

SIX SOLUTIONS FOR THE SEMILINEAR WAVE EQUATION WITH NONLINEARITY CROSSING THREE EIGENVALUES

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ABSTRACT. We get a theorem which shows the existence of at least six solutions for the semilinear wave equation with nonlinearity crossing three eigenvalues. We obtain this result by the variational reduction method and the geometric mapping defined on the finite dimensional subspace. We use a contraction mapping principle to reduce the problem on the infinite dimensional space to that on the finite dimensional subspace. We construct a three-dimensional subspace with three axes spanned by three eigenvalues and a mapping from the finite dimensional subspace to the one-dimensional subspace.

1. Introduction and main result

In this paper we concern with the number of periodic solutions of a semilinear wave equation with Dirichlet boundary condition

$$(1.1) \quad \begin{aligned} u_{tt} - u_{xx} + bu^+ - au^- &= f(x, t) && \text{in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times R, \\ u\left(\pm\frac{\pi}{2}, t\right) &= 0, \\ u \text{ is } \pi\text{-periodic in } t &&& \text{and even in } x \text{ and } t, \end{aligned}$$

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where $u^+ = \max\{0, u\}$.

This type of jumping nonlinear problem is considered by many authors ([1], [2], [3], [4], [5], [6], [7], [8]). McKenna and Walter [8] considered the elliptic case and proved by the Leray-Schauder degree theory that if the operator is elliptic operator in (1.1) and $a < \lambda_1 < \lambda_2 < b$, there exist three weak solutions, where λ_1 and λ_2 are the first eigenvalue and the second one of the elliptic eigenvalue problem. Choi and Jung [1] proved that if $-1 < a < 3 < b < 7$ with $\frac{1}{\sqrt{a+1}} + \frac{1}{\sqrt{b+7}} > 1$ and $f(x, t) = s\phi_{00}$ ($\phi_{00} = \frac{\sqrt{2}}{\pi} \cos x$ is the positive eigenfunction and $s \in R$), then for the case $s > 0$, (1.1) has at least three solutions, one of which is a positive solution, and for the case $s < 0$, (1.1) has at least one solution, which is a negative solution. They proved this result by the variational reduction method. They [4] also proved that if $-5 < a < -1$, $3 < b < 7$ and $f(x, t) = s\phi_{00}$, then there exists $s_0 > 0$ such that if $s \geq s_0$, (1.1) has at least four solutions, one of which is a positive solution, another of which is a negative solution. They got this result by the mapping from the two-dimensional subspace spanned by ϕ_{00} and ϕ_{10} to the space spanned by ϕ_{00} and for the case $s < 0$, (1.1) has at least one solution, which is a negative solution. In this paper we improve this result. To state our result, we need some notations.

The eigenvalue problem

$$(1.2) \quad \begin{aligned} u_{tt} - u_{xx} &= \lambda u && \text{in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times R, \\ u\left(\pm\frac{\pi}{2}, t\right) &= 0, \\ u(x, t) = u(-x, t) &= u(x, -t) = u(x, t + \pi) \end{aligned}$$

has infinitely many eigenvalues

$$\lambda_{mn} = (2n + 1)^2 - 4m^2, \quad m, n = 0, 1, 2, \dots$$

The eigenvalues in the interval $(-15, 9)$ are given by

$$\lambda_{32} = -11 < \lambda_{21} = -7 < \lambda_{10} = -3 < \lambda_{00} = 1 < \lambda_{11} = 5.$$

The corresponding normalized eigenfunctions $\phi_{mn}(x, t)$ ($m, n > 0$) given by

$$\begin{aligned} \phi_{0n} &= \frac{\sqrt{2}}{\pi} \cos(2n + 1)x && \text{for } n \geq 0, \\ \phi_{mn} &= \frac{2}{\pi} \cos 2mt \cos(2n + 1)x && \text{for } m > 0, n \geq 0. \end{aligned}$$

We will look for the π -periodic solutions of (1.1) when $f(x, t) = s\phi_{00}$. Our main result is the following:

THEOREM 1.1. *Let $f(x, t) = s\phi_{00}$ and $-5 < a < -1 = -\lambda_{00}$, $-\lambda_{10} = 3$, $-\lambda_{21} = 7 < b < 11$ and $s > 0$. Then (1.1) has at least six π -periodic solutions.*

The outline of the proof of Theorem 1.1 is as follows: In section 2, we first construct a three-dimensional subspace spanned by three eigenfunctions ϕ_{00} , ϕ_{10} , ϕ_{21} and use the contraction mapping principle to reduce the problem on the infinite dimensional space to that on a three-dimensional subspace. We next construct a mapping from the three-dimensional subspace to the one-dimensional subspace spanned by the eigenfunction ϕ_{00} and try to find the preimages of the mapping. In section 3, we prove Theorem 1.1.

