kr(D, *) = \{0\} \cup \{\frac{f}{g} | 0 \neq f, g \in D[X]$ and there is an $0 \neq h \in D[X]$ such that $(c(f)c(h))^* \subseteq (c(g)c(h))^*\}$. In this paper, we show that D is a $*$-quasi-Prüfer domain if and only if $\bar{D}[X]_{N_*} = Kr(D, *_w)$. As a corollary, we recover Fontana-Jara-Santos's result that D is a Prüfer $*$-multiplication domain if and only if $D[X]_{N_*} = Kr(D, *_w)$.

1. Introduction

Let D be an integral domain with quotient field K, $\bar{D}$ be the integral closure of D in K, X be an indeterminate over D, and $D[X]$ be the polynomial ring over D. For any $f \in D[X]$, we denote by $c_D(f)$ (simply $c(f)$) the ideal of D generated by the coefficients of f. For an ideal A of $D[X]$, let $c_D(A) = \sum_{f \in A} c(f)$ (simply $c_D(A)$ is denoted by $c(A)$).

Let $*$ be a star operation on D. (Definitions related to star operations will be reviewed in the sequel.) Recall that D is a *Prüfer $*$-multiplication*

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domain (P$*$MD) if each nonzero finitely generated ideal I of D is $*_f$-invertible, i.e., $(II^{-1})^{*_f} = D$. A nonzero prime ideal Q of $D[X]$ is an upper to zero in $D[X]$ if $Q \cap D = (0)$. As in [5], we say that D is $*_f$*-quasi-Prüfer* if every upper to zero in $D[X]$ contains an $f \in D[X]$ with $c_D(f)^{*_f} = D$. It is known that D is a P$*$MD if and only if D is an integrally closed $*_f$-quasi-Prüfer domain [14, Theorem 1.1]. Moreover, D is d-quasi-Prüfer if and only if $\bar{D}$ is a Prüfer domain [8, Corollary 6.5.14].

Let $*_c$ be the *e.a.b.* star operation on an integrally closed domain D induced by $*$ (see Lemma 1), and let $Kr(D, *_c)$ be the Kronecker function ring of D with respect to $*_c$. It is known that D is a P$*$MD if and only if $Kr(D, *_c) = D[X]_{N_*}$, where $N_* = \{f \in D[X] | c_D(f)^* = D\}$, [4, Theorem 3.7]. This result provides a generalization of [2, Theorem 4] that D is a Prüfer domain if and only if $D(X) = Kr(D, b)$, where $D(X) = \{\frac{f}{g} | f, g \in D[X],\ 0 \neq g \text{ and } c(g) = D\}$. In [10], Fontana-Loper used an arbitrary star operation to define the Kronecker function ring (see Lemma 2). Using this notion of Kronecker function rings, in [9, Theorem 3.1], Fontana-Jara-Santos showed that D is a P$*$MD if and only if $D[X]_{N_*} = Kr(D, *_w)$.

In this paper, we also use this Kronecker function ring to characterize $*_f$-quasi-Prüfer domains. Precisely, we show that D is a $*_f$-quasi-Prüfer domain if and only if $\bar{D}[X]_{N_*} = Kr(D, *_w)$. As a corollary, we recover Fontana-Jara-Santos's result [9, Theorem 3.1], because $D[X]_{N_*} \cap K = D$ and $Kr(D, *_w)$ is integrally closed.

We next review some definitions and notations related to star operations. Let $\mathbf{F}(D)$ (resp., $\mathbf{f}(D)$) be the set of nonzero fractional (resp., nonzero finitely generated fractional) ideals of D. A mapping $I \mapsto I^*$ of $\mathbf{F}(D)$ into $\mathbf{F}(D)$ is called a *star operation* on D if the following three conditions are satisfied for all $0 \neq a \in K$ and $I, J \in \mathbf{F}(D)$:

(1) $(aD)^* = aD$ and $(aI)^* = aI^*$,
(2) $I \subseteq I^*$; $I \subseteq J$ implies $I^* \subseteq J^*$ and
(3) $(I^*)^* = I^*$.

