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FIXED POINT THEOREMS IN FUZZY METRIC SPACES FOR MAPPINGS WITH SOME CONTRACTIVE TYPE CONDITIONS

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Abstract. In this paper, we derive some fixed point theorems in fuzzy metric spaces for self mappings satisfying different contractive type conditions. Some of these theorems generalize some results of Wairojjana et al. *(Fixed Point Theory and Applications (2015) 2015:69)*. Several examples in support of the theorems are also presented here.

1. Introduction and preliminaries

The concept of fuzzy metric spaces was introduced by Kramosil and Michalek [23] in 1975. The results of Banach and Edelstein [3], [14] plays an effective role in several fixed point theorems. The study of fixed point theory in fuzzy metric spaces was started with the work of Grabiec in 1988 [18], by extending the existing results to fuzzy metric spaces. Further, a new notion of complete fuzzy metric space was introduced by George and Veeramani [16], [17] by defining a Hausdorff and first countable topology for some specific fuzzy metric space. In this paper, we follow this definition of completeness.

The concept of altering distance was earlier introduced by M.S Khan et al. [22] in 1984, with a view to enhance the metric fixed point theory.

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Recently many outstanding research workers have developed several interesting results on fixed point theory in relation with different operators (refer to [1], [2], [4]- [10], [12], [15], [19], [20], [21], [24], [26]- [33], [35]- [38]) as well as fuzzy fixed point theory (refer to [11], [13], [25], [34], [39], [40], [41],etc) with many applications in different branches of applied science. In this paper, our aim is to derive some fixed point theorems in fuzzy metric spaces using mappings with different contractive type conditions defined by altering distance function. Some well known results are also generalized here with suitable examples.

Definition 1.1. [37] A mapping $T : [0,1] \times [0,1] \longrightarrow [0,1]$ is called a triangular norm or t-norm if
(i) $T(a,1) = a$ for all $a \in [0,1]$,
(ii) $T(a,b) = T(b,a)$ for all $a,b \in [0,1]$,
(iii) $a \geq b$,$c \geq d \Rightarrow T(a,c) \geq T(b,d), \forall\ a,b,c,d \in [0,1]$,
(iv) $T(a,T(b,c)) = T(T(a,b),c), \forall\ a,b,c \in [0,1]$.

$T_m(a,b) = \min(a,b)$, $T_p(a,b) = a.b$, $T_L(a,b) = \max(a+b-1,0)$ are some commonly used t-norms.

Definition 1.2. [16] For an arbitrary set X, let T be a continuous t-norm and M be a fuzzy set on $X^2 \times (0,\infty)$. The 3-tuple (X,M,T) is called a fuzzy metric space if :
(i) $M(a,b,s) > 0$, for all $a,b \in X, s > 0$,
(ii) $M(a,b,s) = 1$, for all $s > 0 \Leftrightarrow a = b$,
(iii) $M(a,b,s) = M(b,a,s)$, for all $a,b \in X, s > 0$,
(iv) $T(M(a,b,s), M(b,c,p)) \leq M(a,c,s+p)$,
for all $a,b,c \in X, s,p > 0$,
(v) $M(a,b,.) : (0,\infty) \longrightarrow [0,1]$ is continuous for all $a,b \in X$.

Example 1.3. For $X = \mathbb{R}$, taking the usual metric $d(x,y) = |x-y|$ and the fuzzy t-norm $M(x,y,t) = \frac{t}{t+d(x,y)}$,$t \in (0,\infty)$, we have, (X,M,T) is a fuzzy metric space with respect to the t-norm $T_p(x,y) = x.y,\ \ x,y \in X$

Lemma 1.4. [18] *With respect to* t, $M(x,y,t)$ *is a non-decreasing function for all* x,y *in* X,

Definition 1.5. [12] [16] [18] For a fuzzy metric space (X,M,T),
(i) A sequence $\{a_p\}$ in X is said to be a Cauchy sequence if for any $\epsilon \in (0,1)$, $t > 0$, there exists a positive integer k satisfying $M(a_p,a_q,t) >$

$1-\epsilon$, $\forall\ p,q \geq k$

(ii) A sequence $\{a_p\}$ in X converges to a if for any $\epsilon \in (0,1)$, $t > 0$, there exists a positive integer k satisfying $M(a_p, a, t) > 1-\epsilon$, $\forall\ p \geq k$.

(iii) X is said to be complete if and only if every Cauchy sequence converges in X.

(iv) X is called compact if and only if every sequence in X has a convergent subsequence.

Definition 1.6. [38] Let $\phi : [0,\infty[\longrightarrow [0,\infty[$ be a mapping. If
(i) ϕ is strictly decreasing and left continuous,
(ii) $\phi(\lambda) = 0$ if and only if $\lambda = 1$
i.e, $\lim_{\phi \longrightarrow 1-} \phi(1) = 0$.

then, the function ϕ is called an altering distance function.

In [38], Shen et.al, proved the following fixed point theorem in complete fuzzy metric spaces.

Theorem 1.7. [38] *For a complete fuzzy metric space (X, M, T), let f be a self map on X. Let ϕ be the altering distance function and k be a function from $(0,\infty)$ into $(0,1)$. If for any $t > 0$, f satisfies the following condition:*

$$\phi(M(f(x), f(y), t)) \leq k(t).\phi(M(x,y,t)) \tag{1}$$

$x, y \in X$ and $x \neq y$, then there exists a unique fixed point for f in X.

The following theorem by Wairojjana et al. comes as a generalization of the Theorem 1.7

Theorem 1.8. [41] *For a complete fuzzy metric space (X, M, T), let f be a self map on X. If there exist functions $k_1,\ k_2 : (0,\infty) \longrightarrow [0,1),\ k_3 : (0,\infty) \longrightarrow (0,1), \sum_{i=1}^{3} k_i(t) < 1$, and an altering distance function ϕ such that the following condition is satisfied:*

$$\begin{aligned}\phi(M(f(x), f(y), t)) \leq k_1(t)\cdot\phi(M(x, f(x), t)) + k_2(t)\cdot\phi(M(y, f(y), t))\\ +k_3(t)\cdot\phi(M(x,y,t))\end{aligned} \tag{2}$$

$x, y \in X$, $x \neq y$, and $t > 0$, then there exists a unique fixed point for f.

