Korean J. Math. **25** (2017), No. 1, pp. 61–70
https://doi.org/10.11568/kjm.2017.25.1.61

ON STATISTICALLY SEQUENTIALLY QUOTIENT MAPS

V. Renukadevi* and B. Prakash†

Abstract. In this paper, we introduce the concept of statistically sequentially quotient map which is a generalization of sequence covering map and discuss the relation with covering maps by some examples. Using this concept, we give an affirmative answer for a question by Fucai Lin and Shou Lin.

1. Introduction

Finding the internal characterizations of certain images of metric spaces is one of the central questions in general topology [8–10,12,16,22]. In 1971, Siwiec [20] introduced the concept of sequence covering maps which is closely related to the question about compact-covering and s-images of metric spaces. Lin and Yan in [15] proved that each sequence-covering and compact map on metric spaces is an 1-sequence covering map. Later Lin Fucai and Lin Shou in [13] proved that each sequence-covering and boundary-compact map on metric spaces is an 1-sequence covering map and posed Question 1 below. In [14], they answered this

Received November 3, 2016. Revised February 9, 2017. Accepted February 28, 2017.

2010 Mathematics Subject Classification: 54C10, 54D30, 54D55, 97I30, 54E40, 54A20.

Key words and phrases: sequence covering, 1-sequence covering, sequentially quotient, sn-network, boundary compact map.

† The research is supported by the Council of Scientific & Industrial Research Fellowship in Sciences (CSIR, New Delhi) for Meritorious Students, India.

∗ Corresponding author.

question. In this paper, we consider a similar question for statistically sequentially quotient and boundary-compact maps and prove the main result Theorem 1.

Question 1.1. [13] *Let $f : X \to Y$ be a sequence-covering and boundary-compact map. Is f an 1-sequence-covering map, if X is a space with a point-countable base or a developable space?*

Theorem 1. *Let $f : X \to Y$ be a statistically sequentially quotient and boundary-compact map. Suppose also that at least one of the following conditions holds:*
(1) *X has a point-countable base;*
(2) *X is a developable space.*
Then f is an 1-sequence-covering map.

Throughout this paper, all spaces are T_2, all maps are continuous and onto, and $\mathbb{N}$ is the set of positive integers. $x_n \to x$ denote a sequence $\{x_n\}$ converging to x. Let X be a space and $P \subset X$. A sequence $\{x_n\}$ converging to x in X is eventually in P if $\{x_n \mid n > k\} \cup \{x\} \subset P$ for some $k \in \mathbb{N}$; it is frequently in P if $\{x_{n_k}\}$ is eventually in P for some subsequence $\{x_{n_k}\}$ of $\{x_n\}$. Let $\mathcal{P}$ be a family of subsets of X. Then $\cup\mathcal{P}$ and $\cap\mathcal{P}$ denote the union $\cup\{P \mid P \in \mathcal{P}\}$ and the intersection $\cap\{P \mid P \in \mathcal{P}\}$, respectively.

Definition 1.2. Let X be a space and $P \subset X$.

(a) Let $x \in P$. P is called a *sequential neighborhood* [6] of x in X if whenever $\{x_n\}$ is a sequence converging to the point x, then $\{x_n\}$ is eventually in P.
(b) P is called a *sequentially open* [6] subset in X if P is a sequential neighborhood of x in X for each $x \in P$.

Definition 1.3. Let (X, τ) be a topological space. We define a *sequential closure-topology* σ_τ [6] on X as follows: $O \in \sigma_\tau$ if and only if O is a sequentially open subset in (X, τ). The topological space (X, σ_τ) is denoted by σX.

Definition 1.4. Let $\mathcal{P} = \cup\{\mathcal{P}_x \mid x \in X\}$ be a cover of a space X such that for each $x \in X$, the following conditions (a) and (b) are satisfied:

(a) $\mathcal{P}_x$ is a *network* at x in X, i.e., $x \in \cap\mathcal{P}_x$ and for each neighborhood U of x in $X, P \subset U$ for some $P \in \mathcal{P}_x$;
(b) If $U, V \in \mathcal{P}_x$, then $W \subset U \cap V$ for some $W \in \mathcal{P}_x$.