It is well known that the mapping $I \mapsto I^{*_f} = \cup\{J^* | J \subseteq I$ and $J \in \mathbf{f}(D)\}$ is a star operation on D. The $*_w$-operation is a star operation on D defined by setting $I^{*_w} = \{x \in K | xJ \subseteq I$ for some $J \in \mathbf{f}(D)$ with $J^* = D\}$ for all $I \in \mathbf{F}(D)$. A star operation $*$ on D is said to be of *finite character* if $*_f = *$. Clearly, $(*_f)_f = *_f$ and $*_w = (*_f)_w = (*_w)_f$; so $*_f$ and $*_w$ are of finite character. The most well-known examples of star operations are the d-, v-, t-, and w-operations. The d-operation is just the identity function on $\mathbf{F}(D)$; so $d = d_f = d_w$. The v-operation is defined by $I^v = (I^{-1})^{-1}$, where $I^{-1} = \{x \in K | xI \subseteq D\}$, while the t-operation (resp., w-operation) is defined by $t = v_f$ (resp., $w = v_w$).

An $I \in \mathbf{F}(D)$ is called a $*$-*ideal* if $I^* = I$. A $*$-ideal is called a *maximal* $*$-*ideal* if it is maximal among the proper integral $*$-ideals of D. Let $*$-Max(D) denote the set of maximal $*$-ideals of D. It is well known that a maximal $*_f$-ideal is a prime ideal; each integral $*_f$-ideal is contained in a maximal $*_f$-ideal; $*_f$-Max$(D) \neq \emptyset$ if D is not a field; and $*_f$-Max$(D) = *_w$-Max(D) [1, Theorem 2.16]. An $I \in \mathbf{F}(D)$ is said to be $*$-*invertible* if $(II^{-1})^* = D$. Clearly, $I \in \mathbf{F}(D)$ is $*_f$-invertible if and only if $II^{-1} \nsubseteq P$ for all $P \in *_f$-Max(D). As in [3, page 224], we say that an overring R of D is $*$-*linked* over D if $I^* = D$ implies $(IR)^v = R$ for all $I \in \mathbf{f}(D)$. A valuation overring V of D is a $*$-*valuation overring* of D if $I^* \subseteq IV$ for all $I \in \mathbf{f}(D)$. Obviously, $*$-valuation overrings of D are $*$-linked over D, but $*$-linked vauation overrings need not be $*$-valuation overrings (see the paragraph after Lemma 1).

For any two star-operations $*_1, *_2$ on D, we mean by $*_1 \leq *_2$ that $I^{*_1} \subseteq I^{*_2}$ for all $I \in \mathbf{F}(D)$. We know that if $*_1 \leq *_2$, then $(*_1)_f \leq (*_2)_f$ and $(*_1)_w \leq (*_2)_w$. Also, $*_w \leq *_f \leq *$ and $d \leq * \leq v$ for any star operation $*$ on D; hence $d \leq *_f \leq t$ and $d \leq *_w \leq w$. Clearly, each t-ideal is a $*_f$-ideal, and thus each maximal $*_f$-ideal is a t-ideal if and only if $*_w = w$. For more on basic properties of star operations, see [3], [11], or [13, Sections 32 and 34].

2. Kronecker function rings

Let D be an integral domain with quotient field K. A star operation $*$ on D is said to be *endlich arithmetisch brauchbar* (*e.a.b.*) if, for all $A, B, C \in \mathbf{f}(D)$, $(AB)^* \subseteq (AC)^*$ implies $B^* \subseteq C^*$. Obviously, $*$ is an *e.a.b.* star operation if and only if $*_f$ is an *e.a.b.* star operation. Let $*$ be an *e.a.b.* star operation on D. The *Kronecker function ring* of D with respect to $*$ is an integral domain

$$\mathrm{Kr}(D, *) = \{\frac{f}{g} | f, g \in D[X], g \neq 0, \text{ and } c(f) \subseteq c(g)^*\}.$$

It is well known that $\mathrm{Kr}(D, *)$ is a Bezout domain and $\mathrm{Kr}(D, *) \cap K = D$ [13, Theorem 32.7]. Hence if D admits an *e.a.b.* star-operation, then D

is integrally closed [13, Corollary 32.8]. Conversely, if D is integrally closed, then the b-operation on D defined by $I^b = \cap\{IV | V$ is a valuation overring of $D\}$ for all $I \in \mathbf{F}(D)$ is an *e.a.b.* star operation of finite character on D such that $b \leq *$ for any *e.a.b.* star operation $*$ on D [13, Theorem 32.7 and Corollary 32.14]. More generally, we have

LEMMA 1. ([4, Lemma 3.1]). *Let D be an integrally closed domain and $\{V_\alpha\}$ be the set of $*$-linked valuation overrings of D. Then the map $*_c : \mathbf{F}(D) \to \mathbf{F}(D)$, given by $I \mapsto I^{*_c} = \cap_\alpha IV_\alpha$, is an e.a.b. star operation of finite character on D such that $*_w = (*_c)_w \leq *_c$ and $*_f$-Max$(D) = *_c$-Max(D). In particular, $d_c = b$.*