Taking a minimum condition, Wairojjana et al.,also proved the following result.

THEOREM 1.9. *For a complete fuzzy metric space* (X, M, T)*, let* T *be a triangular norm and* f *be a self map on* X*. If there exist* $k_1, k_2 : (0, \infty) \longrightarrow (0, 1)$*, and an altering distance function* ϕ *such that the following condition holds:*

$$\begin{aligned}
&\phi(M(f(x), f(y), t)) \\
\leq &k_1(t) \cdot \min\{\phi(M(x, y, t)), \phi(M(x, f(x), t)), \phi(M(x, f(y), 2t)), \\
&\phi(M(y, f(y), t))\} + k_2(t)\phi(M(f(x), y, 2t))
\end{aligned} \tag{3}$$

$x, y \in X,\ x \neq y$ *and* $t > 0$*, then there exists a unique point for* f*.*

2. Results and discussion

Considering the function $k : (0, \infty) \longrightarrow (0, 1]$, in Theorem 1.8, we take the case when $k(t) = 1$. Using a condition as in [38], we proved the following:

THEOREM 2.1. *Let* (X, M, T) *be a complete fuzzy metric space and* ϕ *be the altering distance function. Let* f *be a self-map on* X *satisfying*

$$\phi(M(f(x), f(y), t)) \leq \phi(M(x, y, t)), \forall\ x, y \in X, t > 0. \tag{4}$$

For $x_0 \in X$*, let the sequence* $\{x_n\}$ *in* X *is constructed by:* $x_{n+1} = f(x_n), n = 0, 1, 2....$
Then $x_n \longrightarrow p$ *in* X*,* p *being the unique fixed point of* f *if and only if for any two sub sequences* $\{x_{p_n}\}$ *and* $\{x_{q_n}\}$ *with* $x_{p_n} \neq x_{q_n} \forall\ n$*, we have that,*

$$\begin{aligned}
Q_n = \frac{\phi(M(f(x_{p_n}), f(x_{q_n}), t))}{\phi(M(x_{p_n}, x_{q_n}, t))} \longrightarrow 1 \\
\text{only if } \phi(M(x_{p_n}, x_{q_n}, t)) \longrightarrow 0 \text{ as } n \longrightarrow \infty
\end{aligned} \tag{5}$$

Proof. First we assume that the sequence $\{x_n\}$ converges to the unique fixed point p of f. Let $\{x_{p_n}\}$ and $\{x_{q_n}\}$ be any two sub sequences of $\{x_n\}$.

Then

$$\lim_{n\longrightarrow\infty} \phi(M(x_{p_n}, x_{q_n}, t))) = \phi(M(p,p,t)) = \phi(1) = 0$$

$$\text{and } Q_n = \frac{\phi(M(f(x_{p_n}), f(x_{q_n}), t))}{\phi(M(x_{p_n}, x_{q_n}, t))}$$

$$\longrightarrow \frac{\phi(M(f(p), f(p), t))}{\phi(M(p,p,t))} = \frac{\phi(M(p,p,t))}{\phi(M(p,p,t))} \quad (\text{since } f(p) = p)$$

$$= 1 \text{ as } n \longrightarrow \infty$$

So, the condition (5) is satisfied.
Conversely, for $x_0 \in X,\ f(x_n) = x_{n+1},\ n = 0, 1, 2...$
Let $\phi_n = \phi(M(x_n, x_{n+1}, t)),\ n = 0, 1, 2...$ .Then,

$$\begin{aligned}\phi_{n+1} &= \phi(M(f(x_n), f(x_{n+1}), t)) \\ &\leq \phi(M(x_n, x_{n+1}, t)) \quad (\text{by}(4)) \\ &= \phi_n\end{aligned}$$

Thus $\{\phi_n\}$ is a decreasing sequence of non negative real numbers and so, is convergent to some real, say $\epsilon \geq 0$.
Assume that $\epsilon > 0$. Taking $p_n = n$ and $q_n = n + 1$,

$$\begin{aligned}&\phi(M(x_{p_n}, x_{q_n}, t)) \\ &= \phi(M(x_n, x_{n+1}, t)) = \phi_n \longrightarrow \epsilon > 0.\end{aligned}$$

$$\text{But } Q_n = \frac{\phi(M(f(x_n), f(x_{n+1}), t))}{\phi(M(x_n, x_{n+1}, t))}$$

$$= \frac{\phi(M(x_{n+1}, x_{n+2}, t))}{\phi(M(x_n, x_{n+1}, t))} \longrightarrow 1 \text{ as } n \longrightarrow \infty,$$

violating the given condition (5).
Hence $\epsilon = 0$, i.e, $\phi_n \longrightarrow 0$

(6) $$i.e., M(x_n, x_{n+1}, t) \longrightarrow 1 \text{ as } n \longrightarrow \infty.$$

Now, to show that the sequence $\{x_n\}$ is Cauchy :
If not, $\exists\ 0 < \epsilon < 1$ and two increasing sequences of integers $\{a_n\}$ and $\{b_n\}$ such that for every $n \in \mathbb{N} \cup \{0\}$, we have $a_n > b_n > n$

$$\begin{aligned}&M(x_{a_n}, x_{b_n}, t) \leq 1 - \epsilon \text{ and} \\ &M(x_{a_n - 1}, x_{b_n}, t) > 1 - \epsilon,\ t > 0.\end{aligned}$$

Let $S_n(t) = M(x_{a_n}, x_{b_n}, t)$ for each $n \in \mathbb{N} \cup \{0\}$.