(i) $\mathcal{P}$ is called an *sn-network* [7] of X if each element of $\mathcal{P}_x$ is a sequential neighborhood of x for each $x \in X$, where $\mathcal{P}_x$ is called an sn-network at x in X. In this paper, when we say an sn f-countable space Y, it is always assumed that Y has an sn-network $\mathcal{P} = \bigcup\{\mathcal{P}_y \mid y \in Y\}$ such that $\mathcal{P}_y$ is countable and closed under finite intersections for each point $y \in Y$.

(ii) $\mathcal{P}$ is called a *weak base* [2] of X if whenever $G \subset X$, G is open in X if and only if for each $x \in G$, there exists $P \in \mathcal{P}_x$ such that $P \subset G$.

Definition 1.5. [17] Let A be a subset of a space X. We call an open family $\mathcal{N}$ of subsets of X is an *external base* of A in X if for any $x \in A$ and open subset U with $x \in U$ there is a $V \in \mathcal{N}$ such that $x \in V \subset U$.

Similarly, we can define an *externally weak base* for a subset A of a space X.

Definition 1.6. Let $f : X \to Y$ be a map.

(a) f is a *boundary compact map* [13] if $\partial f^{-1}(y)$ is compact in X for every $y \in Y$.

(b) f is *sequence covering* [12] if for every convergent sequence S in Y, there is a convergent sequence L in X such that $f(L) = S$. Equivalently, if whenever $\{y_n\}$ is a convergent sequence in Y, there is a convergent sequence $\{x_n\}$ in X with each $x_n \in f^{-1}(y_n)$ [20].

(c) f is *sequentially quotient* [12] if for every convergent sequence S in Y, there is a convergent sequence L in X such that $f(L)$ is an infinite subsequence of S. Equivalently, if whenever $\{y_n\}$ is a convergent sequence in Y, there is a convergent sequence $\{x_k\}$ in X with each $x_k \in f^{-1}(y_{n_k})$ [20].

(d) f is *1-sequence covering* [11] if for each $y \in Y$, there is $x \in f^{-1}(y)$ such that whenever $\{y_n\}$ is a sequence converging to y in Y, there is a sequence $\{x_n\}$ converging to x in X with each $x_n \in f^{-1}(y_n)$.

Definition 1.7. [5, 19] If $K \subset \mathbb{N}$, then K_n will denote the set $\{k \in K, k \leq n\}$ and $|K_n|$ stands for the cardinality of K_n. The *natural density* of K is defined by $d(K) = \lim\limits_{n\to\infty} \frac{|K_n|}{n}$, if limit exists. And K is called *statistically dense* [4] if $d(K) = 1$.

Definition 1.8. A subsequence S of the sequence L is called *statistically dense* [4] in L if the set of all indices of elements from S is statistically dense.

2. Statistically Sequentially Quotient Maps

In this section, we introduce statistically sequentially quotient maps and give their properties. A map $f : X \to Y$ is said to be a *statistically sequentially quotient* map if for given $y_n \to y$ in Y, there exist $x_k \to x$, $x \in f^{-1}(y)$ and $x_k \in f^{-1}(y_{n_k})$ such that $d(K) = 0$ where $K = \{n \in \mathbb{N} \mid x_k \notin f^{-1}(y_n)$ for all $k \in \mathbb{N}\}$. That is, $\{f(x_n)\}$ is statistically dense in $\{y_n\}$, since $d(K) + d(N \setminus K) = 1$ [18].

Proposition 2.1. *Let $f : X \to Y$ be a map and $g = f|_{\sigma X} : \sigma X \to \sigma Y$. Then f is a statistically sequentially quotient if and only if g is statistically sequentially quotient.*

Proposition 2.2. *Let $f : X \to Y$ and $g : Y \to Z$ be any two maps. Then the following hold.*
(a) *If f and g are statistically sequentially quotient, then $f \circ g$ is statistically sequentially quotient.*
(b) *If $f \circ g$ is statistically sequentially quotient, then g is statistically sequentially quotient.*

Proposition 2.3. *The following hold:*
(a) *Finite product of statistically sequentially quotient map is statistically sequentially quotient.*
(b) *Statistically sequentially quotient maps are hereditarily statistically sequentially quotient maps.*

Proof.

