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LATTICE ORDERED SOFT NEAR RINGS

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Abstract. Keeping in view the expediency of soft sets in algebraic structures and as a mathematical approach to vagueness, in this paper the concept of lattice ordered soft near rings is introduced. Different properties of lattice ordered soft near rings by using some operations of soft sets are investigated. The concept of idealistic soft near rings with respect to lattice ordered soft near ring homomorphisms is deliberated.

1. Introduction

In daily life human beings face different kinds of uncertainties in fields like environmental sciences, economics, social sciences, engineering and medicine etc. To tackle such uncertainties different kinds of tools are used, such as fuzzy sets [27] and rough sets [17] etc. Although these tools are very affective [2, 12] but have their own limitations. In 1999, Molodtsov [16] came to front and introduced the concept of soft sets, which solved these uncertainties more effectively.

Molodtsov [16] when proposed the idea of soft set theory, he pointed out some basic results, and also predicted its future work. After that Maji et al [14] defined some basic operations on soft sets and after that applied the concept of soft sets in decision making problems [15]. But

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later on Ali et al [3] proved that the operations defined in [14] have some shortcomings. In algebraic structures Aktas and Cagman [1] were the first who introduced the concept of soft groups. Aslam and Qurashi [5] also contributed in soft groups. Feng et al [8] initiated the concept of soft semirings and Sezgin et al [19] worked on soft near rings. Jun [11] initiated the idea of soft BCK/BCI-algebras. Ali et al [4] worked on lattice ordered soft sets and applied this concept in daily life problems. More recently Ma et al [13] applied the concept of soft sets in decision making methods based on certain hybrid soft set models. Zhan and others did a spade work in algebraic structures, decision making methods, fuzzy sets and rough sets by using the concept of soft sets [24–26]. For more applications of soft sets one can also see [6, 9, 20, 21, 23].

In this paper the concept of lattice ordered soft near rings is introduced. Different properties of lattice ordered soft near rings by using the binary operations are investigated. The concept of idealistic soft near rings with respect to lattice ordered soft near ring homomorphisms is discussed.

An algebraic system $(\tilde{N}, +, \cdot)$ is known to be right near ring (NR) if it satisfies the following axioms:

1. $(\tilde{N}, +)$ froms a group (not necessarily abelian),
2. $(\tilde{N}, \cdot)$ forms a semigroup,
3. $(a + b) \cdot c = a \cdot c + b \cdot c,\ \forall a, b, c \in \tilde{N}$.

Note that $a \cdot 0 = 0$ & $a \cdot (-b) = -a \cdot b$, but $0 \cdot b \neq 0$, for a, $b \in \tilde{N}$, where $\tilde{N}$ will always denote a NR, if otherwise stated. From now onward instead of writing $a \cdot b$ we will write ab.

An element $d \in \tilde{N}$ is called distributive if $\forall\ n, n_1 \in \tilde{N}$, $d(n + n_1) = dn + dn_1$. We denote and define $\tilde{N}d = \{d \in \tilde{N} : d \text{ is distributive}\}$. Let $(M, +) \leq (\tilde{N}, +)$ for $\tilde{N}$ with for all $a, b \in M \Rightarrow ab \in M$. Then M is known as subnear ring $(subNR)$ of $\tilde{N}$. A normal subgroup I of $(\tilde{N}, +)$ is called an ideal of $\tilde{N}$ if $I\tilde{N}$ is subset of $\tilde{N}$, we write it as $I\tilde{N} \subseteq I$, and $\forall n, n_1 \in \tilde{N},\ i \in I,\ n(n_1 + i) - nn_1 \in I$. Then we write $I \lhd \tilde{N}$. Let $\tilde{N}_1$ and $\tilde{N}_2$ be two NRs, $g : \tilde{N}_1 \to \tilde{N}_2$ is called a NR homomorphism if $\forall n_1, n_2 \in \tilde{N}\ g(n_1 + n_2) = g(n_1) + g(n_2)$ and $g(n_1n_2) = g(n_1)g(n_2)$ [18]. For all undefined concepts about prime N-ideals, we refer to [10].

Definition 1.1. [16] Let $\tilde{N}$ be a preliminary universe set and the parameter set be E. Let $P(\tilde{N})$ represent the power set of $\tilde{N}$ and $A \subseteq E$.

Then a pair (F, A) is known as soft set (SS) over $\tilde{N}$, where F is a mapping $F : A \longrightarrow P(\tilde{N})$.

In other sense a SS over $\tilde{N}$ is a parametrized family of subsets of the universe $\tilde{N}$, for $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the SS (F, A).

Throughout in this section collection of all soft sets over $\tilde{N}$ will be represented by $SS(\tilde{N})$, E_N will denote a parameter set and $A_N, B_N \subseteq E_N$, if otherwise stated.

Definition 1.2. [14] Let (F, A_N), $(G, B_N) \in SS(\tilde{N})$. (F, A_N) is known a soft subset of (G, B_N) if

1) $A_N \subseteq B_N$ &

2) $\forall\ e \in A_N$, $F(e)$ and $G(e)$ are the same approximations.

This relation is represented by $(F, A_N) \subseteq (G, B_N)$. Similarly (F, A_N) is known a soft super set of (G, B_N) if (G, B_N) is a soft subset of (F, A_N). This relationship is represented by $(F, A_N) \supseteq (G, B_N)$.

Definition 1.3. [14] If (F, A_N) is a soft subset of (G, B_N) and (G, B_N) is soft subset of (F, A_N), then (F, A_N) and (G, B_N) are said to be equal. It is represented by $(F, A_N) = (G, B_N)$.

Definition 1.4. [14] $(F, A_N) \in SS(\tilde{N})$ is known to be a null SS represented by ϕ if $\forall\ e \in A_N \Rightarrow F(e) = \emptyset$ (null-set).

Definition 1.5. [14] $(F, A_N) \in SS(\tilde{N})$ is known to be an absolute SS represented by $\acute{A}_N$ if $\forall\ e \in A_N \Rightarrow F(e) = \tilde{N}$.

Definition 1.6. [8] Let $(F, A_N) \in SS(\tilde{N})$. Support of (F, A_N) is denoted and defined by $Supp(F, A_N) = \{e \in A_N : F(e) \neq \emptyset\}$.

Definition 1.7. [14] Let (F, A_N), $(G, B_N) \in SS(\tilde{N})$. The binary operation AND of (F, A_N) and (G, B_N) is represented by $(F, A_N)\widetilde{\wedge}(G, B_N) = (H,\ A_N \times B_N)$ and defined by $H(\alpha, b) = F(\alpha) \cap G(b)\ \forall\ (\alpha, b) \in A_N \times B_N$.

Definition 1.8. [14] Let (F, A_N), $(G, B_N) \in SS(\tilde{N})$. The binary operation OR of (F, A_N) and (G, B_N) is represented by $(F, A_N)\widetilde{\vee}(G, B_N) = (H, A_N \times B_N)$ and defined as $H(\alpha, b) = F(\alpha) \cup G(b)\ \forall\ (\alpha, b) \in A_N \times B_N$.

