

A STUDY ON THE PREREQUISITE LEARNING THROUGH COOPERATIVE LEARNING

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ABSTRACT. Cooperation is an essential element in mathematics education with independence. We observe cooperative learning and apply it to the education spot. We conducted cooperative learning experiment with students who were not ready for the prerequisite learning of college mathematics. We try to make up the prerequisite learning through collaborative learning to them. We discuss how cooperative learning affects the students who were not ready for the prerequisite learning of college mathematics.

1. Introduction

The area of mathematics of College Scholastic Ability Test consists of mathematics type Ga and mathematics type Na. Most middle-level universities except a few high-level universities select new students by screening test admitting mathematics type Na in sciences track. Although students got addition mark in mathematics type Ga, those who take mathematics type Na and enter a school are increasing because it is easy to get high scores in mathematics type Na. But this causes many problems not only to students themselves but also to the department.

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Especially, since the department of mathematics education is the department of educating middle or high school teacher who majors in mathematics, the area of mathematics has considerable influence upon this department more than any other department. Students who take mathematics type Na and enter a school were not ready for the prerequisite learning of college mathematics. So they become to have difficulty in majoring in mathematics, lose interest, and occur to give up even studying. In spite of above fact, increasing number of students are choosing mathematics type Na because the contents of mathematics type Ga is much and difficult and it is advantageous to students to choose mathematics type Na in order to get good scores. Thus, we became to feel the necessity to make up for the prerequisite learning to relieve the difficulty for such students' major.

Our department tries to make students study the prerequisite learning of college mathematics, improve learning ability, and encourage their desires to learn. So we hope to adapt themselves to the department of mathematics education well. Because to open a class for these students is difficult, we try to make up for the prerequisite learning through cooperative learning by organizing experimental group separately. By doing that, we have them follow the curriculum of the department well. The objective of this study is to get satisfactory achievements not only in students themselves but also in the department.

2. Theoretical background

Cooperation is a traditional form of life begun with human history. Long ago it was applied to learning activity. On the evidence of Talmud, a saying goes that everybody must make learning friends to learn. About the first century Quintillion mentioned, "Students teach each other and can get many gains." Roman phylosopher Seneca said, "They learn two times when they teach," and about the 17th century Comenius said, "Everybody gets the effect of learning by teaching or learning other students." In the 1700s, Lancaster and Bell applied cooperative learning extensively in England. And in 1806 Lancaster School was established in New York and cooperative learning was introduced into the United States. Because cooperation is the form of human living for long time like this, cooperative learning is not a new idea. But the introduction into the classroom by form of class was achieved in the late 19th century

[4].

Cooperative learning structure removes the recognizable and definitive weakness which competitive learning structure and individual learning structure have. It is on the basis of the development psychology which became clear in child's intellectual development research and research achievements of society psychology about mutual action of group. And it is a theory that Deutch formed newly in the 1940s. After that, in the early twenty century Parker who was the principal of public school in Quincy Massachusetts advocated cooperative learning passionately and practiced. Every year over thirty thousand visited the school to see his cooperative class. His cooperative learning prevailed American education of twenty century turning point. Dewey succeeded him and emphasized cooperative learning, too [3].

After that, many researchers tested the effects by experimenting this in the classroom spot and expanded as an education motion with the conviction. Especially this study boom was active from the late 1970s. Devries and Edward at college of Johns Hopkins started experimental study of cooperative learning systematically and the study of cooperative learning was expanded much by Jonhson & Johnson, Slavin. And Aronson at University of California, Santa Cruz, developed Jigsaw model which was the cooperative learning method. Sharon, Lazarowitz at University of Tel Aviv in Israel, Kagan at University of California, Riverside, Hughes in Canada, and Hjertaker in Norway participated the study of cooperative learning and became to arise worldwide interest.

In cooperative learning structure, a common learning objective is given to small group and members become to help each other and learn to achieve it. Cooperative learning structure has positive interdependence, i.e. people become to recognize that they themselves can succeed only if other people succeed. Thus they strive to get the beneficial results to all of themselves. They are positively linked as fate each other and share their gains. Hence they have friendly friendship relation. Because individual's success is up to success of group, recognition of objective is clear and both individual and group have the accountability. The occasions to divide the project are numerous. There is time when rule and process are clear but they can give considerable adaptability. At this time, social function is much accounted of.