$$\begin{aligned}\text{Then } 1-\epsilon \geq S_n(t) &\geq T(M(x_{a_n}, x_{a_n-1}, t/2), M(x_{a_n-1}, x_{b_n}, t/2))\\ &\geq T(M(x_{a_n}, x_{a_n-1}, t/2), 1-\epsilon)\\ &\longrightarrow T(1, 1-\epsilon) \ \ (\text{by}(6))\\ &= 1-\epsilon \text{ as } n \longrightarrow \infty\end{aligned}$$

Thus, $S_n(t) \longrightarrow 1-\epsilon$ as $n \longrightarrow \infty$

Therefore, $\phi(S_n(t)) = \phi(M(x_{a_n}, x_{b_n}, t)) \longrightarrow \phi(1-\epsilon)$ as $n \longrightarrow \infty$

$$\text{But} \quad Q_n = \frac{\phi(M(x_{a_n+1}, x_{b_n+1}, t))}{\phi(M(x_{a_n}, x_{b_n}, t))} \longrightarrow 1 \text{ as } n \longrightarrow \infty,$$

which again violates the condition (5).

Therefore, $\{x_n\}$ is a Cauchy sequence in X. Since (X, M, T) is complete, so, $\exists\, p \in X$ such that $x_n \longrightarrow p$.

$$\begin{aligned}\text{Now,} \quad 0 \leq \phi(M(f(x_n), f(p), t)) &= \phi(M(x_{n+1}, f(p), t))\\ &\leq \phi(M(x_n, p, t))\end{aligned}$$

Taking $n \longrightarrow \infty$,

$$\begin{aligned}\phi(M(p, f(p), t)) &\leq \phi(M(p, p, t)) = \phi(1) = 0\\ &\Rightarrow \phi(M(p, f(p), t)) = 0\\ &\Rightarrow M(p, f(p), t) = 1\\ &\Rightarrow f(p) = p,\end{aligned}$$

showing that p is a fixed point for f.

To show the uniqueness :
If p_1 is another fixed point for f,

$$\phi(M(p, p_1, t)) = \phi(M(f(p), f(p_1), t)) < \phi(M(p, p_1, t))$$

a contradiction. So, p is the unique fixed point for f. □

Example 2.2. Let $X = [0, 1]$ and $d(x, y) = |x - y| \ \forall\, x, y \in X$. Then (X, d) is a complete metric space. Let M be a fuzzy set on $X^2 \times [0, \infty[$ given by $M(x, y, t) = \dfrac{t}{t + d(x, y)}$, if $t > 0$ and $M(x, y, 0) = 0$.

Then (X, M, T) is a complete fuzzy metric space with respect to the t-norm : $T(a, b) = \min\{a, b\}$, $a, b \in [0, 1]$. We take $f : X \longrightarrow X$ given by $f(x) = \dfrac{x}{7}\ \forall\ x \in X$ and $\phi(\lambda) = 1 - \lambda$.

$$\begin{aligned} \text{Now,}\ \ M(f(x), f(y), t) =& \frac{t}{t + \frac{1}{7}|x - y|} \\ &= M(x, y, 7t),\ t > 0 \\ &\geq M(x, y, t) \\ &\text{(as } M(x, y, .) \text{ is non-decreasing)} \end{aligned}$$

So, $\phi(M(f(x), f(y), t)) \leq \phi(M(x, y, t))$

For $x_0 \in [0, 1]$, we consider the sequence $\{\dfrac{x_0}{7^n}\}$ which is obtained by: $x_{n+1} = f(x_n), n = 0, 1, 2, 3....$. Now, for any two sub sequences of $\{\dfrac{x_0}{7^n}\}$, the condition (5) is obviously satisfied.
Hence f has a unique fixed point which is clearly 0, here.

Omitting the completeness condition in Theorem 2.1, we get

THEOREM 2.3. *Let (X, M, T) be a fuzzy metric space and ϕ be the altering distance function. Let f be a self map on X satisfying*
(i) $\phi(M(f(x), f(y), t)) \leq k_1(x, y, t)\phi(M(x, f(x), t)) + k_2(x, y, t)\phi(M(y, f(y), t)) + k_3(x, y, t)\phi(M(x, y, t))$, *where $k_1,\ k_2,\ k_3$ are functions from $X^2 \times (0, \infty)$ to $(0, 1)$ such that $0 < k_1 + k_2 + k_3 < 1,\ k_i = k_i(x, y, t)$.*
(ii) f is continuous at some point $p \in X$
(iii) $\exists$ a point $z \in X$ such that the sequence of iterates $\{f^n(z)\}$ has a subsequence $\{f^{n_k}(z)\}$ converging to p.
Then p is the unique fixed point for f.

Proof. Let $x_0 = z \in X$. we define

$$x_{n+1} = f(x_n),\ n = 0, 1, 2.....$$

If $f^n(x_0) = f^{n+1}(x_0)$ for some n, then there exists $y \in X$ such that $y = f^k(x_0)$ for all $k \geq n$. [In that case, y is the unique fixed point.]

So, we assume that $f^n(x_0) \neq f^{n+1}(x_0)$, for any n.
By (iii), $\lim_{k\longrightarrow\infty} f^{n_k}(z) = p$, i.e, $\lim_{k\longrightarrow\infty} x_{n_k} = p$.
Since, f is continuous at p, (by(ii)), so,

$$\begin{aligned} f(p) &= \lim_{k\longrightarrow\infty} f^{n_k+1}(z) = \lim_{k\longrightarrow\infty} f^{n_k+1}(x_0) \\ &= \lim_{k\longrightarrow\infty} x_{n_k+1} \end{aligned}$$