We now give an example of $*$-linked valuation overrings that are not $*$-valuation overrings. Let X, y be indeterminates over the field $\mathbb{Q}$ of rational numbers, $K = \mathbb{Q}(y)$, $V = K[\![X]\!]$ be the power series ring, and $D = \mathbb{Q} + XK[\![X]\!]$. Clearly, D is an integrally closed quasi-local domain whose maximal ideal is a v-ideal, and hence each overring of D is t-linked over D. If every valuation overring V of D is a t-valuation overring, then $I^t \subseteq IV$, and so $I^t \subseteq \cap\{IV \mid V$ is a valuation overring of $D\} = I^b$ for all $I \in \mathbf{f}(D)$. Hence $v_f = t = b$ because $b \leq t$, and so v is an *e.a.b.* star operation on D. Thus every $I \in \mathbf{f}(D)$ is v-invertible [13, Theorem 34.6], and since the maximal ideal of D is a v-ideal, I is invertible. But if we let $I = (X, yX)$, then I is not invertible, a contradiction. Therefore there is a (t-linked) valuation overring of D that is not a t-valuation overring.

Let $*$ be a star operation on D. An $x \in K$ is said to be *$*$-integral over* D if $xJ^* \subseteq J^*$ for some $J \in \mathbf{f}(D)$. Let $D^{[*]} = \{x \in K | x$ is $*$-integral over $D\}$; then $D^{[*]}$, called the $*$-integral closure of D, is an integrally closed overring of D [17, Theorems 2.3 and 2.8]. We say that D is $*$-integrally closed if $D^{[*]} = D$. In [10], Fontana and Loper used an arbitrary star operation to define a Kronecker function ring.

LEMMA 2. ([10, Theorem 5.1, Proposition 4.5(2), and Corollary 3.5]) *Let $*$ be a star operation on D, and let $Kr(D, *) = \{0\} \cup \{\frac{f}{g} | 0 \neq f, g \in D[X]$ and there is an $0 \neq h \in D[X]$ such that $(c(f)c(h))^* \subseteq (c(g)c(h))^*\}$. Then $Kr(D, *)$ is a Bezout domain with quotient field $K(X)$ and $Kr(D, *) \cap K = D^{[*]}$.*

Clearly, if $*$ is *e.a.b.*, then the $Kr(D, *)$ of Lemma 2 is the usual Kronecker function ring (so we use the same notation $Kr(D, *)$). It is clear that $Kr(D, *) = Kr(D, *_f)$ and if $*_1 \leq *_2$ are star operations on D, then $Kr(D, *_1) \subseteq Kr(D, *_2)$; in particular, $Kr(D, d) \subseteq Kr(D, w) \subseteq$

$Kr(D,t) = Kr(D,v)$. For more on $Kr(D,*)$, see Fontana-Loper's interesting survey article [12].

Assume that D is $*$-integrally closed, and let $I^{*_a} = \cup\{JKr(D,*) \cap K | J \in \mathbf{f}(D)$ and $J \subseteq I\}$ for each $I \in \mathbf{F}(D)$. Then the map $*_a : \mathbf{F}(D) \to \mathbf{F}(D)$, given by $I \mapsto I^{*_a}$, is an *e.a.b.* star operation of finite character on D [10, Proposition 4.5 and Corollary 5.2]. It is known that $Kr(D,*) = Kr(D,*_a)$ and $I^{*_a} = IKr(D,*) \cap K = \cap\{IV_\beta | V_\beta$ is a $*$-valuation overring of $D\}$ for each $I \in \mathbf{F}(D)$ [10]; hence $*_c \leq *_a$ since $*$-valuation overrings are $*$-linked, and so $Kr(D,*_c) \subseteq Kr(D,*_a)$.