Davidson [2] defined cooperative learning as a learning technique designed to participate actively by students in the learning process through

investigation and discussion. Slavin [12] presented two hypothetical conditions which are on the basis of cooperative learning theory and distinguished from other learning and able to improve learning achievements. First, cooperative group must have the common objective. Only if it is that way, the members ask each other, render help, and improve positive interdependence, which can improve the effect of achievement. Second, the success of group as individual accountability must be estimated by average of members' ability. Otherwise, because excellent one or two members finish the assignments of the group and inferior members are isolated, inferior students can't have the motive of achievement.

Johnson & Johnson [5] summarized reasons which the effect of achievement in cooperative learning structure is bigger than that in other learning structure through extensive study as seven different kinds.

-Regardless of patterns of learning assignments, the structure of cooperative learning is more effective for learning achievement in most assignments which consist of acquirement of concept, problem-solution, and memory.

-Discussion process in cooperative learning improves high-level thinking rather than in competitive or individual learning structure.

-Members to participate in cooperative learning cause more disputes and troubles in information, opinion, and idea. And such disputes make motivation of achievement, memory, and width of understanding deep and broad.

-In the discussion of members to participate in the cooperative learning, repetition of a lot of information, new information, explanation, synthesis, and rationalization are mentioned. Such linguistic performance is easy to remember and keep for long time and so it increases the achievement of studies.

-There are control about member's learning, feedback, support, and encouragement in the group of cooperative learning.

-Because group of cooperative learning is organized inhomogeneously in the level of study and gender, many intellectual and definitive nutrition is provided through diverse aspects. This point affects school achievement, too.

-Cooperative learning increases the motive to desire to learn and encourage achievement. The motive to desire to learn in responsibility for peers does not appear in other learning structure and this motive of learning

affects school achievement.

Established studies which were related to cooperative study between 2000 and 2009 were mostly the studies in elementary or middle school [11] and established studies done in university [7, 10] were only 6% [9]. Established form about cooperative learning done in university was applied to all the students of department and several groups were subjects of experiment. However, cooperative learning done in this study was applied to one experimental group. Because all the students in the department were not subject of prerequisite study, the program suitable to subject was implemented and deficient units were supplemented.

This study investigate whether the effects which are explained theoretically appear in cooperative learning. And we observe whether cooperative learning supplement students' prerequisite learning of college mathematics.

3. A method of study

Learning model which combined cooperative learning and individual learning was applied to students to supplement the prerequisite learning. This learning model is similar to TAI (Team-assisted Individualization) model [8] which combines individual learning program with cooperative motive, which is developed in intention to increase the learning effects. TAI model applied cooperative motive to all the students by making several groups while this program made one experimental group for students requiring the prerequisite learning and applied cooperative motive to them.

Students to supplement the prerequisite learning are not all the students of the department and so to open the class is difficult. Special program was necessary for some students and program for cooperative learning was implemented to them. Here, it is the program that students who are not ready for the prerequisite learning study and discuss at the same classroom for the same learning objective at the same time. We try to supplement students' prerequisite learning through cooperative learning.

3.1. The subject of study and the process of study.

The subject of this study is the freshmen of the department of mathematics education, who are not ready for the prerequisite learning. They took mathematics type Na in College Scholastic Ability Test. Experiment period was from March 26, 2012 to June 25, 2012 and they studied two hours every week. They studied the integration unit which occupies many parts in college mathematics among contents of mathematics type Ga. Since they didn't study mathematics type Ga, they studied the concept with internet lecture.

To implement the individual accountability which is one of fundamental concept organizing cooperative learning [6], we assigned them work to do. Whenever they study, we assigned a leader and made him(or her) charge activities of that day and lead them. They made whole notebook and individual notebook. All the students recorded individual notebook. And they put down concept and problem solution in their notebooks. But we decided a writer and made him write whole writing of that day. They put down their own feeling in whole notebook.

Process of cooperative learning is divided into three different kinds and implemented.

- Grasping the core
- Sharing experiences
- Development of concept and derivation of formula

(1) Grasping the core

It is necessary for students to know the core to accomplish the learning objective. After listening internet lecture, they thought important concept of unit and the prerequisite part. After that, we made them put down the concept and problem solution in the notebook. And we made them review the important concept whenever they were stuck with solving the problem. To be familiar with the important concept, they solved from the easy problem to apply the concept directly. They were able to organize the concept and the theory which they learned in solving such easy problems.

(2) Sharing the experiences

After solving the problems, students talked each other and shared how to solve. They talked the experiences of the time when they can't solve the problems. They recognized that other friends had the same thoughts

as themselves about their unknown and difficult parts, too. So they encouraged and were helped by asking each other. One student solved the problem intuitively and another student solved it by proving. They shared each solution and improved mathematical thinking.