(a) Let $\prod_{i=1}^{\mathcal{N}} f_i : \prod_{i=1}^{\mathcal{N}} X_i \to \prod_{i=1}^{\mathcal{N}} Y_i$ be a map where each $f_i : X_i \to Y_i$ is statistically sequentially quotient map for $i = 1, 2, 3, ...\mathcal{N}$. Let $\{(y_{i,n})\}_{n\in\mathbb{N}}$ be a sequence converges to (y_i) in $\prod_{i=1}^{\mathcal{N}} Y_i$. Then each $\{y_{i,n}\}$ is a sequence converges to y_i in Y_i. Since each f_i is a statistically sequentially quotient map, there exists a sequence $\{x_{i,k}\}$ converges to x_i such that $f_i(x_{i,k}) = y_{i,n_k}$.
Take $(x_i) \in \prod_{i=1}^{\mathcal{N}} X_i$. Then $\{(x_{i,k})\}$ converges to (x_i). And for each $i = 1, 2, 3, ...\mathcal{N}$, $\mathcal{N}_i = \{n_k \in \mathbb{N}/x_{i,k} \in f_i^{-1}(y_{i,n_k})\}$ is statistically dense in $\mathbb{N}$. By Remark 1.1 (3) in [23], $\mathcal{N}' = \cap\mathcal{N}_i$ is statistically dense in $\mathbb{N}$. That implies a sequence $\{(x_{i,k})\}_{n\in\mathbb{N}}$ is converges to (x_i) and $f((x_{i,k}))$ is statistically dense in $(y_{i,n})$. Therefore, $\prod_{i=1}^{\mathcal{N}} f_i$ is a statistically sequentially quotient map.

(b) Let $f : X \to Y$ be a statistically sequentially quotient map and H be a subspace of Y. Take $g = f|_{f^{-1}(H)}$ such that $g : f^{-1}(H) \to H$ be a map.
Given a sequence $\{y_n\}$ converging to y in H, there exists a sequence $x_k \in f^{-1}(y_{n_k}) \in f^{-1}(H)$ such that (x_k) converges to $x \in f^{-1}(y) \in f^{-1}(H)$, since f is statistically sequentially quotient map and $\{y_n\}$ converges to y in Y. Therefore, g is a statistically sequentially quotient map.

□

We observe the following implications.
sequence covering map $\Longrightarrow$ statistically sequentially quotient map $\Longrightarrow$ sequentially quotient map.

But none of the reverse implications need not be true as shown by the following examples.

EXAMPLE 2.4. Let $X = (\bigoplus_{\alpha \in I} S_\alpha) \bigoplus I$ where I is the closed unit interval with usual topology and $S_\alpha = \{x_{\alpha,n}/n \in \mathbb{N}\} \bigcup \{x_\alpha\}$ with the topology defined as follows:

(i) Each point $x_{\alpha,n}$ is open
(ii) Open set containing x_α is of the form $\{x_{\alpha,n}/n \geq n_0\} \bigcup \{x_\alpha\}$ for some $n_0 \in \mathbb{N}$,

and Y be the space obtained from X by identifying the limit point of S_α with α. Take Y as a quotient topology that is open sets of Y are as follows:

(i) Each point $x_{\alpha,n}$ is open
(ii) open set U containing α is of the form $\{x_{\alpha,n}/n \geq n_0\} \bigcup \{x_\alpha\} \bigcup U'$ where U' is open neighborhood of α in I and $n_0 \in \mathbb{N}$.

Let $f : X \to Y$ be the map defined by

$$f(x) = \begin{cases} x & if\ x = x_{\alpha,n} \in S_\alpha \\ \alpha & if\ x = x_\alpha \in S_\alpha \\ \alpha & if\ x = \alpha \in I \end{cases}$$

(a) f is sequentially quotient
Let S be a non-trivial convergent sequence in Y with its limit y. Clearly $y \in I$. Take $S \cap S_y = S'$ and $S \cap I = S''$. Either S' or S'' must be infinite, since S is a non-trivial convergent sequence. Then the infinite sequence S' or S'' is a convergent sequence in X

with its image is a subsequence of S. Therefore, f is sequentially quotient map.