Definition 1.9. [14] Let (F, A_N), $(G, B_N) \in SS(\tilde{N})$. Then union of (F, A_N) and (G, B_N) is represented by $(F, A_N)\widetilde{\cup}_B(G, B_N) = (H, C_N)$, where $C_N = A_N \cup B_N$ and $\forall\ \varsigma \in C_N$

$$H(\varsigma)=\begin{cases} F(\varsigma) & \text{if } \varsigma\in A_N-B_N\\ G(\varsigma) & \text{if } \varsigma\in B_N-A_N\\ F(\varsigma)\cup G(\varsigma) & \text{if } \varsigma\in A_N\cap B_N\end{cases}$$

Definition 1.10. [14] Let (F,A_N), $(G,B_N)\in SS(\tilde{N})$ such that $C_N=A_N\cap B_N\neq\emptyset$. Intersection of (F,A_N) and (G,B_N) is represented by $(F,A_N)\widetilde{\cap}(G,B_N)=(H,C_N)$ and defined by $H(\varsigma)=F(\varsigma)$ or $G(\varsigma)$ $\forall$ $\varsigma\in C_N$.

Definition 1.11. [3] Let (F,A_N), $(G,B_N)\in SS(\tilde{N})$ such that $C_N=A_N\cap B_N\neq\emptyset$. Restricted union of (F,A_N) and (G,B_N) is represented by $(F,A_N)\widetilde{\cup_R}(G,B_N)=(L,C_N)$ and defined by $L(\varsigma)=F(\varsigma)\cup G(\varsigma)$ $\forall$ $\varsigma\in C_N$.

Definition 1.12. [3] Let (F,A_N), $(G,B_N)\in SS(\tilde{N})$ such that $C_N=A_N\cap B_N\neq\emptyset$. Restricted intersection of (F,A_N) and (G,B_N) is represented by $(F,A_N)\widetilde{\cap_R}(G,B_N)=(H,C_N)$ and defined by $H(\varsigma)=F(\varsigma)\cap G(\varsigma)$ $\forall$ $\varsigma\in C_N$.

Definition 1.13. [3] Let (F,A_N), $(G,B_N)\in SS(\tilde{N})$. Extended intersection of (F,A_N) and (G,B_N) is represented by $(F,A_N)\widetilde{\cap_E}(G,B_N)=(H,C_N)$, where $C_N=A_N\cup B_N$ and $\forall$ $\varsigma\in C_N$ and defined as

$$H(\varsigma)=\begin{cases} F(\varsigma) & \text{if } \varsigma\in A_N-B_N\\ G(\varsigma) & \text{if } \varsigma\in B_N-A_N\\ F(\varsigma)\cap G(\varsigma) & \text{if } \varsigma\in A_N\cap B_N\end{cases}$$

Definition 1.14. [7] A partially ordered set (POS) $\pounds$ is said to be a lattice iff for every $\alpha,\flat\in\pounds$, $sup\{\alpha,\flat\}$, $inf\{\alpha,\flat\}\in\pounds$.

In our next study E_N will always denote a lattice, if otherwise stated.

Definition 1.15. [4] A SS (F,A_N) is known as lattice ordered soft set $(LOSS)$ over $\tilde{N}$ (anti-lattice ordered soft sets $(ALOSS)$ over $\tilde{N}$) if $a_1\preceq\alpha_2\Rightarrow F(\alpha_1)\subseteq F(\alpha_2)$ $(F(\alpha_1)\supseteq F(\alpha_2))$, $\forall$ $\alpha_1,\ \alpha_2\in A_N$.

Definition 1.16. [19] Let (F,A_N) be a non-null SS over $\tilde{N}$. Then (F,A_N) is known as soft near ring (SNR) over $\tilde{N}$ if $F(a)$ is the $subNR$ of $\tilde{N}$, $\forall$ $\alpha\in Supp(F,A_N)$.

From now onward, if stated otherwise, collection of all soft near rings over $\tilde{N}$ will be denoted by $SNR(\tilde{N})$.

Definition 1.17. [19] Let (F,A_N), $(G,B_N)\in SNR(\tilde{N})$. Then (F,A_N) is known sub soft near ring $(subSNR)$ of (G,B_N) if $A_N\subseteq B_N$ and $F(\alpha)$ is a $subNR$ of $G(\alpha)$, $\forall$ $\alpha\in Supp(F,A_N)$.

2. Lattice Ordered Soft Near Rings

In this section we define lattice ordered soft near rings, anti-lattice ordered soft near rings and discuss the associated results.

Definition 2.1. A soft set (F, A_N) over $\tilde{N}$ is said to be lattice (anti-lattice) ordered soft near ring over $\tilde{N}$ iff
(1) $F(\alpha)$ is a $subNR$ of $\tilde{N}$ $\forall\ \alpha \in A_N$,
(2) If $\alpha_1 \preccurlyeq \alpha_2$ then $F(\alpha_1) \subseteq F(\alpha_2)$, $(F(\alpha_1) \supseteq F(\alpha_2))$, for any α_1, $\alpha_2 \in A_{N,}$.

Definition 2.2. From now onward $LOSNR(\tilde{N})$ will denote the collection of all lattice ordered soft near rings over $\tilde{N}$ and $ALOSNR(\tilde{N})$ will denote the collection of all anti-lattice ordered soft near rings over $\tilde{N}$.

Definition 2.3. Let (F, A_N), $(G, B_N) \in LOSNR(\tilde{N})$. (F, A_N) is said to be LOS $subNR$ of $(G,\ B_N)$ if it satisfies the following conditions:
(1) $A_N \subseteq B_N$
(2) $F(\alpha)$ is $subNR$ of $G(b)$ for $\alpha \in Supp(F, A_N)$.