2. Reduction to the three dimensional subspace

Let Q be the square $[-\frac{\pi}{2}, \frac{\pi}{2}] \times [-\frac{\pi}{2}, \frac{\pi}{2}]$ and H be the Hilbert space defined by

$$H = \{u \in L^2(Q) \mid u \text{ is even in } x \text{ and } t\}.$$

Then the set of functions $\{\phi_{mn} \mid m, n = 0, 1, 2, \dots\}$ is an orthonormal basis in H . Problem (1.1) is equivalent to the problem

$$(2.1) \quad u_{tt} - u_{xx} + bu^+ - au^- = s\phi_1 \quad \text{in } H.$$

Let V be the three-dimensional subspace of H spanned by $\{\phi_{00}, \phi_{10}, \phi_{21}\}$ and W be the orthogonal complement of V in H . Let P be an orthogonal projection from H onto V . Then any element $u \in H$ can be expressed by $u = v + w$, where $v = Pu$, $w = (I - P)u$. Hence (2.1) is equivalent to a system

$$(2.2) \quad v_{tt} - v_{xx} + P(b(v+w)^+ - a(v+w)^-) = s\phi_{00},$$

$$(2.3) \quad w_{tt} - w_{xx} + (I - P)(b(v+w)^+ - a(v+w)^-) = 0.$$

LEMMA 2.1. *Assume that $f(x, t) = s\phi_{00}$ and $-5 < a < -1 = -\lambda_{00}$, $-\lambda_{10} = 3$, $-\lambda_{21} = 7 < b < 11$. Then for fixed $v \in V$, (2.3) has a unique solution $w = \theta(v)$. Furthermore $\theta(v)$ is Lipschitz continuous in terms of v .*

Proof. We shall use the contraction mapping principle. Let $\delta = \frac{a+b}{2}$. Then (2.3) can be rewritten as

$$(-D_{tt} + D_{xx} - \delta)w = (I - P)(b(v + w)^+ - a(v + w)^- - \delta(v + w))$$

or

$$(2.4) \quad w = (-D_{tt} + D_{xx} - \delta)^{-1}(I - P)(b(v + w)^+ - a(v + w)^- - \delta(v + w)).$$

The operator $(-D_{tt} + D_{xx} - \delta)^{-1}(I - P)$ is a self adjoint compact map from $(I - P)H$ into itself. The operator L^2 norm $\|(-D_{tt} + D_{xx} - \delta)^{-1}(I - P)\|$ is $\max\{\frac{1}{|-5-\delta|}, \frac{1}{|11-\delta|}\}$. We note that

$$\begin{aligned} & |(b(v + w_1)^+ - a(v + w_1)^- - \delta(v + w_1)) - (b(v + w_2)^+ - a(v + w_2)^- - \delta(v + w_2))| \\ &= |((b - \delta)(v + w_1)^+ - (a - \delta)(v + w_1)^-) - ((b - \delta)(v + w_2)^+ - (a - \delta)(v + w_2)^-)| \\ &= |((b - \delta)(v + w_1)^+ - (b - \delta)(v + w_1)^-) - ((b - \delta)(v + w_2)^+ - (b - \delta)(v + w_2)^-)| \\ &\leq |b - \delta| \|w_1 - w_2\|. \end{aligned}$$

Thus we have

$$\begin{aligned} & \|(b(v + w_1)^+ - a(v + w_1)^- - \delta(v + w_1)) - (b(v + w_2)^+ - a(v + w_2)^- - \delta(v + w_2))\|_{L^2(Q)} \\ &\leq |b - \delta| \|w_1 - w_2\|_{L^2(Q)}. \end{aligned}$$