By (i),

$$\begin{aligned}\phi(M(f(x_{n-1}),f(x_n),t)) \leq& k_1\phi(M(x_{n-1},f(x_{n-1}),t))\\ &+k_2\phi(M(x_n,f(x_n),t))+k_3\phi(M(x_{n-1},x_n,t))\\ \Rightarrow \phi(M(x_n,x_{n+1},t)) \leq& k_1\phi(M(x_{n-1},x_n,t))\\ &+k_2\phi(M(x_n,x_{n+1},t))+k_3\phi(M(x_{n-1},x_n,t))\\ \Rightarrow \phi(M(x_n,x_{n+1},t)) \leq& (\frac{k_1+k_3}{1-k_2})\phi(M(x_{n-1},x_n,t))\\ \Rightarrow \phi(M(x_n,x_{n+1},t)) <& \phi(M(x_{n-1},x_n,t)) \qquad (7)\end{aligned}$$

So, $\phi(M(f(x_{n_k+1}),f(f(p)),t)) < \phi(M(x_{n_k+1},f(p),t))$

Since, ϕ is left continuous and M is continuous,

$$\begin{aligned}\phi(M(\lim_{k\longrightarrow\infty} x_{n_k+2},f^2(p),t)) \leq& \phi(M(\lim_{k\longrightarrow\infty} x_{n_k+1},f(p),t))\\ =&\phi(M(f(p),f(p),t))\\ =&0\end{aligned}$$

$$\Rightarrow \lim_{k\longrightarrow\infty} x_{n_k+2} = f^2(p) \qquad (8)$$

Again,

$$\begin{aligned}\phi(M(p,f(p),t)) =& \lim_{k\longrightarrow\infty}\phi(M(x_{n_k},x_{n_k+1},t))\\ =& \lim_{k\longrightarrow\infty}\phi(M(x_{n_k+1},x_{n_k+2},t))\\ =&\phi(M(f(p),f^2(p),t))\ (by (8)) \qquad (9)\end{aligned}$$

If $f(p) \neq p$, then from (7),
$\phi(M(f(p),f^2(p),t)) < \phi(M(p,f(p),t))$,
a contradiction to (9). Therefore, $f(p) = p$, showing that p is a fixed point for f.
Uniqueness follows easily from (7). □

EXAMPLE 2.4. Let $X = [0,1]$ and $d(x,y) = |e^x - e^y|$ for every $x, y \in X$. Then (X,d) is an incomplete metric space. Let M be a fuzzy set on $X^2 \times [0,\infty[$ given by $M(x,y,t) = \dfrac{t}{t+d(x,y)}$, $t > 0$ and $M(x,y,0) = 0$.

Let, $T(x,y) = xy$. Then (X,M,T) is an incomplete fuzzy metric space. We take $\phi(\lambda) = 1-\lambda$, $\lambda \in [0,1]$ and $f : X \longrightarrow X$ is defined by $f(x) = \dfrac{2+x}{3}$.
Clearly, f is continuous at the point $1 \in X$.

Let $z = x_0 \in X$ and we consider the sequence of iterates $\{f^n(x_0)\}$ having a subsequence $\{f^{n_k}(x_0)\}$, where $n_k = 2k-1,\ k \in N$

$$\text{Now,} \quad \lim_{k \longrightarrow \infty} f^{n_k}(x_0) = 1$$

$$\text{We take,} \quad k_1(x,y,t) = \begin{cases} \dfrac{1+\frac{t}{e^{\frac{2+x}{3}}+e^x}}{1+\frac{t}{e^{\frac{2+x}{3}}-e^{\frac{2+y}{3}}}}, & \text{if } x > y \\ \dfrac{1+\frac{t}{e^{\frac{2+x}{3}}+e^x}}{1+\frac{t}{e^{\frac{2+y}{3}}-e^{\frac{2+x}{3}}}}, & \text{if } y > x \end{cases}$$

$$\text{and} \quad k_2(x,y,t) = k_3(x,y,t) = \frac{1-k_1(x,y,t)}{3}$$

$$\text{Again,} \quad \phi(M(f(x),f(y),t)) = \begin{cases} \frac{e^{\frac{2+x}{3}}-e^{\frac{2+y}{3}}}{t+(e^{\frac{2+x}{3}}-e^{\frac{2+y}{3}})}, & \text{if } x > y \\ \frac{e^{\frac{2+y}{3}}-e^{\frac{2+x}{3}}}{t+(e^{\frac{2+y}{3}}-e^{\frac{2+x}{3}})}, & \text{if } y > x \end{cases}$$

$$\phi(M(x,f(x),t)) = \frac{e^{\frac{2+x}{3}} - e^x}{t + e^{\frac{2+x}{3}} - e^x}$$

Similarly,

$$\phi(M(y,f(y),t)) = \frac{e^{\frac{2+y}{3}} - e^y}{t + e^{\frac{2+y}{3}} - e^y}$$

$$\text{and } \phi(M(x,y,t)) = \begin{cases} \frac{e^x-e^y}{t+e^x-e^y}, & \text{for } x > y \\ \frac{e^y-e^x}{t+e^y-e^x}, & \text{for } y > x \end{cases}$$

$$\begin{aligned} \text{Now,} \quad & k_1(x,y,t)\phi(M(x,f(x),t)) + k_2(x,y,t)\phi(M(y,f(y),t)) \\ & +k_3(x,y,t)\phi(M(x,y,t)) \\ & \geq \phi(M(f(x),f(y),t)) \end{aligned}$$

So, all the conditions of Theorem 2.3 are satisfied and f has a unique fixed point. Clearly, 1 is the unique fixed point for f here.