Proposition 3. *If D is $*$-integrally closed, then $Kr(D,*_c) = Kr(D,*_a)$ if and only if each $*$-linked valuation overring of D is a $*$-valuation overring. In this case, $*_c = *_a$.*

Proof. ($\Rightarrow$) Let V be a $*$-linked valuation overring of D that is not a $*$-valuation overring. Then there exists a $J \in \mathbf{f}(D)$ such that $J^* \nsubseteq JV$. So $JV \subsetneq J^*V$, and hence $J^{*_c} = \cap\{JV_\alpha | V_\alpha$ is a $*$-linked valuation overring of $D\} \subsetneq \cap_\alpha J^*V_\alpha \subseteq \cap\{J^*V_\beta | V_\beta$ is a $*$-valuation overring of $D\} = \cap_\beta JV_\beta = J^{*_a}$. Thus $Kr(D,*_c) \subsetneq Kr(D,*_a)$ [13, Theorem 32.7]. ($\Leftarrow$) Conversely, assume that each $*$-linked valuation overring of D is a $*$-valuation overring. Then $I^{*_c} = I^{*_a}$ for all $I \in \mathbf{F}(D)$, and thus $*_c = *_a$ and $Kr(D,*_c) = Kr(D,*_a)$. $\square$

3. A new characterization of $*$-quasi-Prüfer domains

Let D be an integral domain with quotient field K, $\bar{D}$ be the integral closure of D in K, X be an indeterminate over D, and $D[X]$ be the polynomial ring over D. Let $*$ be a star operation on D and $N_* = \{f \in D[X] | c(f)^* = D\}$.

It is clear that D is a $*_f$-quasi-Prüfer domain if and only if $c(Q)^{*_f} = D$ for each upper to zero Q in $D[X]$. In particular, a t-quasi-Prüfer domain is exactly the same as the notion of a *UMT-domain* [15, Theorem 1.4]. Also, as in [8, page 210], we say that D is a *quasi-Prüfer domain* if for each prime ideal P of D, if Q is a prime ideal of $D[X]$ with $Q \subseteq PD[X]$, then $Q = (Q \cap D)D[X]$. Hence d-quasi-Prüfer domains are just the quasi-Prüfer domains [5, Theorem 1.1]. It is known that a $*_f$-quasi-Prüfer domain is a UMT-domain (Lemma 4((1) $\Rightarrow$ (5)). For useful characterizations of UMT-domains, see [7].

We next recall some characterizations of $*$-quasi-Prüfer domains, which are essential in the proof of the main result (Theorem 5) of this paper.

LEMMA 4. *The following statements are equivalent for a star operation $*$ on D.*

(1) *D is a $*_f$-quasi-Prüfer-domain.*
(2) *The integral closure of $D[X]_{N_*}$ is a Prüfer domain.*
(3) *$D[X]_{N_*}$ is a quasi-Prüfer domain.*
(4) *D_P is a quasi-Prüfer domain for each maximal $*_f$-ideal P of D.*
(5) *D is a UMT-domain and each maximal $*_f$-ideal of D is a t-ideal.*
(6) *For each $0 \neq f \in D[X]$, there is a $0 \neq g \in K[X]$ such that $c_D(fg)^* = D$.*

Proof. (1) $\Leftrightarrow$ (2) $\Leftrightarrow$ (3) $\Leftrightarrow$ (4) $\Leftrightarrow$ (5) [5, Theorem 2.16].

(1) $\Rightarrow$ (6) Let $f = f_1^{e_1} \cdots f_k^{e_k}$, where $f_i \in K[X]$, $f_iK[X]$ is a prime, and $f_iK[X] \neq f_jK[X]$ for $i \neq j$. Then $fK[X] \cap D[X] = (f_1^{e_1}K[X] \cap \cdots \cap f_k^{e_k}K[X]) \cap D[X] = (f_1^{e_1}K[X] \cap D[X]) \cap \cdots \cap (f_k^{e_k}K[X] \cap D[X])$. Note that $f_iK[X] \cap D[X]$ is an upper to zero in $D[X]$; so there is a $0 \neq g_i \in K[X]$ such that $c_D(f_ig_i)^* = D$ by the definition of a $*_f$-quasi-Prüfer domain. Clearly, $c_D(f_i^{e_i}g_i^{e_i})^* = D$. Hence if we set $g = g_1^{e_1} \cdots g_k^{e_k}$, then $c_D(fg)^* = (c_D(f_1^{e_1}g_1^{e_1}) \cdots c_D(f_k^{e_k}g_k^{e_k}))^* = D$.

(6) $\Rightarrow$ (1) Let Q be an upper to zero in $D[X]$. Then $Q = fK[X] \cap D[X]$ for some $0 \neq f \in D[X]$ and f irreducible in $K[X]$, and by (6), there is a $0 \neq g \in K[X]$ such that $c(fg)^* = D$. Clearly, $fg \in Q$. Thus D is $*_f$-quasi-Prüfer. □

Obviously, $\bar{D}[X]_{N_*}$ is the integral closure of $D[X]_{N_*}$; so D is a $*_f$-quasi-Prüfer domain if and only if $\bar{D}[X]_{N_*}$ is a Prüfer domain by Lemma 4((1) $\Leftrightarrow$ (2)). We are now ready to prove the main result of this paper, which gives a new characterization of $*_f$-quasi-Prüfer domains including UMT-domains.