Example 2.4. Consider $\tilde{N} = \mathbb{Z}_8 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}\}$ with $\overline{0} \preccurlyeq \overline{1} \preccurlyeq \overline{2} \preccurlyeq \overline{3} \preccurlyeq \overline{4} \preccurlyeq \overline{5} \preccurlyeq \overline{6} \preccurlyeq \overline{7}$. Take $A_N = \{\overline{1}, \overline{2}, \overline{4}\}$ define $F : A_N \longrightarrow P(\tilde{N})$ by
$F(\alpha) = \{n \in \tilde{N} : \alpha n = \overline{0}\}$, then $(F, A_N) = \{F(\overline{1}) = \{\overline{0}\},\ F(\overline{2}) = \{\overline{0}, \overline{4}\}, F(\overline{4}) = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}\}\}$ is $LOSNR$.
Now let $B_N = \{\overline{1}, \overline{2}, \overline{4}\}$ and let define $G : B_N \longrightarrow P(\tilde{N})$ by
$G(b) = \{n \in \tilde{N} :\ bn \in \{\overline{0}, \overline{4}\}\}$, then $G(\overline{1}) = \{\overline{0}, \overline{4}\}$, $G(\overline{2}) = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}\}$, $G(\overline{4}) = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}\}, \Longrightarrow (\ G, B_N) = \{G(\overline{1}) = \{\overline{0}, \overline{4}\}$, $G(\overline{2}) = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}\}$, $G(\overline{4}) = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}\}\}$ is $LOSNR$.
Now for $\overline{1} \in Supp(F.A_N)$, $F(\overline{1}) = \{\overline{0}\}$ is $subNR$ of $G(\overline{1}) = \big\{\overline{0}, \overline{4}\big\}$, for $\overline{2} \in Supp(F, A_N)$, $F(\overline{2}) = \big\{\overline{0}, \overline{4}\big\}$ is $subNR$ of $G(\overline{2}) = \big\{\overline{0}, \overline{2}, \overline{4}, \overline{6}\big\}$, for $\overline{4} \in Supp(F, A_N)$, $F(\overline{4}) = \big\{\overline{0}, \overline{2}, \overline{4}, \overline{6}\big\}$ is $subNR$ of $G(\overline{4}) = \big\{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}\big\}$ with $A_N \subseteq B_N$. So (F, A_N) is a LOS $subNR$ of (G, B_N).

Theorem 2.5. *Let* (F, A_N), $(G, B_N) \in LOSNR(\tilde{N})$. *Then,* (F, A_N) $\tilde{\wedge}(G, B_N) \in LOSNR(\tilde{N})$, *whenever* $(F, A_N)\tilde{\wedge}(G, B_N)$ *is non-null* $SS(\tilde{N})$.

Proof. As by Definition 1.7 we know that $(F, A)_N\tilde{\wedge}(G, B_N) = (H, C_N)$ with $C_N = A_N \times B_N$. Then for any $\alpha \in A_N$, $b \in B_N$ and for $H(\alpha, b) = F(\alpha) \cap G(b)$, where $F(\alpha)$ and $G(b)$ are $subNR$ over $\tilde{N}$. As $(H, A_N \cap B_N)$ is non-empty then $F(\alpha)\cap G(b)\neq\emptyset$. Also we know that intersection of any number of $subNR$s of $\tilde{N}$ is $subNR$ over $\tilde{N}$. So $H(\alpha,b)=F(\alpha)\cap G(b)$ is $subNR$ over $\tilde{N}$. Hence $(H,A_N\widetilde{\cap}B_N)$ is SNR over $\tilde{N}$.

Now we have to show that $(H,A_N\widetilde{\cap}B_N)$ contains lattice order. Since A_N, $B_N\subseteq E_N$ so partial order is promoted into A_N and B_N from E_N. Now for $\alpha_1\preceq_{A_N}\alpha_2$ we have $F(\alpha_1)\subseteq F(\alpha_2)\ \forall\,\alpha_1,\alpha_2\in A_N$, for $b_1\preceq_{B_N}b_2$ we have $G(b_1)\subseteq G(b_2)\ \ \forall\ b_1,\ b_2\in B_N$. Now $\preceq$ is the partial order on C_N which is transferred by partial orders on A_N and B_N, therefore for any $(\alpha_1,b_1),(\alpha_2,b_2)\in C_N$ if $(\alpha_1,b_1)\preceq(\alpha_2,b_2)$ then $F(\alpha_1)\subseteq F(\alpha_2)$ and $G(b_1)\subseteq G(b_2)$ implies $F(\alpha_1)\cap G(b_1)\subseteq F(\alpha_2)\cap G(b_2)\Rightarrow H(\alpha_1,b_1)\subseteq H(\alpha_2,b_2)$.

Hence $(F,A_N)\widetilde{\wedge}(G,B_N)\in LOSNR(\tilde{N})$. □

Remark 2.6. The binary operation OR of two $LOSNR$s may or may not be a $LOSNR$.

Example 2.7. Consider the nearring $\tilde{N}=\mathbb{Z}_8=\{\overline{0},\overline{1},\overline{2},\overline{3},\overline{4},\overline{5},\overline{6},\overline{7}\}$ with $\overline{0}\preceq\overline{1}\preceq\overline{2}\preceq\overline{3}\preceq\overline{4}\preceq\overline{5}\preceq\overline{6}\preceq\overline{7}$. Let $A_N=\{\overline{2},\overline{4}\}$ and define $F:A_N\to P(\tilde{N})$ by

$H(\alpha)=\{n\in\tilde{N}:\alpha n=\overline{0}\}$, then $(F,A_N)=\{F(\overline{2})=\{\overline{0},\overline{4}\},\ F(\overline{4})=\{\overline{0},\overline{2},\overline{4},\overline{6}\}\}$.

Now take $B_N=\{\overline{1},\overline{2},\overline{4}\}$ and define $G:B_N\to P(\tilde{N})$ by

$G(b)=\{n\in\tilde{N}:bn\in\{\overline{0},\overline{4}\}\}$, then $(G,B_N)=\{\ G(\overline{1})=\{\overline{0},\overline{4}\}$, $G(\overline{2})=\{\overline{0},\overline{2},\overline{4},\overline{6}\},\ G(\overline{4})=\tilde{N}\}$. Here $(F,A_N),(G,B_N)\in LOSNR(\tilde{N})$.

Now $A_N\times B_N=\{(\overline{2},\overline{1}),(\overline{2},\overline{2}),(\overline{2},\overline{4}),(\overline{4},\overline{1}),(\overline{4},\overline{2}),(\overline{4},\overline{4})\}$, then $(L,C_N)=\{L(\overline{2},\overline{1})=\{\overline{0},\overline{4}\},\ L(\overline{2},\overline{2})=\{\overline{0},\overline{2},\overline{4},\overline{6}\},\ \ L(\overline{2},\overline{4})=\{\tilde{N}\},\ \ L(\overline{4},\overline{1})=\{\overline{0},\overline{2},\overline{4},\overline{6}\},\ L(\overline{4},\overline{2})=\{\overline{0},\overline{2},\overline{4},\overline{6}\},\ \ L(\overline{4},\overline{4})=N\notin LOSNR(\tilde{N})$.

Theorem 2.8. *Let* $(F,A_N),(G,B_N)\in LOSNR(\tilde{N})$. *Then,* $(F,A_N)\widetilde{\vee}(G,B_N)\in LOSNR(\tilde{N})$, *whenever* $(F,A_N)\widetilde{\vee}(G,B_N)$ *is non-null* $SS(\tilde{N})$ *and either* $F(\alpha)\subseteq G(b)$ *or* $G(b)\subseteq F(\alpha)$.

Proof. For any $(\alpha,b)\in A_N\times B_N$ we consider that $F(\alpha)\subseteq G(b)$. By Definition 1.8 we know that $(F,A_N)\widetilde{\vee}(G,B_N)=(L,C_N)$, $C_N=A_N\times B_N$, then for any $(\alpha,b)\in A_N\times B_N$ we have $L(\alpha,b)=F(\alpha)\cup G(b)$, where $F(\alpha)$ and $G(b)$ are $subNR$ over $\tilde{N}$.