(b) f is not statistically sequentially quotient
Let $\{\alpha_n\}$ be a sequence in I converging to $\alpha \in I$. Define the sequence $S = \{y_n\}$ in Y in the following way:

$$y_n = \begin{cases} x_{\alpha,n} & if\ n\ is\ even \\ \alpha_n & if\ n\ is\ odd \end{cases}.$$

Then $\{y_n\}$ converges to $\alpha \in Y$. Consider $S \cap S_\alpha$ and $S \cap I$ in X. We have $d(\{n/y_n \in S \cap S_\alpha\}) = \frac{1}{2}$, $d(\{n/y_n \in S \cap I) = \frac{1}{2}$ and $S \cap S_\alpha$ converges to x_α, $S \cap I$ converges to α. Since X is Hausdorff, we conclude that there is no sequence S' in X such that $f(S')$ is statistically dence subsequence of S. Therefore, f is not statistically sequentially quotient.

Example 2.5. Let $\wedge = \{K/d(K) = 1, K$ is a subsequence of $\mathbb{N}$ obtained by deleting infinitely many elements$\}$ and $S_\alpha = \{x_{\alpha,n}/n \in \mathbb{N}\} \bigcup \{x_\alpha\}$ be a topological space as defined in Example 2.4, where $\alpha \in \wedge$. Let $X = \bigoplus_{\alpha \in \wedge} S_\alpha$ and $Y = \{y_n/n \in \mathbb{N}\} \bigcup \{y\}$. Then Y be a topological space as defined in S_α and $f : X \to Y$ defined by $f(x_{\alpha,n}) = \alpha(n)$ and $f(x_\alpha) = y$ where $\alpha(n)$ is an n^{th} element in the sequence α.

(1) f is a statistically sequentially quotient
Let S be a non-trivial convergent sequences in Y with its limit y. If S is a statistically dense subsequence of Y, then there is an element $\alpha \in \wedge$ such that $f(S_\alpha)$ is a statistically dense subsequence of S. If S is a non-statistically dense subsequence of Y, then there is an element $\alpha \in \wedge$ such that $f(S_\alpha) \cap S = S$ that is $S' = f^{-1}(f(S_\alpha) \cap S) \cap S_\alpha$ is a convergent sequence in X whose image is S. Therefore, f is a statistically sequentially quotient map.

(2) f is not a sequence covering map
Since corresponding to the sequence $\{y_n\}$, there are no sequence in X whose image is $\{y_n\}$. Therefore, f is not a sequence covering map.

convergent sequence with its limit x_α. Then for each $\alpha \in \wedge$, $S_\alpha = \{x_{\alpha,i}, x_\alpha \mid i \in \alpha\}$. Let X be a disjoint union of S_α and Y be a convergent sequence $\{y_n\}$ with its limit y. Then $f : X \to Y$ defined by $f(x_{\alpha,i}) = y_i$ and $f(x_\alpha) = y$ is a statistically sequentially quotient map but not a sequence covering map,

3. Proof of Theorem 1

The proof of Theorem 1 will be divided into Theorems 3.1, 3.2, 3.4 and Lemma 3.3. Let Ω be the set of all topological spaces such that each compact subset $K \subset X$ is metrizable and has a countable neighborhood base in X. In fact, Michael and Nagami in [17] has proved that $X \in \Omega$ if and only if X is the image of some metric space under an open and compact-covering map. It is easy to see that if a space X is developable or has a point-countable base, then $X \in \Omega$. (see [1] and [21], respectively)

THEOREM 3.1. *Let $f : X \to Y$ be a statistically sequentially quotient and boundary compact map, where Y is sn f-countable. For each non-isolated point $y \in Y$, there exists a point $x_y \in \partial f^{-1}(y)$ such that whenever U is an open subset with $x_y \in U$, there exists a $P \in \mathcal{P}_y$ satisfying $P \subset f(U)$.*