A : The area under the curve $y = x(x + 1)(x - 2)$ and x -axis was found but the answer was negative.

B : How did you find it?

A :

$$\int_{-1}^2 x(x + 1)(x - 2)dx = \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\right]_{-1}^2 = -\frac{9}{4}$$

C : Since the area under the curve $y = x(x + 1)(x - 2)$ and x -axis has negative part, we need to take absolute value on the expression.

D : The area to find is

$$\begin{aligned} & \int_{-1}^2 |x(x + 1)(x - 2)|dx \\ &= \int_{-1}^0 x(x + 1)(x - 2)dx - \int_0^2 x(x + 1)(x - 2)dx \\ &= \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\right]_{-1}^0 - \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\right]_0^2 \\ &= \frac{37}{12} \end{aligned}$$

A : That's right.

They were helped each other by thinking together subjects which were hard to think individually. And they became to share easy solutions. By sharing solutions, they felt new, had broad shoulders each other, and were able to make their things. This is the achievement which they can't expect in individual learning. They thought what to get in the unit and solved the problems. We let students who were weak in calculation practice the calculation much before development into complex problem. They became to know several solutions by solving several kinds of problems. They recognized that proficiency was necessary to solve several problems.

(3) Development of concept and derivation of formula

Whenever new concepts were introduced, we planned a study strategy to relate to easy problems. When they discussed, they were able to

develop from simple problems into complex problems. We let complex problems approach systematically. They found the concept by simplifying the problem. They extracted an expression in the process of solving problems and strived to find the answer by using a subordinate concept. However, since to deal with progressive difficult problem is hard, they planned the strategy to develop step by step. They discussed an important concept deeply.

To derive a formula needs time. Since students' levels are various and study is proceeded differently, we let them derive the formula after solving various applied problems. We gave them manipulative problems, simple problems, and familiar problems. So they were helped in solving difficult problems.

When they solved problems so much as to derive the formula and their atmosphere was ripe, they had the time to derive the formula. They derived the formula finding volumes of revolution and the length of curves. But they felt difficulty in deriving the formula. They put down the process of deriving the formula in the notebook. They became to understand the missed concept while they were deriving the formula and to be able to do mathematical thinking deeply. So they became to understand problems which were related with the formula more clearly. Before deriving the formula of finding volumes of revolution, they made a mistake in finding answer about volumes of revolution. But the incorrect number in problem finding volume was decreased after studying derivation process. They felt difficulty in deriving the formula but the process was helpful to them. They recognized that the process was logical process to ensure the concept and it was not wasteful process.

3.2. Analysis of result.

To evaluate experiment, we made students put down the process of learning and observed their reactions through an interview after experiment. We obtained information through a questionnaire and evaluated a major.

Students said that to put down contents and problems in the notebook for themselves was helpful for knowing contents and arranging thinking. They said that to share experience of the time when they had difficult situation got rid of the fear of solving problems. If we didn't conduct experiment, they should make up the supplementary parts by their will. They felt good about cooperative learning and pleasant about solving

problems together and waited for the cooperative learning. They had the common objective to accomplish the unit as they realized that the integration unit is related with college mathematics. So they gave and took help by asking each other and became to have self confidence. As a consequence of above fact, they could increase interdependence and improve the achievement of college mathematics.

Johnson & Johnson presented essential features of cooperative learning structure as four different kinds, which are positive interdependence, individual accountability, face to face promotive interaction, and person and group processing [5]. Questionnaires for experiment group was made by being based on above features and evaluated by five steps about respective items.

1:very bad 2:bad 3:common 4:good 5:very good

Items and scores of questionnaire are as follows.

1. I listen to their friends' opinions well and respect them.(4)
2. I share friends' ideas and do positive interaction with friends.(4.25)
3. I accept learning objective importantly and make an effort to accomplish the objective.(4.25)
4. Several friends' opinions deserve to be accepted in some parts.(3.5)
5. I am responsible for my accountability.(4.25)
6. Motive to desire to study comes out of responsibility for peers.(4.25)
7. I had the confidence because of cooperative learning.(4)
8. Peers' encouragement and support help scholastic performance.(3.75)
9. It is pleasant for me to study through cooperative learning.(4.5)
10. It is convenient for me to study through cooperative learning.(4.25)
11. It is comfortable for me to ask friends.(4.5)
12. I can sympathize with friends in hard parts and find my comfort in the process of settling difficulty.(4.5)
13. To study with friends is more efficient than to study by myself.(3.75)
14. Friends help me to study well.(3.25)
15. I became more familiar with friends through cooperative learning.(4.5)
16. Cooperative learning is more helpful than individual learning to me.(4.25)