Since $|b - \delta| \leq \min\{|-5 - \delta|, |11 - \delta|\}$, the right hand side of (2.4) defines a Lipschitz mapping from W into itself with Lipschitz constant $r < 1$. By the contraction mapping principle, for fixed $v \in V$, there is a unique $w \in W$ which solves (2.4). If $\theta(v)$ denotes the unique $w \in (I - P)L^2(\Omega)$ which solves (2.4), we claim that θ is Lipschitz continuous in terms of v . In fact, if $w_1 = \theta(v_1)$ and $w_2 = \theta(v_2)$, then

$$\begin{aligned} & \|w_1 - w_2\|_{L^2(\Omega)} \\ &= \|(-D_{tt} + D_{xx} - \delta)^{-1}(I - P)((b(v_1 + w_1)^+ - a(v_1 + w_1)^- - \delta(v_1 + w_1)) \\ &\quad - (b(v_2 + w_2)^+ - a(v_2 + w_2)^- - \delta(v_2 + w_2)))\|_{L^2(\Omega)} \\ &\leq r \|(v_1 + w_1) - (v_2 + w_2)\|_{L^2(\Omega)} \\ &\leq r(\|v_1 - v_2\|_{L^2(\Omega)} + \|w_1 - w_2\|_{L^2(\Omega)}). \end{aligned}$$

Hence

$$\|w_1 - w_2\|_{L^2(Q)} \leq C \|v_1 - v_2\|_{L^2(Q)} \quad C = \frac{r}{1 - r}.$$

Thus θ is Lipschitz continuous in terms of v . □

By Lemma 2.1, the study of the multiplicity of solutions of (2.1) is reduced to that of the multiplicity of the solutions of the problem

$$(2.5) \quad v_{tt} - v_{xx} + P(b(v + \theta(v))^+ - a(v + \theta(v))^-) = s\phi_{00}$$

defined on a three-dimensional subspace V spanned by $\{\phi_{00}, \phi_{10}, \phi_{21}\}$.

We note that if $v \geq 0$ or $v \leq 0$, then $\theta(v) = 0$. In fact, if $v \geq 0$ and $\theta(v) = 0$, then (2.3) is reduced to

$$(D_{tt} - D_{xx})v + (I - P)(bv^+ - av^-) = 0,$$

which is possible since $v^+ = v$, $v^- = 0$ and $(I - P)(bv^+ - av^-) = 0$.

Let us construct six subspaces of V as follows: Since the subspace V is spanned by $\{\phi_{00}, \phi_{10}, \phi_{21}\}$ and $\phi_{00}(x) > 0$ in Q , there exist a cone C_1 , a small number $\epsilon_1 > 0$, $\epsilon_2 > 0$ defined by

$$C_1 = \{v = c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21} \mid c_1 \geq 0, |c_2| \leq \epsilon_1 c_1, |c_3| \leq \epsilon_2 |(c_1, c_2)|\}$$

so that $v \geq 0$ for all $v \in C_1$. Here (c_1, c_2) with $|c_2| \leq \epsilon_1 |c_1|$ is a plane spanned by ϕ_{00} and ϕ_{10} satisfying $|c_2| \leq \epsilon_1 |c_1|$. Let us define

$$C_2 = \{v = c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21} \mid |c_2| \geq \epsilon_1 |c_1|, c_2 < 0, |c_3| \leq \epsilon_2 |(c_1, c_2)|\},$$

$$C_3 = \{v = c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21} \mid c_1 \leq 0, |c_2| \leq \epsilon_1 |c_1|, |c_3| \leq \epsilon_2 |(c_1, c_2)|\}$$

such that $v \leq 0$ for all $v \in C_3$. Let

$$C_4 = \{v = c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21} \mid |c_2| \geq \epsilon_1 |c_1|, c_2 > 0, |c_3| \leq \epsilon_2 |(c_1, c_2)|\},$$

$$C_5 = \{v = c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21} \mid |c_2| \geq \epsilon_1 |c_1|, |c_3| \geq \epsilon_2 |(c_1, c_2)|, c_3 > 0\},$$

$$C_6 = \{v = c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21} \mid |c_2| \geq \epsilon_1 |c_1|, |c_3| \geq \epsilon_2 |(c_1, c_2)|, c_3 < 0\}.$$

We do not know $\theta(v)$ for all $v \in PH$, but we know that $\theta(v) = 0$ for $v \in C_1 \cup C_3$. We consider the map

$$F : v \longrightarrow F(v) = v_{tt} - v_{xx} + P((b(v + \theta(v)))^+ - a(v + \theta(v))^-).$$