Proof. Suppose not, that is there exists a non-isolated point $y \in Y$ such that for every point $x \in \partial f^{-1}(y)$, there is an open neighborhood U_x of x such that $P \not\subseteq f(U_x)$ for every $P \in \mathcal{P}_y$. Then $\partial f^{-1}(y) \subset \cup\{U_x \mid x \in \partial f^{-1}(y)\}$. Since $\partial f^{-1}(y)$ is compact, there exists a finite subfamily $\mathcal{U} \subset \{U_x \mid x \in \partial f^{-1}(y)\}$ such that $\partial f^{-1}(y) \subset \cup\mathcal{U}$. We denote $\mathcal{U}$ by $\{U_i \mid 1 \leq i \leq n_0\}$. Assume that $\mathcal{P}_y = \{P_n \mid n \in \mathbb{N}\}$ and $\mathcal{W}_y = \{F_n = \bigcap_{i=1}^{n} P_i \mid n \in \mathbb{N}\}$. It is obvious that $\mathcal{W}_y \subset \mathcal{P}_y$ and $F_{n+1} \subset F_n$ for every $n \in \mathbb{N}$. For each $1 \leq m \leq n_0, n \in \mathbb{N}$, it follows that there exists $x_{n,m} \in F_n \setminus f(U_m)$. Denote $y_k = x_{n,m}$ where $k = (n-1)n_0 + m$. Since $\mathcal{P}_y$ is a network at a point y and $F_{n+1} \subset F_n$ for every $n \in \mathbb{N}$, $\{y_n\}$ is a sequence converging to y in Y. Since f is a statistically sequentially quotient map, $\{y_{n_k}\}$ is an image of some sequence $\{x_k\}$ converging to $x \in \partial f^{-1}(y)$ in X. From $x \in \partial f^{-1}(y) \subset \cup\mathcal{U}$ it follows that there exists $1 \leq m_0 \leq n_0$ such that $x \in U_{m_0}$. Therefore, $\{x\} \cup \{x_k \mid k \geq k_o\} \subseteq U_{m_0}$ for some $k_0 \in \mathbb{N}$. Hence $\{y\} \cup \{y_{n_k} \mid n_k \geq k_0\} \subset f(U_{m_0})$. However, we can choose an $n > k_0$ such that $n_k = (n-1)n_0 + m_0 \geq k_0$, $y_{n_k} = x_{n,m_0}$ which implies that $x_{n,m_0} \in f(U_{m_0})$. Suppose there is no $n > k_0$ such that $n_k = (n-1)n_0 + m_0 \geq k_0$, $y_{n_k} \in f(U_{m_0})$. That is, for all $n > k_0$, $n_k = (n-1)n_0 + m_0 \geq k_0$ such that $y_{n_k} \notin f(U_{m_0})$. This implies $\{n_k \in \mathbb{N} \mid n \geq n_k, n_k = (n'-1)n_0 + m_0, n' > k_0\} \subset K_n$ where $k_0 = qn_0 + r$, $n = q_1 n_0 + r_1$ and $K = \{n/y_n \notin f(U_{m_0})\}$. Now $|K_n| > q_1 - q$ implies $d(K) = \lim\limits_{n\to\infty} \frac{|K_n|}{n} > \lim\limits_{q_1\to\infty} \frac{q_1 - q}{q_1 n_0 + r_1} = \frac{1}{n_0}$ which is a contradiction to $d(K) = 0$. Therefore, there exists $n > k_0$ such that $n_k = (n-1)n_0 + m_0 \geq k_0, y_{n_k} = x_{n,m_0}$ and $x_{n,m_0} \in f(U_{m_0})$ which contradicts $x_{n,m_0} \in F_n \setminus f(U_{m_0})$. □

THEOREM 3.2. *Let $f : X \to Y$ be a statistically sequentially quotient and boundary-compact map, where X is first countable. Then Y is snf-countable if and only if f is an 1-sequence-covering map.*

Proof. Necessity. Let y be a non-isolated point in Y. Since Y is snf-countable, by Theorem 3.1, there exists a point $x_y \in \partial f^{-1}(y)$ such that whenever U is an open neighborhood of x_y, there is a $P \in \mathcal{P}_y$ satisfying $P \subset f(U)$. Let $\{B_n \mid n \in \mathbb{N}\}$ be a countable neighborhood base at x_y such that $B_{n+1} \subset B_n$ for each $n \in \mathbb{N}$. Now for each B_n, there exists a $P_n \in \mathcal{P}_y$ such that $P_n \subset f(B_n)$ for each $n \in \mathbb{N}$ which implies that every $f(B_n)$ is a sequential neighborhood of y in Y, since every $P \in \mathcal{P}_y$ is a sequential neighborhood of y.