Now as $A_N\subseteq B_N$, then $A_N\cap B_N=A_N\ \ \Rightarrow(L,A_N\times B_N)=(L,A_N)$ $\Rightarrow F(x)\ \Rightarrow(F,A_N)\widetilde{\vee}(G,B_N)=(F,A_N)$, and $(F,A_N)\in LOSNR(\tilde{N})$.

Hence $(F,A_N)\widetilde{\vee}(G,B_N)\in LOSNR(\tilde{N})$. □

THEOREM 2.9. *Let* $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$. *Then* $(F, A_N)\widetilde{\cap}_R$ $(G, B_N) \in LOSNR(\tilde{N})$, *whenever* $(F, A_N)\widetilde{\cap}_R(G, B_N)$ *is non-null* $SS(\tilde{N})$.

Proof. As by Definition 1.12 we know that $(F, A_N)\widetilde{\cap}_R(G, B_N) = (H, C_N)$, where $C_N = A_N \cap B_N$ with $A_N \cap B_N \neq \emptyset$, then for $\varsigma \in C_N$ we have $H(\varsigma) = F(\varsigma) \cap G(\varsigma)$, where $F(a)$ and $G(b)$ are $subNR$ over $\tilde{N}$. As $(H, A_N \cap B_N)$ is non-empty then $F(\alpha) \cap G(b) \neq \emptyset$, Also we know that intersection of any number of $subNR$s over $\tilde{N}$ is $subNR$ over $\tilde{N}$. So $H(\varsigma) = F(\alpha) \cap G(b)$ is $subNR$ over $\tilde{N}$. Hence $(H, A_N \cap B_N)$ is SNR over $\tilde{N}$.

Now we have to show that $(H, A_N \cap B_N)$ contains lattice ordered. Since A_N, $B_N \subseteq E_N$, so partial order is promoted into A_N and B_N from E_N. Now for $\alpha_1 \preccurlyeq_{A_N} \alpha_2$ we have $F(\alpha_1) \subseteq F(\alpha_2)$ $\forall$ α_1, $\alpha_2 \in A_N$, for $b_1 \preccurlyeq_{B_N} b_2$ we have $G(b_1) \subseteq G(b_2)$ $\forall$ b_1,$b_2 \in B_N$. And for $\varsigma_1 \preccurlyeq_{C_N} \varsigma_2$ we have $F(\varsigma_1) \subseteq F(\varsigma_2)$ and $G(\varsigma_1) \subseteq G(\varsigma_2)$ $\forall$ ς_1, $\varsigma_2 \in C_N$. Also $F(\varsigma_1) \cap G(\varsigma_1) \subseteq F(\varsigma_2) \cap G(\varsigma_2) \Rightarrow H(\varsigma_1) \subseteq H(\varsigma_2)$.

Hence $(F, A_N)\widetilde{\cap}_R(G, B_N) = (H, A_N \cap B_N) \in LOSNR(\tilde{N})$. □

REMARK 2.10. The restricted union of two $LOSNRs$ may or may not be a $LOSNR$.

EXAMPLE 2.11. Consider $\tilde{N} = \mathbb{Z}_6 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}\}$ with $\overline{0} \preccurlyeq \overline{1} \preccurlyeq \overline{2} \preccurlyeq \overline{3} \preccurlyeq \overline{4} \preccurlyeq \overline{5}$. Take $A_N = \{\overline{1}, \overline{2}\}$ define $F : A_N \longrightarrow P(\tilde{N})$ by

$F(\alpha) = \{n \in \tilde{N} : \alpha n = \overline{0}\}$, then $(F, A_N) = \{F(\overline{1}) = \{\overline{0}\},\ F(\overline{2}) = \{\overline{0}, \overline{3}\}\}$ is $LOSNR$.

Now $B_N = \{\overline{2}, \overline{3}\}$ define $G : B_N \longrightarrow P(\tilde{N})$ by

$G(b) = \{n \in \tilde{N} :\ bn \in \{\overline{0}, \overline{2}, \overline{4}\}\}$, then $(G, B_N) = \{G(\overline{2}) = \{\overline{0}, \overline{2}, \overline{4}\}$, $G(\overline{3}) = \{\overline{0}, \overline{2}, \overline{4}\}\}$ is $LOSNR$.

Now let us consider $(F, A_N)\widetilde{\cup}_R(G, B_N)$. Then $(F, A_N)\widetilde{\cup}_R(G, B_N) = (L, A_N \cap B_N)$, where $L(\varsigma) = F(\varsigma) \cup G(\varsigma)$ for $\varsigma \in A_N \cap B_N$.

Now for $2 \in A_N \cap B_N$ we have $L(2) = F(2) \cup G(2) = \left\{\overline{0}, \overline{2}, \overline{3}, \overline{4}\right\}$ is not $subNR$So $(F, A_N)\widetilde{\cup}_R(G, B_N)$ is not $LOSNR$.

THEOREM 2.12. *Let* $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$. *Then* $(F, A_N)\widetilde{\cup}_R$ $(G, B_N) \in LOSNR(\tilde{N})$, *whenever* $(F, A_N)\widetilde{\cup}_R(G, B_N)$ *is non-null* SS *over* $\tilde{N}$ *and one of them is soft subset of the other.*

Proof. Let $(F, A_N) \subseteq (G, B_N)$ with $A_N \subseteq B_N$. As by Definition 1.11 we know that $(F, A_N)\widetilde{\cup}_R(G, B_N) = (H, C_N)$, $C_N = A_N \cap B_N$ with

$A_N \cap B_N \neq \emptyset$. Then for any $\varsigma \in C_N$ we have $H(\varsigma) = F(\alpha) \cup G(b)$, where $F(\alpha)$ and $G(b)$ are subNR over $\tilde{N}$.

Now as $A_N \subseteq B_N$, then $A_N \cap B_N = A_N \Rightarrow (H, A_N \cap B_N) = (H, A_N)$ $\Rightarrow F(x) \Rightarrow (F, A_N) \cup_R (G, B_N) = (F, A_N)$, but $(F, A_N) \in LOSNR(\tilde{N})$.

Hence $(F, A_N) \widetilde{\cup}_R (G, B_N) \in LOSNR(\tilde{N})$. □

THEOREM 2.13. *Let* $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$. *Then* $(F, A_N) \widetilde{\cap}_E$ $(G, B_N) \in LOSNR(\tilde{N})$, *whenever* $(F, A_N) \widetilde{\cap}_E (G, B_N)$ *be non-null* $SS(\tilde{N})$.

THEOREM 2.14. *Let* $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$. *Then* $(F, A_N) \widetilde{\cap}_B$ $(G, B_N) \in LOSNR(\tilde{N})$, *provided* $A_N \cap B_N \neq \emptyset$.