Remark 2.5. Introducing another function $k_4 : X^2 \times (0, \infty) \longrightarrow (0, 1)$, the condition (i) in the Theorem 2.3 can be replaced by:

$$\begin{aligned}\phi(M(f(x), f(y), t)) \leq & k_1\phi(M(x, f(x), t)) + k_2\phi(M(y, f(y), t)) \\ & + k_3\phi(M(x, y, t)) + k_4\frac{\phi(M(y, f(y), t))\,(1 + \phi(M(x, f(x), t)))}{1 + \phi(M(x, y, t))} \\ & \text{with } 0 \leq k_1 + k_2 + k_3 + k_4 < 1\end{aligned}$$

In this case,

$$\begin{aligned}\phi(M(f(x_{n-1}), f(x_n), t)) \leq & k_1\phi(M(x_{n-1}, f(x_{n-1}), t)) \\ & + k_2\phi(M(x_n, f(x_n), t)) + k_3\phi(M(x_{n-1}, x_n, t)) \\ & + k_4\frac{\phi(M(x_n, f(x_n), t))\,(1 + \phi(M(x_{n-1}, f(x_{n-1}), t)))}{1 + \phi(M(x_{n-1}, x_n, t))} \\ & = k_1\phi(M(x_{n-1}, x_n, t)) + k_2\phi(M(x_n, x_{n+1}, t)) \\ & + k_3\phi(M(x_{n-1}, x_n, t)) + k_4\frac{\phi(M(x_n, x_{n+1}, t))\,(1 + \phi(M(x_{n-1}, x_n, t)))}{1 + \phi(M(x_{n-1}, x_n, t))} \\ & = (k_1 + k_3)\phi(M(x_{n-1}, x_n, t)) + (k_2 + k_4)\phi(M(x_n, x_{n+1}, t)) \\ & \Rightarrow \phi(M(x_n, x_{n+1}, t)) \leq \frac{k_1 + k_3}{1 - (k_2 + k_4)}\phi(M(x_n, x_{n+1}, t)) \\ & < \phi(M(x_n, x_{n+1}, t))\end{aligned}$$

The other part is same as in Theorem 2.3.
Similar example as 2.4 can be constructed for this also.

In the following theorem, we use the condition $\lim_{t \longrightarrow \infty} M(x, y, t) = 1$.

Theorem 2.6. *Let (X, M, T) be a complete fuzzy metric space and ϕ be the altering distance function. Let f be a self map on X satisfying the following: for every positive integer n and $t > 0$,*

$$\phi(M(f^n(x), f^n(y), k_n t)) \leq \phi(M(x, y, t)), \tag{10}$$

for all $x, y \in X$, $k_n > 0$ being independent of x and y. If $k_n \longrightarrow 0$ as $n \longrightarrow \infty$ then f has a unique fixed point in X.

Proof. Let $x_0 \in X$. We construct a sequence $\{x_n\}$ in X by $x_1 = f(x_0), x_2 = f(x_1) = f^2(x_0), ..., x_n = f(x_{n-1}) = f^n(x_0), ..., \ n \in \mathbb{N}$
From (10),

$$\phi(M(f^n(x), f^n(y), k_n t)) \leq \phi(M(x, y, t))$$

Since, ϕ is strictly decreasing, so,

$$M(f^n(x), f^n(y), k_n t) \geq M(x, y, t) \tag{11}$$

Again, for $p \in \mathbb{N}$,

$$\begin{aligned}
1 \geq & M(x_n, x_{n+p}, t) \\
\geq & T(T(...T(M(x_n, x_{n+1}, t/p), M(x_{n+1}, x_{n+2}, t/p)...), M(x_{n+p-1}, x_{n+p}, t/p)) \\
\geq & T(T(...T(M(x, x_1, \frac{t}{pk_n}), M(x, x_1, \frac{t}{pk_{n+1}}), ...), M(x, x_1, \frac{t}{pk_{n+p-1}})) \text{ (by (11))} \\
& \longrightarrow T(T(...T(1,1), ...), 1) \text{ as } n \longrightarrow \infty \\
& \text{(Since } k_n \longrightarrow 0 \text{ as } n \longrightarrow \infty \text{ and} \lim_{t \longrightarrow \infty} M(x, y, t) = 1) \\
& = 1
\end{aligned}$$

Thus, $\lim_{n \longrightarrow \infty} M(x_n, x_{n+p}, t) = 1 \ \forall \ t > 0$

So, $\{x_n\}$ is a Cauchy sequence in X.
As X is complete, $\exists \ p \in X$ such that $x_n \longrightarrow p$ as $n \longrightarrow \infty$.
Now,

$$\begin{aligned}
0 \leq \phi(M(x_{n+1}, f(p), t)) &= \phi(M(f(x_n), f(p), t)) \\
&\leq \phi(M(x_n, p, t/k_1)) \text{ (by (10))} \quad \forall \ \ t > 0
\end{aligned}$$

Taking $n \longrightarrow \infty$, we have,

$$\begin{aligned}
& \phi(M(p, f(p), t)) \leq \phi(M(p, p, t/k_1)) = 0 \\
\Rightarrow & \phi(M(p, f(p), t)) = 0 \ \forall \ t > 0 \\
\Rightarrow & p = f(p),
\end{aligned}$$

showing that p is a fixed point for f.
To show uniqueness:
Let $q \in X$ be such that $f(q) = q$. Now,

$$\begin{aligned}
0 \leq \phi(M(p, q, t)) =& \phi(M(f^n(p), f^n(q), t)) \\
\leq & \phi(M(p, q, t/k_n)) \longrightarrow 0 \ \text{ as } n \longrightarrow \infty \ \forall \ t > 0 \\
\Rightarrow \phi(M(p, q, t)) =& 0 \ \ \forall \ t > 0 \\
\Rightarrow & \ p = q.
\end{aligned}$$

Thus, f has the unique fixed point p in X. □

EXAMPLE 2.7. We take the same complete fuzzy metric space (X, M, T) as in Example 2.2. Let f be a self map on X defined by : $f(x) =$

$\dfrac{3+x}{4}$, $x \in X$ and $\phi(\lambda) = 1 - \lambda$, $\lambda \in [0,1]$. For $n \in \mathbb{N}$, let $k_n = \frac{1}{n}$. Then $k_n \longrightarrow 0$ as $n \longrightarrow \infty$.