Suppose that $\{y_n\}$ is a sequence in Y which converges to y. Then for each $n \in \mathbb{N}$, there is an $i_n \in \mathbb{N}$ such that $y_i \in f(B_n)$ for every $i \geq i_n$. Let it $1 < i_n < i_{n+1}$ for every $n \in \mathbb{N}$. Take

$$x_j \in \begin{cases} f^{-1}(y_j), & \text{if } j < i_1 \\ f^{-1}(y_j) \cap B_n, & \text{if } i_n \leq j < i_{n+1} \end{cases}$$

Denote $S = \{x_j \mid j \in \mathbb{N}\}$. It is easy to see that S converges to x_y in X and $f(S) = \{y_n\}$. Therefore, f is an 1-sequence-covering map. Converse part is easy to see. □

LEMMA 3.3. *Let $f : X \to Y$ be a sequentially quotient and boundary-compact map. If $X \in \Omega$, then Y is snf-countable*

Proof. Let y be a non-isolated point for Y. Since $X \in \Omega$ and $\partial f^{-1}(y)$ is non-empty and compact for X, there exist a countable external base $\mathcal{U}$ for $\partial f^{-1}(y)$ in X. Let it be $\mathcal{V} = \{\cup\mathcal{F} \mid$ there is a finite subfamily $\mathcal{F} \subset \mathcal{U}$ with $\partial f^{-1}(y) \subset \cup\mathcal{F}\}$. Obviously, $\mathcal{V}$ is countable. We now prove that $f(\mathcal{V})$ is a countable sn-network at point y.

(1) $f(\mathcal{V})$ is a network at y.

Let $y \in U$. Obviously, $\partial f^{-1}(y) \subset f^{-1}(U)$. For each $x \in \partial f^{-1}(y)$, there exists an $U_x \in \mathcal{U}$ such that $x \in U_x \subset f^{-1}(U)$. Therefore, $\partial f^{-1}(y) \subset \cup\{U_x \mid x \in \partial f^{-1}(y)\}$. Since $\partial f^{-1}(y)$ is compact, it follows that there exists a finite subfamily $\mathcal{F} \subset \{U_x \mid x \in \partial f^{-1}(y)\}$ such that $\partial f^{-1}(y) \subset \cup\mathcal{F} \subset f^{-1}(U)$. It is easy to see that $\cup\mathcal{F} \in \mathcal{V}$ and $y \in f(\cup\mathcal{F}) \subset U$.

(2) For any $P_1, P_2 \in f(\mathcal{V})$, there exists a $P_3 \in f(\mathcal{V})$ such that $P_3 \subset P_1 \cap P_2$.

It is obvious that there exist V_1, V_2 in $\mathcal{V}$ such that $f(V_1) = P_1, f(V_2) = P_2$, respectively. Since $\partial f^{-1}(y) \subset V_1 \cap V_2$, it follows from the similar proof of (1) that there exist a $V_3 \in \mathcal{V}$ such that $\partial f^{-1}(y) \subset V_3 \subset V_1 \cap V_2$. Let $P_3 = f(V_3)$. Thus, $P_3 \subset f(V_1 \cap V_2) \subset f(V_1) \cap f(V_2) = P_1 \cap P_2$.

(3) For each $P \in f(\mathcal{V})$, P is a sequential neighborhood of y.

Let $\{y_n\}$ be any sequence in Y converges to y in Y. Since f is a sequentially quotient map, there exist a sequence $\{x_k\}$ converging to $x \in \partial f^{-1}(y) \subset X$ where $x_k \in f^{-1}(y_{n_k})$. Since $P \in f(\mathcal{V})$, there exists a $V \in \mathcal{V}$ such that $P = f(V)$. Therefore, $\{x_n\}$ is eventually in V, and this implies that $\{y_n\}$ is eventually in P. Suppose not, there exists a subsequence $\{y'_n\}$ such that $y'_n \notin P$ and it converges to y. Then there exists a sequence $\{x'_n\}$ converges to $x' \in \partial f^{-1}(y)$ and it's image is a subsequence of $\{y'_n\}$. Since $x' \in \partial f^{-1}(y) \subset V$, y'_n is frequently in P, which is a contradiction. Therefore, $f(\mathcal{V})$ is a countable $sn-$network at a point y. □