REMARK 2.15. The union of two $LOSNR$s may or may not be a $LOSNR$.

EXAMPLE 2.16. Consider $\tilde{N} = \mathbb{Z}_6 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}\}$ with $\overline{0} \preccurlyeq \overline{1} \preccurlyeq \overline{2} \preccurlyeq \overline{3} \preccurlyeq \overline{4} \preccurlyeq \overline{5}$. Take $A_N = \{\overline{1}, \overline{2}\}$ define $F : A_N \longrightarrow P(\tilde{N})$ by

$F(\alpha) = \{n \in \tilde{N} : \alpha n = \overline{0}\}$, then $(F, A_N) = \{F(\overline{1}) = \{\overline{0}\},\ F(\overline{2}) = \{\overline{0}, \overline{3}\}\}$ is $LOSNR$.

Now $B_N = \{\overline{2}, \overline{3}\}$ define $G : B_N \longrightarrow P(\tilde{N})$ by

$G(b) = \{n \in \tilde{N} :\ bn \in \{\overline{0}, \overline{2}, \overline{4}\}\}$, then $(G, B_N) = \{G(\overline{2}) = \{\overline{0}, \overline{2}, \overline{4}\}$, $G(\overline{3}) = \{\overline{0}, \overline{2}, \overline{4}\}\}$ is $LOSNR$.

Now let us consider $(F, A_N) \widetilde{\cup}_B (G, B_N)$. Then $(F, A_N) \widetilde{\cup}_B (G, B_N) = (L, A_N \cup B_N)$ and define by

$$L(\varsigma) = \begin{cases} F(\varsigma) & \text{if } \varsigma \in A_N - B_N \\ G(\varsigma) & \text{if } \varsigma \in B_N - A_N \\ F(\varsigma) \cap G(\varsigma) & \text{if } \varsigma \in A_N \cap B_N \end{cases} \quad \forall \varsigma \in A_N \cup B_N$$

Now for $\overline{2} \in A_N \cap B_N$ we have $L(\overline{2}) = F(\overline{2}) \cup G(\overline{2}) = \left\{\overline{0}, \overline{2}, \overline{3}, \overline{4}\right\}$ is not *sub*NR. So $(F, A_N) \widetilde{\cup}_B (G, B_N)$ is not $LOSNR$.

THEOREM 2.17. *Let* $(F, A_N), (G, B_N) \in LOSNR(\tilde{N})$. *Then* $(F, A_N) \widetilde{\cup}_B$ $(G, B_N) \in LOSNR(\tilde{N})$, *whenever* $(F, A_N) \widetilde{\cup}_B (G, B_N)$ *is non-null* $SS(\tilde{N})$ *and* $A_N \cap B_N = \emptyset$ *or one of them is soft subset of other.*

Proof. As by Definition 1.9 $(F, A_N) \widetilde{\cup}_B (G, B_N) = (L, C_N)$, where $C_N = A_N \cup B_N$. Now for any $\varsigma \in C_N$

$$L(\varsigma) = \begin{cases} F(\varsigma) & \text{if } \varsigma \in A_N - B_N \\ G(\varsigma) & \text{if } \varsigma \in B_N - A_N \\ F(\varsigma) \cup G(\varsigma) & \text{if } \varsigma \in A_N \cap B_N \end{cases}, \text{ where } F(\varsigma) \text{ and } G(\varsigma)$$

are *sub*NR of $\tilde{N}$. As $A_N \cap B_N = \emptyset$ so we have

$$L(\varsigma)=\begin{cases} F(\varsigma) & \text{if } \varsigma\in A_N-B_N\\ G(v) & \text{if } \varsigma\in B_N-A_N\end{cases}$$

When $\varsigma\in A_N-B_N$, then $\Rightarrow L(\varsigma)=F(\varsigma)\Rightarrow (L,C_N)=(F,C_N)$, $(F,A_N)\in LOSNR(\tilde{N})$. So $(L,C_N)\in LOSNR(\tilde{N})$.

When $\varsigma\in B_N-A_N$, then $L(\varsigma)=G(\varsigma)\Rightarrow (L,C_N)=(G,B_N)$, $(G,B_N)\in LOSNR(\tilde{N})$. So $(L,C_N)\in LOSNR(\tilde{N})$.

Now consider $(F,A_N)\subseteq(G,B_N)$ with $A_N\subseteq B_N\Rightarrow L(\varsigma)=G(v)\Rightarrow (L,C_N)=(G,B_N)$. $(G,B_N)\in LOSNR(\tilde{N})\Rightarrow (L,C_N)\in LOSNR(\tilde{N})$, hence $(F,A_N)\widetilde{\cup}_B(G,B_N)\in LOSNR(\tilde{N})$. □

3. Lattice Ordered Idealistic Soft Near Rings

Definition 3.1. Let $(F,A_N)\in LOSNR(\tilde{N})$. A non-empty SS (M,I) over $\tilde{N}$ is known the $LOISNR$ over $\tilde{N}$ represented by $(M,I)\lhd(F,A_N)$ if the following conditions are satisfied:

(1) $I\subseteq A_N$,

(2) $\forall\ \alpha\in Supp(M,I)$, $M(\alpha)\lhd F(\alpha)$ and $\forall\ \alpha_1,\ \alpha_2\in\alpha_N$ with $\alpha_1\preceq\alpha_2$, $M(\alpha_1)\preceq M(\alpha_2)$.

From now onward collection of all lattice ordered idealistic soft near rings over $\tilde{N}$ will be represented by $LOISNR(\tilde{N})$.

Example 3.2. Let $\tilde{N}=\mathbb{Z}_6=\{\overline{0},\overline{1},\overline{2},\overline{3},\overline{4},\overline{5}\}$ with $\overline{0}\preceq\overline{1}\preceq\overline{2}\preceq\overline{3}\preceq\overline{4}\preceq\overline{5}$. Now $I=\{\overline{1},\overline{2}\}$ and define $M:I\to P(\tilde{N})$ by $M(I)=\{n\in\tilde{N}:in=\overline{0}\}$, then $(M,I)=\{M(\overline{1})=\{\overline{0}\},\ M(\overline{2})=\{\overline{0},\overline{3}\}\}$ $\Rightarrow M(\overline{1})\subseteq M(\overline{2})$. Hence $(M,I)\in LOISNR(\tilde{N})$.

Theorem 3.3. *Let* $(M_1,I_1),(M_2,I_2)\in LOISNR(\tilde{N})$. *Then* $(M_1,I_1)\widetilde{\wedge}(M_2,I_2)\in LOISNR(\tilde{N})$, *whenever* $(M_1,I_1)\widetilde{\wedge}(M_2,I_2)$ *is non-null* $SS(\tilde{N})$.