THEOREM 5. *Let $*$ be a star operation on D and $Kr(D, *_w)$ be as in Lemma 2. Then D is a $*_f$-quasi-Prüfer domain if and only if $\bar{D}[X]_{N_*} = Kr(D, *_w)$.*

Proof. ($\Rightarrow$) We first note that if D is $*_f$-quasi-Prüfer, then D is a UMT-domain and $*_w = w$ by Lemma 4((1) $\Rightarrow$ (5)); so $N_* = N_v$. For convenience, we let $R = D^{[*_w]}$.

Let $N_v(R) = \{f \in R[X] | c_R(f)^v = R\}$. Then R is a PvMD and $\bar{D}[X]_{N_v} = R[X]_{N_*} = R[X]_{N_v(R)}$ [6, Theorem 2.6]. Hence $R[X]_{N_v(R)}$ is a

Bezout domain [16, Theorem 3.7], and thus each overring of $R[X]_{N_v(R)}$ is a quotient ring of $R[X]_{N_v(R)}$ [13, Theorem 27.5]. Note that $D[X]_{N_*} \subseteq Kr(D, *_w)$ and $Kr(D, *_w)$ is integrally closed; so $\bar{D}[X]_{N_*} \subseteq Kr(D, *_w)$. Thus $Kr(D, *_w)$ is a quotient ring of $R[X]_{N_v(R)}$ (and hence of $R[X]$).

Let $S = \{f \in R[X] | \frac{1}{f} \in Kr(D, *_w)\}$. Clearly, $Kr(D, *_w) = R[X]_S$, and hence $f \in S$ if and only if there exists an $0 \neq h \in D[X]$ with $c_D(h)^{*_w} \subseteq (c_D(f)c_D(h))^{*_w}$. Since D is $*_f$-quasi-Prüfer, there exists a $0 \neq g \in K[X]$ such that $c_D(hg)^{*_w} = D$ by Lemma 4; hence $c_D(hg) \subseteq (c_D(h)c_D(g))^{*_w} \subseteq (c_D(f)c_D(h)c_D(g))^{*_w}$. Also, since R is a PvMD and $N_* \subseteq N_v(R)$ [3, Theorem 4.1], we have $(c_R(h)c_R(g))^w = c_R(hg)^w = R$. Hence by [3, Lemma 2.3],

$$\begin{aligned} c_D(hg) &\subseteq (c_D(f)c_D(h)c_D(g))^{*_w} \\ &= (c_D(f)c_D(h)c_D(g))D[X]_{N_*} \cap K \\ &\subseteq (c_R(f)c_R(h)c_R(g))R[X]_{N_v(R)} \cap K \\ &= (c_R(f)c_R(h)c_R(g))^w \\ &= (c_R(f)c_R(hg))^w \\ &= c_R(f)^w; \end{aligned}$$

so $R = c_R(hg)^v = (c_D(hg)R)^v \subseteq c_R(f)^v \subseteq R$. Hence $c_R(f)^v = R$ that is $f \in N_v(R)$, and thus $Kr(D, *_w) \subseteq R[X]_{N_v(R)} = \bar{D}[X]_{N_*}$. Thus $Kr(D, *_w) = \bar{D}[X]_{N_*}$.

($\Leftarrow$) Note that $\bar{D}[X]_{N_*}$ is the integral closure of $D[X]_{N_*}$ and $Kr(D, *_w)$ is a Bezout domain. Thus D is a $*_f$-quasi-Prüfer domain by Lemma 4((3) $\Rightarrow$ (1)). □

Recall that $d_w = d$ and $v_w = w$; so the following two corollaries are immediate consequences of Theorem 5.

Corollary 6. *D is a quasi-Prüfer domain if and only if $\bar{D}(X) = Kr(D, d)$.*

Corollary 7. *D is a UMT-domain if and only if $\bar{D}[X]_{N_v} = Kr(D, w)$.*

It is known that D is a P$*$MD if and only if D is an integrally closed $*_f$-quasi-Prüfer domain. Also, $D[X]_{N_*} \cap K = D$. Hence by Theorem 5, we have

Corollary 8. ([9, Theorem 3.1]) *D is a P$*$MD if and only if $D[X]_{N_*} = Kr(D, *_w)$.*

Corollary 9. *D is a Prüfer domain if and only if $D(X) = Kr(D, d)$.*

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