THEOREM 3.4. *Let $f : X \to Y$ be a statistically sequentially quotient and boundary-compact map. If $X \in \Omega$, then f is an 1-sequence-covering map.*

Proof. From Lemma 3.3, it follows that Y is snf-countable. Therefore, f is an 1-sequence covering map, since $\partial f^{-1}(y)$ is compact subset of $X \in \Omega$, by Theorem 3.2. □

References

[1] B. Alleche, A. Arhangel'skii and J. Calbrix, *Weak developments and metrization*, Topology Appl. (2000), 23–38.

[2] A. Arhangel'skii, *Mappings and spaces*, Russian Math. Surveys **21** (1966), 115–162.

[3] J. R. Boone and F. Siwiec, *Sequentially quotient mappings*, Czech. Math. J. **26** (1976), 174–182.

[4] G. Di Maio and Lj. D. R. Kočinac, *Statistical convergence in topology*, Topology Appl. **156** (2008), 28–45.

[5] H. Fast, *Sur la convergence statistique*, Colloq. Math. **2** (1951), 241–244.

[6] S. P. Franklin, *Spaces in which sequences suffice*, Fund. Math. **57** (1965), 107–115.

[7] Y. Ge, *On sn-metrizable spaces*, Acta Math. Sinica **45** (2002), 355-360(in chinese).

[8] G. Gruenhage, E. Michael and Y. Tanaka, *Spaces determined by point-countable covers*, Pacific J. Math. **113** (1984), 303-332.

[9] Z. Li, *Images of locally compact metric spaces*, Acta Math. Hungar. **99** (2003), 81–88.
[10] J. Li, andS. Jiang, *On sequence-covering* $K-$*mappings*, Indian J. Pure Appl. Math. **34** (2003), 397–403.
[11] S. Lin, *On sequence-covering s-maps*, Adv. Math. (in Chinese) **25** (1996), 548–551.
[12] S. Lin, *Point-countable covers and sequence-covering Mappings* (in Chinese): Science Press, Beijing, 2002
[13] F. Lin and S. Lin, *On Sequence-covering boundary compact maps of metric spaces*, Adv. Math(China) **39**(1) (2010), 71–78.
[14] F. Lin and S. Lin, *Sequence-covering maps on generalized metric spaces*, Houston J. Math. **40**(3) (2014), 927–943.
[15] S. Lin and P. Yan, *Sequence-covering maps of metric spaces*, Topology Appl. **109** (2001), 301–314.
[16] C. Liu and S. Lin, *On countable-to-one maps*, Topology Appl. **154** (2007), 449–454.
[17] E. A. Michael and K. Nagami, *Compact-covering images of metric spaces*, Proc. Amer. Math. Soc. **37** (1973), 260–266.
[18] I. M. Niven, H. S. Zuckerman and H.L. Montgomery, *An introduction to the theory of numbers*, John wiley and sons. inc., 1991
[19] I. J. Schoenberg, *The Integrability of certain function and related summability methods*, Amer. Math. Monthly **66** (1959), 361–375.
[20] F. Siwiec, *Sequence-covering and countably bi-quotient maps*, General Topology Appl. **1** (1971), 143–154.
[21] V. V. Tkachuk, *Monolithic spaces and D-spaces revisited*, Topology Appl. **156** (2009), 840–846.
[22] P. Yan, S. Lin and S. Jiang, *Metrizability is preserved by closed sequence-covering maps*, Acta Math. Sinica **47**(1) (2004), 87–90.
[23] Z. Tang and F. Lin, *Statistical versions of sequential and Fréchet Urysohn spaces*, Adv. Math. (china) **44** (2015), 945–954.

V. Renukadevi
Center for Research and Post Graduate Department of Mathematics
Ayya Nadar Janaki Ammal College(Autonomous)
Sivakasi 626 124, Tamil Nadu, India
E-mail: renu_siva2003@yahoo.com

B. Prakash
Center for Research and Post Graduate Department of Mathematics
Ayya Nadar Janaki Ammal College(Autonomous)
Sivakasi 626 124, Tamil Nadu, India
E-mail: prakashphd101@gmail.com