Proof. As by Definition 1.7 we know that $(M_1,I_1)\widetilde{\wedge}(M_2,I_2)=(M,I)$ where $I=I_1\times I_2$ with $I_1\cap I_2\neq\emptyset$. Then for $\alpha_1\in I_1$, $\alpha_2\in I_2$ and for $(\alpha_1,\alpha_2)\in I_1\times I_2$ we have $M(\alpha_1,\alpha_2)=M_1(\alpha_1)\cap M_2(\alpha_2)$, where $M_1(\alpha_1)$ and $M_2(\alpha_2)$ are both ideals of $\tilde{N}$.

As (M,I) is non-empty, then $M_1(\alpha_1)\cap M_2(\alpha_2)\neq\emptyset$. Since the intersection of any number of ideals is a ideal $\tilde{N}$, $M(\alpha_1,\alpha_2)=M_1(\alpha_1)\cap M_2(\alpha_2)$ is ideal over $\tilde{N}$. Hence (M,I) is ideal $SS(\tilde{N})$.

Now we have to show that (M,I) contains lattice order. Since I_1, $I_2\subseteq E_N$, so partial order is promoted into I_1 and I_2 from E_N.

Now for $\alpha_1 \preceq_{I_1} \alpha_2$, we have $M_1(\alpha_1) \subseteq M_1(\alpha_2)$, $\forall\ b_1,\ b_2 \in I_2$, for $b_1 \preceq_{I_2} b_2$ we have $M_2(b_1) \subseteq M(b_2)$, $\forall\ b_1,\ b_2 \in I_2$. Now $\preceq$ is the partial order on I which is promoted by partial order on I_1 and I_2, therefore for any $(\alpha_1, b_1), (\alpha_2, b_2) \in I_1 \times I_2$ we have $M_1(\alpha_1) \subseteq M_1(\alpha_2)$ and $M_2(b_1) \subseteq M(b_2)$ this implies $M_1(\alpha_1) \cap M_2(b_1) \subseteq M_1(\alpha_2) \cap M_2(b_2) \Rightarrow M(\alpha_1, b_1) \subseteq M(\alpha_2, b_2)$.

Hence $(M_1, I_1)\widetilde{\wedge}(M_2, I_2) \in LOISNR(\tilde{N})$. □

Theorem 3.4. *Let* $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$. *Then* $(M_1, I_1)\widetilde{\vee}$ $(M_2, I_2) \in LOISNR(\tilde{N})$, *whenever* $(M_1, I_1)\widetilde{\vee}(M_2, I_2)$ *is non-null* $SS(\tilde{N})$ *and either* $M_1(\alpha_1) \subseteq M_2(\alpha_2)$ *or* $M_2(\alpha_2) \subseteq M_1(\alpha_1)$.

Proof. For any $(\alpha_1, \alpha_2) \in I_1 \times I_2$ we consider that $M_1(\alpha_1) \subseteq M_2(\alpha_2)$. By Definition 1.8 $(M_1, I_1)\widetilde{\vee}(M_2, I_2) = (M, I)$, where $I = I_1 \times I_2$ and for any $(\alpha_1, \alpha_2) \in I_1 \times I_2$ we have $M(\alpha_1, \alpha_2) = M_1(\alpha_1) \cup M_2(\alpha_2)$. As $M_1(\alpha_1) \subseteq M_2(\alpha_2)$, $M_1(\alpha_1) \cup M_2(\alpha_2) = M_2(\alpha_1) \Rightarrow (M_2, I_2) = (M, I)$, $(M_2, I_2) \in LOISNR(\tilde{N})$. So $(M, I) \in LOISNR(\tilde{N})$.

It follows that $(M_1, I_1)\widetilde{\vee}(M_2, I_2) = (M, I) \in LOISNR(\tilde{N})$. □

Theorem 3.5. *Let* $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$. *Then* $(M_1, I_1)\widetilde{\cap}_R$ $(M_2, I_2) \in LOISNR(\tilde{N})$, *whenever* $(M_1, I_1)\widetilde{\cap}_R(M_2, I_2)$ *is a non-null* SS *over* $\tilde{N}$.

Proof. By Definition 1.12 $(M_1, I_1)\widetilde{\cap}_R(M_2, I_2) = (M, I)$, as $M(\alpha) = M_1(\alpha) \cap M_2(\alpha)$, where $I = I_1 \cap I_2$, $\forall\ \alpha \in I$. Since $I_1 \subseteq A_N$ and $I_2 \subseteq A_N$, it is obvious that $I \subseteq A_N$. Then for $\alpha \in I$ we have $M(\alpha) = M_1(\alpha) \cap M_2(\alpha)$. We deduce that the non-empty sets $M_1(\alpha)$ and $M_2(\alpha)$ are both ideals of $F(\alpha)$. Since the intersection of any number of ideals of $\tilde{N}$ is an ideal of $\tilde{N}$, $M(\alpha) = M_1(\alpha) \cap M_2(\alpha)$ is an ideal of $\tilde{N}$, therefore $(M_1, I_1)\widetilde{\cap}_R(M_2, I_2)$ is an ideal SS over $\tilde{N}$.

Now we have to show that (M, I) contains lattice ordered. Since I_1, $I_2 \subseteq E_N$ so partial order is promoted into I_1 and I_2 from E_N. Now for $\alpha_1 \preceq_{I_1} \alpha_2$ we have $M_1(\alpha_1) \subseteq M_1(\alpha_2)$ $\forall\ b_1,\ b_2 \in I_1$, for $b_1 \preceq_{I_2} b_2$ we have $M_2(b_1) \subseteq M_2(b_2)$ $\forall\ b_1,\ b_2 \in I_2$ and for $\varsigma_1 \preceq_I \varsigma_2$ we have $M_1(\varsigma_1) \subseteq M_1(\varsigma_2)$ and $M_2(\varsigma_1) \subseteq M_2(\varsigma_2)$ $\forall\ \varsigma_1,\ \varsigma_2 \in I$ also $M_1(\varsigma_1) \cap M_2(\varsigma_1) \subseteq M_1(\varsigma_2) \cap M_2(\varsigma_2) \Rightarrow M(\varsigma_1) \subseteq M(\varsigma_2)$.

Hence $(M_1, I_1)\widetilde{\cap}_R(M_2, I_2) = (M, I) \in LOISNR(\tilde{N})$. □

Theorem 3.6. *Let* $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$. *Then* $(M_1, I_1)\widetilde{\cup}_R$ $(M_2, I_2) \in LOISNR(\tilde{N})$, *whenever* $(M_1, I_1)\widetilde{\cup}_R(M_2, I_2)$ *is non-null* $SS(\tilde{N})$ *and one of them is soft subset of other.*

Proof. By Definition 1.11 we know that $(M_1, I_1)\widetilde{\cup}_R(M_2, I_2) = (M, I)$, where $I = I_1 \cap I_2$ with $I_1 \cap I_2 = \emptyset$, for $\alpha \in I$. $M(\alpha) = M_1(\alpha) \cup M_2(\alpha)$, where $M_1(\alpha)$ and $M_2(\alpha)$ are both ideals of $\tilde{N}$.