For $x \neq y$,

$$\begin{aligned}\phi(M(f^n(x), f^n(y), k_n t)) =& \frac{|\frac{x-y}{4^n}|}{\frac{t}{n} + |\frac{x-y}{4^n}|} \\ =& \frac{|x-y|}{\frac{4^n}{n}t + |x-y|} < \frac{|x-y|}{t + |x-y|} = \phi(M(x,y,t))\end{aligned}$$

For $x = y$, $\phi(M(f^n(x), f^n(y), k_n t)) = \phi(M(x,y,t))$.
Thus, the condition of Theorem 2.6 is satisfied and so, f has a unique fixed point.
Clearly, 1 is the unique fixed point for f in X.

Corollary 2.8. *Let (X, M, T) be a complete metric space, ϕ be the altering distance function and f be a self map on X satisfying*
$\phi(M(f(x), f(y), kt)) \leq \phi(M(x,y,t)) \ \forall \ x, y \in X, \ t > 0$
and $0 < k < 1$.
Then, f has a unique fixed point in X.

Now, instead of taking the constants $k_n (n \in \mathbb{N})$ in Theorem 2.6, we take $k(t)$ as a function of t and prove the following:

Theorem 2.9. *Let (X, M, T) be a complete fuzzy metric space, ϕ be the altering distance function and f be a self map on X. Let $k : (0,\infty) \longrightarrow (0,\infty)$ be a bijective function such that for each $t \in (0,\infty)$, $\{k^n(t)\}$ is a decreasing sequence with $\lim_{n\longrightarrow\infty} k^n(t) = 0$. If f satisfies:*

$$\phi(M(f^n(x), f^n(y), k^n(t))) \leq \phi(M(x,y,t)) \tag{12}$$

for all $x, y \in X$ and $t > 0$, $n \in \mathbb{N}$, then f has a unique fixed point in X.

Proof. For $x_0 \in X$, we construct the sequence $\{x_n\}$ in X as in Theorem 2.6.

Now,

$$\begin{aligned}0 \leq& \phi(M(x_n, x_{n+m}, t)) \\ \leq& \phi(M(x, x_m, k^{-n}t)) \text{ (Replacing } t \text{ by } k^{-n}(t) \text{in (12))} \\ & \text{for each } m \in \mathbb{N}.\end{aligned}$$

Again, $\lim\limits_{n\longrightarrow\infty} \phi(M(x, x_m, k^{-n}(t))) = 0$ (Since ϕ is left continuous) $\forall\ t > 0$

$$\text{So,}\quad \phi(M(x_n, x_{n+m}, t)) \longrightarrow 0 \text{ as } n \longrightarrow \infty$$
$$\Rightarrow M(x_n, x_{n+m}, t) \longrightarrow 1 \text{ as } n \longrightarrow \infty\ \ \forall\ t > 0$$

Thus $\{x_n\}$ is a Cauchy sequence in X and so, converges to some $p \in X$.

$$\begin{aligned}\text{Now,}\quad 0 \le \phi(M(x_{n+1}, f(p), t)) &= \phi(M(f(x_n), f(p), t))\\ &\le \phi(M(x_n, p, k^{-1}(t)))\ \ \forall\ t > 0\end{aligned}$$

Taking $n \longrightarrow \infty$,

$$\begin{aligned}&\phi(M(p, f(p), t)) \le \phi(M(p, p, k^{-1}(t))) = 0\ \forall\ t > 0\\ \Rightarrow &M(p, f(p), t) = 1\ \ \forall\ t > 0\\ \Rightarrow &f(p) = p.\end{aligned}$$

showing that p is a fixed point for f.
Uniqueness can be shown in a similar manner. □

EXAMPLE 2.10. For the same fuzzy metric space (X, M, T) and the same altering distance function as in Example 2.2, let $k : (0, \infty) \longrightarrow (0, \infty)$ be defined by: $k(t) = \frac{t}{2},\ \ t > 0$. Then $k(t)$ is bijective. Also, for each t, $\{k^n(t)\}$ is a decreasing sequence and $\lim_{n\longrightarrow\infty} k^n(t) = \lim_{n\longrightarrow\infty} \frac{t}{2^n} = 0$.
We take $f : X \longrightarrow X$ as $f(x) = \frac{2+x}{3}$.
So, $f^n(x) = \frac{2}{3} + \frac{2}{3^2} + \ldots.. + \frac{2}{3^n} + \frac{x}{3^n}$, and

$$\begin{aligned}\phi(M(f^n(x), f^n(y), k^n(t))) =&\frac{|x-y|}{\frac{3^n}{2^n}t + |x-y|}\\ <&\frac{|x-y|}{t + |x-y|}\\ =&\phi(M(x, y, t))\qquad \forall\ x, y \in X,\ t \in (0, \infty),\ n \in \mathbb{N}\end{aligned}$$

Thus, all the conditions of Theorem 2.9 are satisfied and so, f has a unique fixed point which is clearly 1 here.

The following theorem can be taken as a generalization of Theorem 1.9 by Wairojjana et al. [41] :

THEOREM 2.11. *Let (X, M, T) be a complete fuzzy metric space, f be a self map on X, and ϕ be the altering distance function. If there* *exist* $k_1, k_2 : (0,\infty) \longrightarrow (0,1)$ *such that* $k_1(t) + k_2(t) < 1$*, and*

$$(13)\quad \begin{aligned}&\phi(M(f(x), f(y), t))\\ \le& k_1(t)\cdot min\{\phi(M(x,y,t)), \phi(M(x,f(x),t)), \phi(M(x,f(y),2t)),\\ &\phi(M(y,f(y),t))\} + k_2(t)\cdot max\{\phi(M(x,y,t)), \phi(M(x,f(x),t)),\\ &\phi(M(f(x),y,2t)), \phi(M(y,f(y),t))\},\ x,y\in X,\ x\neq y,\ t>0,\end{aligned}$$

then f *has a unique fixed point.*

Proof. For $x_0 \in X$, we consider the sequence $\{x_n\}$ in X, where $x_{n+1} = f(x_n),\ n = 0,1,2,....$
If there exists $n_0 \in \mathbb{N}\cup\{0\}$ such that $x_{n_0+1} = x_{n_0}$, then x_{n_0} will be a fixed point for f. So, we assume that $x_n \neq x_{n+1}$ for any n. Then
$0 < M(x_n, x_{n+1}, t) < 1,\ t > 0,\ n \in \mathbb{N}\cup\{0\}$.
Substituting $x = x_{n-1}, y = x_n$ in (13), we have,