In questionnaire, the respective numbers indicates the following. Numbers from 1 to 4 indicate item of interdependence, numbers from 5 to 8 indicate item of individual accountability, numbers from 9 to 12 indicate item of face to face promotive interaction, and numbers from 13 to 16

Item	Positive inter-dependence	Individual accountability	Face to face promotive interaction	Personal and group processing
Average score	4.00	4.06	4.44	3.94

TABLE 1. The average of items in questionnaire

indicate item of personal and group process. Average score of the respective items is shown in Table 1. Table 1 shows that students estimate the respective items positively. But they have neglected their accountabilities when their schedule overlapped with events held by school or there were private affairs. They have chatted and the atmosphere of study has been distracted because it was not regular class.

To evaluate how students' cooperative learning influences major, we compared the grades of the final exam of calculus and linear algebra. We averaged grades of experimental group and comparative group and then compared two grades. Here, comparative group consists of students who already studied the prerequisite learning and took a course together.

Calculus is the subject which is directly related with the prerequisite learning. The average of calculus of experimental group is 82.50 while the average of comparative group is 80.00. Table 2 shows this fact. This presents that students of experimental group got over the difficulties, which came from the fact that they were not ready for the prerequisite learning at the time they enter a school.

The average of linear algebra of experimental group is 87.25 while the average of comparative group is 80.93. Table 3 shows this fact. Linear algebra is the subject which is not directly related with the prerequisite learning. But the difference of grades in linear algebra differed more greatly than in calculus. This can be analyzed that the will and enthusiasm for the study which was formed owing to cooperative learning were reflected more strongly to experimental group and the difference of grades differed greatly. Since the students of experimental group were not ready for the prerequisite learning at the time they enter a school, they were placed at a disadvantageous position. But they could have better grades than students of comparative group because of supplementation of the prerequisite learning.

Group	Experimental group	Comparative group
Average grade	82.50	80.00

TABLE 2. The average of calculus of two groups

Group	Experimental group	Comparative group
Average grade	87.25	80.93

TABLE 3. The average of linear algebra of two groups

4. Conclusion

Because students to choose mathematics Na in College Scholastic Ability Test increases gradually and they feel difficulty in major, this department became to feel the need for supplementing the prerequisite learning of college mathematics for these students. So we implemented cooperative learning for them. If we didn't make this kind of program, they should make up the prerequisite learning by themselves. This was not easy to them. It was desirable that make-up program is prepared through cooperative learning.

Students of experimental group knew that the parts they didn't study were related with college mathematics and students of comparative group already studied. So they asked a question each other, helped, and studied with the common objective, which was that they would finish the necessary unit. Regardless of the difference of ability, they became to study with the positive personal relation. So they became to get the effect of positive interdependence which appeared in the cooperative learning.[1] As a consequence, average of major of students in experimental group was higher than that of major of students in comparative group. This is a consistent result with Slavin's assertion that positive achievement effect of all cooperative learning depends on objective of group and individual accountability. And as a consequence of questionnaire, students showed many positive aspects in social and psychological aspects which cooperative learning influences. So they became to have many positive attitude in college mathematics by escaping from the fear for not studying mathematics Ga. When cooperative learning, they got to feel the pleasure in the process of asking each other and finding answer.

Cooperative learning is the model to supplement students' weakness and absorb their merits. Through cooperative learning, they strived to solve the problems somehow with friends and improve the mathematical thinking. They got to the will to study and felt happy. As a consequence of implementing cooperative learning, students who were not ready for the prerequisite learning were not isolated from the related subject and could improve abilities which were fit to their own levels. Better students could accomplish high-level learning objective according to their abilities. By supplementing of the prerequisite learning through cooperative learning, we try to the supplement the weakness of selection system and educate talented men to cope with curriculum of the department. It is the way to improve the satisfaction not only for themselves but also for the department.

If selection test is the test satisfying the requirement of department, the different kinds of learning program would be developed. It is desirable to study the prerequisite learning through cooperative learning because it feels keenly the necessity of the prerequisite learning for the major in the department of mathematics or mathematics education of school permitting mathematics Na. After the experiment period, it is necessary to set conditions for studying deficient units. It is the way that they do not to fall behind and study happily. We should trace and observe scores of students in experimental group. And the constant interest and study for maximizing the effect of the cooperative learning is necessary.

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