Since $I_1 \subseteq I_2$, then $I_1 = I_1 \cap I_2 \Rightarrow (M_1, I_1) = (M, I) \Rightarrow (M_1, I_1) \cup_R (M_2, I_2) = (M_1, I_1)$, $(M_1, I_1) \in LOISNR(\tilde{N})$. So $(M, I) \in LOISNR(\tilde{N})$. Hence, $(M_1, I_1) \cup_R (M_2, I_2) \in LOISNR(\tilde{N})$. □

THEOREM 3.7. *Let $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$. Then $(M_1, I_1) \cap_B (M_2, I_2) \in LOISNR(\tilde{N})$, provided $I_1 \cap I_2 \neq \emptyset$.*

THEOREM 3.8. *Let $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$. Then $(M_1, I_1)\widetilde{\cup}_B (M_2, I_2) \in LOISNR(\tilde{N})$, whenever $(M_1, I_1)\widetilde{\cup}_B(M_2, I_2)$ is non-null $SS(\tilde{N})$ and $I_1 \cap I_2 = \emptyset$ or if $(M_1, I_1) \subseteq (M_2, I_2)$ or $(M_2, I_2) \subseteq (M_1, I_1)$.*

Proof. By Definition 1.9 we know that $(M_1, I_1)\widetilde{\cup}_B(M_2, I_2) = (M, I)$, where $I = I_1 \cup I_2$, $\forall\ \alpha \in I$

$$M(\alpha) = \begin{cases} M_1(\alpha) & \text{if } \alpha \in I_1 - I_2 \\ M_2(\alpha) & \text{if } \alpha \in I_2 - I_1 \\ M_1(\alpha) \cup M_2(\alpha) & \text{if } \alpha \in I_1 \cap I_2 \end{cases},$$ where $M_1(\alpha)$ and $M_2(\alpha)$ are ideals of $\tilde{N}$. As $I_1 \cap I_2 = \emptyset$ so we have

$$M(\alpha) = \begin{cases} M_1(\alpha) & \text{if } \alpha \in I_1 - I_2 \\ M_2(\alpha) & \text{if } \alpha \in I_2 - I_1 \end{cases}$$

When $\alpha \in I_1 - I_2 \Rightarrow M(\alpha) = M_1(\alpha) \Rightarrow (M_1, I_1) = (M, I)$, $(M_1, I_1) \in LOISNR(\tilde{N})$. So $(M, I) \in LOISNR(\tilde{N})$.

When $\alpha \in I_2 - I_1 \Rightarrow M(\alpha) = M_2(\alpha) \Rightarrow (M_2, I_2) = (M, I)$, $(M_2, I_2) \in LOISNR(\tilde{N})$. So $(M, I) \in LOISNR(\tilde{N})$.

Now consider $(M_1, I_1) \cup_E (M_2, I_2)$ with $I_1 \subseteq I_2 \Rightarrow M(\alpha) = M_1(\alpha) \Rightarrow (M_1, I_1) = (M, I)$, $(M_1, I_1) \in LOISNR(\tilde{N}) \Rightarrow (M, I) \in LOISNR(\tilde{N})$.

Hence $(M_1, I_1)\widetilde{\cup}_B(M_2, I_2) \in LOISNR(\tilde{N})$. □

THEOREM 3.9. *Let $(M_1, I_1), (M_2, I_2) \in LOISNR(\tilde{N})$. Then $(M_1, I_1)\widetilde{\cap}_E (M_2, I_2) \in LOISNR(\tilde{N})$, whenever $(M_1, I_1)\widetilde{\cap}_E(M_2, I_2)$ is non-null $SS(\tilde{N})$ and $I_1 \cap I_2 = \emptyset$.*

Proof. By Definition 1.13 we know that $(M_1, I_1)\widetilde{\cap}_E(M_2, I_2) = (M, I)$, where $I = I_1 \cup I_2$, $\forall\ \alpha \in I$

$$M(\alpha) = \begin{cases} M_1(\alpha) & \text{if } \alpha \in I_1 - I_2 \\ M_2(\alpha) & \text{if } \alpha \in I_2 - I_1 \\ M_1(\alpha) \cap M_2(\alpha) & \text{if } \alpha \in I_1 \cap I_2 \end{cases},$$ where $M_1(\alpha)$ and $M_2(\alpha)$ are ideals of $\tilde{N}$. As $I_1 \cap I_2 = \emptyset$ so we have

$$M(\alpha)=\begin{cases} M_1(\alpha) & \text{if } \alpha\in I_1-I_2\\ M_2(\alpha) & \text{if } \alpha\in I_2-I_1\end{cases}$$

When $\alpha \in I_1-I_2 \Rightarrow M(\alpha)=M_1(\alpha) \Rightarrow (M_1,I_1)=(M,I)$, $(M_1,I_1)\in LOISNR(\tilde{N})$. So $(M,I)\in LOISNR(\tilde{N})$.

Since $\alpha \in I_2-I_1 \Rightarrow M(\alpha)=M_2(\alpha) \Rightarrow (M_2,I_2)=(M,I)$, $(M_2,I_2)\in LOISNR(\tilde{N})$. So is $(M,I)\in LOISNR(\tilde{N})$.

It follows that $(M_1,I_1)\widetilde{\cap}_E(M_2,I_2)$ is $LOISNR$ over (F,A_N). □

4. Lattice Ordered Soft Near Ring Homomorphism

DEFINITION 4.1. Let (F,A_N) and (G,B_N) be two $LOSNR$ over $\tilde{N}_1$ and $\tilde{N}_2$ respectively and $f:\tilde{N}_1\to\tilde{N}_2$ and $h:A_N\to B_N$ be two functions. We say that (f,h) is lattice ordered soft near ring homomorphism ($LOSNRH$) from (F,A_N) to (G,B_N) if it satisfies the following conditions

(1) $f:\tilde{N}_1\to\tilde{N}_2$ is a near ring homomorphism (NRH).
(2) $h:A_N\to B_N$ is an onto mapping.
(3) $f(F(\alpha))=G(h(\alpha))\ \forall\ \alpha\in A_N$.
(4) $\forall\ \alpha_1,\ \alpha_2\in A_N$ with $\alpha_1\preceq\alpha_2$ we have $f(F(\alpha_1)\preceq f(F(\alpha_2))$.

EXAMPLE 4.2. Let $\tilde{N}_1=3Z_6, \tilde{N}_2=Z_6=\{\overline{0},\overline{1},\overline{2},\overline{3},\overline{4},\overline{5}\}, A_N=\{\alpha_1,\alpha_2,\alpha_3\}$ with $\alpha_1\preceq\alpha_2\preceq\alpha_3$ and $B_N=\{\beta_1,\beta_2,\beta_3\}$ with $\beta_1\preceq\beta_2\preceq\beta_3$.