(14)

$$\begin{aligned}&\phi(M(x_n, x_{n+1}, t))\\ \le& k_1(t)\cdot \min\{\phi(M(x_{n-1},x_n,t)), \phi(M(x_{n-1},x_n,t)), \phi(M(x_{n-1},x_{n+1},2t)),\\ &\phi(M(x_n,x_{n+1},t))\} + k_2(t)\cdot\max\{\phi(M(x_{n-1},x_n,t)), \phi(M(x_{n-1},x_n,t)),\\ &\phi(M(x_n,x_n,2t)), \phi(M(x_n,x_{n+1},t))\}\\ =& k_1(t)\cdot\min\{\phi(M(x_{n-1},x_n,t)), \phi(M(x_{n-1},x_{n+1},2t)), \phi(M(x_n,x_{n+1},t))\}\\ &+ k_2(t)\cdot\max\{\phi(M(x_{n-1},x_n,t)), \phi(M(x_n,x_{n+1},t))\}\\ \le& k_1(t)\cdot\min\{\phi(M(x_{n-1},x_n,t)), \phi(T(M(x_{n-1},x_n,t)), (M(x_n,x_{n+1},t))),\\ &\phi(M(x_n,x_{n+1},t))\} + k_2(t)\cdot\max\{\phi(M(x_{n-1},x_n,t)),\\ &\phi(M(x_n,x_{n+1},t))\}\end{aligned}$$

Again,

$$\begin{aligned}&\phi(a) \le \phi(T(a,b)) \text{ and } \phi(b) \le \phi(T(a,b))\\ &(\text{Since, } a = T(a,1) \ge T(a,b) \text{ and } b = T(b,1) \ge T(a,b))\\ \text{So,}\quad &\min\{\phi(a), \phi(b), \phi(T(a,b))\} = \min\{\phi(a),\phi(b)\} \qquad \forall\ a,b\in[0,1].\end{aligned}$$

$$\begin{aligned}\text{Hence, (14)} \Rightarrow &\phi(M(x_n, x_{n+1}, t))\\ \le& k_1(t)\cdot\min\{\phi(M(x_{n-1},x_n,t)), \phi(M(x_n,x_{n+1},t))\}\\ &+ k_2(t)\cdot\max\{\phi(M(x_{n-1},x_n,t)), \phi(M(x_n,x_{n+1},t))\}\end{aligned}$$

If $\min\{\phi(M(x_{n-1},x_n,t)),\phi(M(x_n,x_{n+1},t))\} = \phi(M(x_{n-1},x_n,t))$, then,

$$\begin{aligned}\phi(M(x_n,x_{n+1},t)) \leq & k_1(t)\phi(M(x_{n-1},x_n,t)) + k_2(t)\phi(M(x_n,x_{n+1},t))\\ \Rightarrow \phi(M(x_n,x_{n+1},t)) \leq & \frac{k_1(t)}{1-k_2(t)}\phi(M(x_{n-1},x_n,t))\\ < & \phi(M(x_{n-1},x_n,t)), \quad \text{a contradiction.}\end{aligned}$$

So, $\min\{\phi(M(x_{n-1},x_n,t)),\phi(M(x_n,x_{n+1},t))\} = \phi(M(x_n,x_{n+1},t)$.

$$\begin{aligned}\text{Then, } \phi(M(x_n,x_{n+1},t) \leq & \frac{k_2(t)}{1-k_1(t)}\phi(M(x_{n-1},x_n,t))\\ < & \phi(M(x_{n-1},x_n,t))\end{aligned}$$

Thus, $\phi(M(x_n,x_{n+1},t)) < \phi(M(x_{n-1},x_n,t))$.

Since ϕ is strictly decreasing, so, it follows that $\{M(x_n,x_{n+1},t)\}_n$ is strictly increasing for every $t > 0$.

Now, proceeding as in Theorem 2.1 together with the given condition(13), it can be easily shown that f has a unique fixed point in X. □

Example 2.12. Let $X = \{P,Q,R,S\} \subseteq \mathbb{R}^2$, where P,Q and R are the points $(1,1),(2,1)$ and $(2,5)$ respectively and S is an arbitrary point on the unit circle with centre at R. Let $f : X \longrightarrow X$ be defined by: $f(P) = f(Q) = f(S) = P, \quad f(R) = Q$.
Let $M(x,y,t) = \frac{t}{t+d(x,y)}, \quad t > 0$, where $d(x,y)$ is the Euclidean distance in $\mathbb{R}^2$. The altering distance ϕ is given by $\phi(\lambda) = 1-\lambda, \quad \lambda \in [0,1]$. With $T(a,b) = \min\{a,b\}$, (X,M,T) becomes a complete fuzzy metric space. Let $k_1,k_2 : (0,\infty) \longrightarrow (0,1)$ be defined by:

$$k_2(t) = \begin{cases} \frac{1}{1+\frac{t}{7}}, & t \in (0,1] \\ \frac{t}{t+\frac{1}{7}}, & t \in (1,\infty) \end{cases}$$

and $k_1(t) = (1-k_2(t))/2$
Then, $k_1(t) + k_2(t) < 1$.
Now, it can be shown that every pair of points $(P,Q),(P,S),(Q,S),(P,R)$, (Q,R) and (R,S) satisfies the condition (13).
Hence by Theorem 2.11 the function has a unique fixed point.

Next, we obtain the following generalization of Theorem 1.9 :

Theorem 2.13. *Let (X,M,T) be a complete fuzzy metric space, f be a self map on X, and ϕ be the altering distance function. If there* *exist functions* $k_1, k_2, k_3, k_4 : (0, \infty) \longrightarrow (0, 1)$ *with* $k_1(t) + k_2(t) + k_3(t) + k_4(t) < 1$, *such that*

$$
\begin{aligned}
(15)\quad & \phi(M(f(x), f(y), t)) \\
&\leq k_1(t) \cdot min\{\phi(M(x, f(x), t)), \phi(M(x, y, t))\} \\
&\quad + k_2(t) \cdot min\{\phi(M(y, f(y), t)), \phi(M(x, f(x), t))\} \\
&\quad + k_3(t) \cdot min\{\phi(M(x, y, t)), \phi(M(x, f(y), 2t)), \phi(M(y, f(y), t))\} \\
&\quad + k_4(t)\phi(M(f(x), y, 2t)),\ x, y \in X,\ x \neq y,\ t > 0
\end{aligned}
$$

then f *has a unique fixed point.*

Proof. For $x_0 \in X$, we take the sequence $\{x_n\}$ with $x_{n+1} = f(x_n)$ as earlier. Taking $x_n \neq x_{n+1}$, for any n i.e., $0 < M(x_n, x_{n+1}, t) < 1, n \in \mathbb{N} \cup \{0\},\ t > 0$, we have,

$$
\begin{aligned}
& \phi(M(x_n, x_{n+1}, t)) \\
\leq\ & k_1(t) \cdot \min\{\phi(M(x_{n-1}, x_n, t)), \phi(M(x_{n-1}, x_n, t))\} \\
& +k_2(t) \cdot \min\{\phi(M(x_n, x_{n+1}, t)), \phi(M(x_{n-1}, x_n, t))\} \\
& +k_3(t) \cdot \min\{\phi(M(x_{n-1}, x_n, t)), \phi(M(x_{n-1}, x_{n+1}, 2t)), \phi(M(x_n, x_{n+1}, t))\} \\
& +k_4(t)\phi(M(x_n, x_n, t)) \\
\leq\ & k_1(t) \cdot \phi(M(x_{n-1}, x_n, t)) + k_2(t) \cdot \min\{\phi(M(x_n, x_{n+1}, t)), \phi(M(x_{n-1}, x_n, t))\} \\
& +k_3(t) \cdot \min\{\phi(M(x_{n-1}, x_n, t)), \phi(T(M(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t))), \\
& \quad \phi(M(x_n, x_{n+1}, t))\}
\end{aligned}
$$

Again,

$$\min\{\phi(a), \phi(b), \phi(T(a, b))\} = \min\{\phi(a), \phi(b)\} \qquad \forall\ a, b \in [0, 1]$$

Therefore,

$$
\begin{aligned}
\phi(M(x_n, x_{n+1}, t)) \leq & k_1(t) \cdot \phi(M(x_{n-1}, x_n, t)) + k_2(t) \cdot \min\{\phi(M(x_n, x_{n+1}, t)), \\
& \phi(M(x_{n-1}, x_n, t))\} + k_3(t) \cdot \min\{\phi(M(x_n, x_{n+1}, t)), \phi(M(x_{n-1}, x_n, t))\}
\end{aligned}
$$

If $\min\{\phi(M(x_n, x_{n+1}, t)),\phi(M(x_{n-1}, x_n, t))\} = \phi(M(x_n, x_{n+1}, t))$,

$$\begin{aligned} \text{then}\quad \phi(M(x_n, x_{n+1}, t)) \leq& k_1(t)\cdot\phi(M(x_{n-1}, x_n, t)) + k_2(t)\cdot\phi(M(x_n, x_{n+1}, t)) \\ &+ k_3(t)\cdot\phi(M(x_n, x_{n+1}, t)) \\ \Rightarrow \phi(M(x_n, x_{n+1}, t)) \leq& \frac{k_1(t)}{1-k_2(t)-k_3(t)}\phi(M(x_{n-1}, x_n, t)) \\ <& \ \phi(M(x_{n-1}, x_n, t)) \end{aligned}$$

If $\min\{\phi(M(x_n, x_{n+1}, t)),\phi(M(x_{n-1}, x_n, t))\} = \phi(M(x_{n-1}, x_n, t))$,

$$\begin{aligned} \text{then}\quad \phi(M(x_n, x_{n+1}, t)) \leq& (k_1(t) + k_2(t) + k_3(t))\cdot\phi(M(x_{n-1}, x_n, t)) \\ <& \ \phi(M(x_{n-1}, x_n, t)) \end{aligned}$$

Thus, in both cases, $\phi(M(x_n, x_{n+1}, t)) \leq \phi(M(x_{n-1}, x_n, t))$.
So, proceeding as in the previous theorem, f has a unique fixed point. □

EXAMPLE 2.14. We consider the same fuzzy metric space (X, M, T) together with the self map f and the altering distance function ϕ as Example 2.12. Here we take:

$$k_4(t) = \begin{cases} \frac{7}{t+7}, & t \in (0, 1] \\ \frac{t}{t+\frac{1}{7}}, & t \in (1, \infty) \end{cases}$$

and $k_1(t) = k_2(t) = k_3(t) = \frac{1-k_4(t)}{3}$
Then every pair of points satisfies the condition (15). Hence by Theorem 2.13, f has a unique fixed point.

3. Conclusion

In this paper, we have obtained different fixed point theorems for fuzzy metric spaces with several contractive type mappings. In 2016, Lael and Heidarpour [24] introduced a new class of generalized nonexpansive mappings and obtained some fixed point theorems in Banach spaces. Here we can pose the following problem:

Can we obtain similar type of results as in Theorem 2.1, 2.3, etc., in case of nonexpansive mappings in fuzzy metric spaces ?

Similar problems can also be raised for other types of mappings on fuzzy metric spaces.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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