Now define $f:\tilde{N}_1\to\tilde{N}_2$ by $f(n)=3n$,
$h:A_N\to B_N$ by $h(\alpha_i)=\beta_i$ for $1\le i\le 3$,
$F:A\to P(\tilde{N}_1)$ by $F(\alpha_1)=\{\overline{0}\}, F(\alpha_2)=F(\alpha_3)=\tilde{N}_1$ and
$G:A\to P(\tilde{N}_2)$ by $G(\beta_1)=\{\overline{0}\}, G(\beta_2)=G(\beta_3)=3\tilde{N}_2$.
Then (f,h) is $LOSNRH$ from (F,A_N) to (G,B_N).

THEOREM 4.3. *Let (F,A_N), $(G,B_N)\in LOSNR(\tilde{N}_1)$. If $f:\tilde{N}_1\to\tilde{N}_2$ be a NRH, then $(f(F),A_N)$, $(f(G),B_N)\in LOSNR(\tilde{N}_2)$ and if (G,B_N) is LOS subNR of (F,A_N), then $(f(G),B_N)$ is LOS subNR of $(f(F),A_N)$.*

Proof. Given that (F,A_N), $(G,B_N)\in LOSNR(\tilde{N}_1)$. Then we have $\forall\ \alpha_1,\ \alpha_2\in Supp(F,A_N)$, $\alpha_1\preceq\alpha_2$ we have $F(\alpha_1)\subseteq F(\alpha_2)$. Similarly $\forall\ b_1,b_2\in Supp(G,B_N)$, $b_1\preceq b_2$ we have $G(b_1)\subseteq G(b_2)$, where we have $G(b_1)$, $G(b_2)$, $F(\alpha_1)$ and $F(\alpha_2)$ are all $subNR$ over $\tilde{N}_1$.

Now $f(F) : A_N \to P(\tilde{N}_2)$ is defined by $f(F)(\alpha) = g(F(\alpha))\ \forall\ \alpha \in A_N$ and $f(G) : B_N \to P(\tilde{N}_2)$ is defined by $f(G)(b) = f(G(b))\ \forall\ b \in B_N$.

Now consider $m_1,\ m_2 \in F(\alpha_1)$, as we have $F(\alpha_1)$ is subNR of $\tilde{N}_1$, then we have $m_1 - m_2 \in F(\alpha_1)$, $m_1m_2 \in F(\alpha_1)$. Let $F(m_1),\ F(m_2) \in g((\alpha_1))$.

Now consider $m_1 - m_2 \in F(\alpha_1) \Rightarrow f(m_1 - m_2) \in f(F(\alpha_1) \Rightarrow f(m_1) - f(m_2) \in f(F(\alpha_1))$ and $m_1m_2 \in F(\alpha_1) \Rightarrow f(m_1m_2) \in f(F(\alpha_1) \Rightarrow f(m_1)f(m_2) \in f(F(\alpha_1))$.

As $f(m_1) - f(m_2) \in f(F(\alpha_1))$ and $f(m_1)f(m_2) \in f(F(\alpha_1))$. So $f(F(\alpha_1))$ is *subNR* of $\tilde{N}_2$, in similar way we have $f(F(\alpha_1))$ is *subNR* of $\tilde{N}_2$.

Hence $\forall\ \alpha \in A_N,\ f(F(\alpha))$ is *subNR* of $\tilde{N}_2$. So we have $(f(F), A_N)$ is SNR over $\tilde{N}_2$. Similarly we have $(f(G), B_N)$ is SNR over $\tilde{N}_2$, as we have (G, B_N) is *LOS subNR* of (F, A_N), so $\forall\ b \in B_N,\ G(b)$ is *subNR* of $F(b)$, therefore $f(G)(b))$ is *subNR* of $f(F(b))$ and It follows that $(f(G), B_N)$ is soft *subNR* of $(f(F), A_N)$.

As it is given that (F, A_N), $(G, B_N) \in LOSNR(\tilde{N}_1)$, then by Definition we have $\forall\ \alpha_1, \alpha_2 \in A_N,\ \alpha_1 \preceq \alpha_2$ we have $F(\alpha_1) \subseteq F(\alpha_2)$. Similarly $\forall\ b_1,\ b_2 \in B_N$, $b_1 \preceq b_2$ we have $G(b_1) \subseteq G(b_2)$.

Now let $m_1 \in F(\alpha_1) \Rightarrow g(m_1) \in g(F(\alpha_1))$. Now as $F(\alpha_1) \subseteq F(\alpha_2)$ and $m_1 \in F(\alpha_1) \Rightarrow m_1 \in F(\alpha_2) \Rightarrow f(m_1) \in f(F(\alpha_2))$. As for $f(m_1) \in f(F(\alpha_1))$ we have $f(m_1) \in f(F(\alpha_2) \Rightarrow f(F(\alpha_1) \subseteq f(F(\alpha_2))$.

This implies that $(f(F, A_N)$ is $LOSNR(\tilde{N}_2)$ and similarly we have $(f(G), B_N) \in LOSNR(\tilde{N}_2)$ such that $(f(F, A_N)$ is LOS subNR of $(f(G), B_N)$. □

THEOREM 4.4. *Let (F, A_N) and (G, B_N) is $LOSNR$ over $\tilde{N}_1$ and $\tilde{N}_2$ respectively, where $\tilde{N}_1$ and $\tilde{N}_2$ are two NRs and (f, h) is a $LOSNRH$ from (F, A_N) to (G, B_N). If (F, A_N) is $LOISNR$ over $\tilde{N}_1$, then (G, B_N) is $LOISNR$ over $\tilde{N}_2$.*

Proof. Given that $(F, A_N) \in LOISNR(\tilde{N}_1)$, then by we have $\forall$, α_1, $\alpha_2 \in A_N$, $\alpha_1 \preceq_{A_N} \alpha_2$ we have $F(\alpha_1) \subseteq F(\alpha_2)$, where we have $F(\alpha_1)$ and $F(\alpha_2)$ are ideal of $\tilde{N}_1$. Also as it is given that $(G, B_N) \in LOSNR(\tilde{N}_2)$, then $\forall\ b_1,\ b_2 \in B_N,\ b_1 \preceq_{B_N} b_2$ we have $G(b_1) \subseteq G(b_2)$, where we have $G(b_1)$ and $G(b_2)$ are ideal of $\tilde{N}_2$. Also as (f, h) is a $LOSNRH$ from (F, A_N) to (G, B_N), then for any $b_1 \in B_N$ there exists an element $\alpha_1 \in A_N$ for which we have $b_1 = h(\alpha_1) \Rightarrow G(b_1) = G(h(\alpha_1) = f(F(\alpha_1))$. As $F(\alpha_1)$ is ideal of $\tilde{N}_1$, so $f(F(\alpha_1))$ is a ideal NR of $G(h(b_1))$. Similarly for any $b_2 \in B_N$, there exists an element $\alpha_2 \in A_N$ for which we have $b_2 = h(\alpha_2)$ and $f(F(\alpha_2))$ is an ideal of $G(h(b_2))$. Thus, $(G, B_N) \in LOISNR(\tilde{N}_2)$. □

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