If $v \in C_1$, then $v \geq 0$ and

$$F(v) = (b + 1)c_1\phi_{00} + (b - 3)c_2\phi_{10} + (b - 7)c_3\phi_{21}.$$

The image of $c_1\phi_{00} + c_2\phi_{10} \pm c_3\phi_{21}$, $|c_2| \leq \epsilon_1 |c_1|$, $c_1 > 0$, $|c_3| \leq \epsilon_2 |(c_1, c_2)|$ can be explicitly calculated and they are

$$(b + 1)c_1\phi_{00} + (b - 3)c_2\phi_{10} \pm (b - 7)c_3\phi_{21},$$

$$|c_2| \leq \epsilon_1 |c_1|, c_1 > 0, |c_3| \leq \epsilon_2 |(c_1, c_2)|$$

or

$$d_1\phi_{00} + d_2\phi_{10} \pm d_3\phi_{21}, d_1 > 0, |d_2| \leq \frac{b - 3}{b + 1}\epsilon_1 d_1,$$

$$|d_3| \leq (b-7)\epsilon_2 \left| \left(\frac{d_1}{b+1}, \epsilon_1 \frac{d_1}{b+1} \right) \right|$$

. Thus F maps C_1 into the cone

$$D_1 = \{d_1\phi_{00} + d_2\phi_{10} + d_3\phi_{21} \mid d_1 > 0, \quad |d_2| \leq \frac{b-3}{b+1}\epsilon_1 d_1,$$

$$|d_3| \leq (b-7)\epsilon_2 \left| \left(\frac{d_1}{b+1}, \epsilon_1 \frac{d_1}{b+1} \right) \right|\}.$$

Similarly F maps C_3 into the cone

$$D_3 = \{d_1\phi_{00} + d_2\phi_{10} + d_3\phi_{21} \mid d_1 < 0, \quad |d_2| \leq \left| \frac{a-3}{a+1}\epsilon_1 d_1 \right|,$$

$$|d_3| \leq \left| (a-7)\epsilon_2 \left(\frac{d_1}{a+1}, \epsilon_1 \frac{d_1}{a+1} \right) \right|\}.$$

3. Proof of Theorem 1.1

$F(v) = s\phi_{00}$, $s > 0$, has one solution $\frac{s\phi_{00}}{b+1}$ in C_1 and has one solution $\frac{s\phi_1}{a+1}$ in C_3 . We shall find the other solutions in the complements of $C_1 \cup C_3$ of the map $F(v) = s\phi_{00}$ for $s > 0$. We need a lemma.

LEMMA 3.1. *There exist $p_1, p_2 > 0$ such that*

$$(i) \quad (F(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{20}), \phi_{00}) \geq p_1|c_2|.$$

$$(ii) \quad (F(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{20}), \phi_{00}) \geq p_2|c_3|.$$

Proof. (i)

$$\begin{aligned} & F(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{20}) \\ &= (D_{tt} - D_{xx})(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21}) \\ & \quad + P((b(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21}) + \theta(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21}))^+ \\ & \quad - a(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21} + \theta(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21}))^-). \end{aligned}$$

If $u = c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21} + \theta(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21})$, then

$$\begin{aligned} & (F(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21}), \phi_{00}) \\ &= ((D_{tt} - D_{xx} - 1)(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21}) + P(bu^+ - au^- + u), \phi_{00}). \end{aligned}$$

Since $(D_{tt} - D_{xx} - 1)\phi_{00} = 0$ and $D_{tt} - D_{xx}$ is self adjoint, $((D_{tt} - D_{xx} - 1)(c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21}), \phi_{00}) = 0$. We note that

$$bu^+ - au^- + u = (b+1)u^+ - (a+1)u^- \geq \gamma|u|,$$

where $\gamma = \min\{b+1, -a-1\} > 0$. Thus

$$(bu^+ - au^- + u, \phi_{00}) \geq \gamma \int_Q |u| \phi_{00}.$$

Thus there exists $p_1 > 0$ such that $\gamma\phi_{00} > p_1|\phi_{10}|$, so that

$$\gamma \int_Q |u| \phi_{00} \geq p_1 \int_Q |u| |\phi_{10}| \geq p_1 \int_Q u \phi_{10} = p_1 |(u, \phi_{10})| = p_1 |c_2|.$$

(ii) We also have that

$$\gamma \int_Q |u| \phi_{00} \geq p_2 \int_Q |u| |\phi_{21}| \geq p_2 \int_Q u \phi_{21} = p_2 |(u, \phi_{21})| = p_2 |c_3|,$$

for some $p_2 > 0$ such that $\gamma\phi_{00} \geq p_2|\phi_{21}|$. \square

Now we are looking for the preimages of the mapping $F(v) = s\phi_{00}$, for $s > 0$, in the complement of $C_1 \cup C_3$. Let us consider the image under F of $c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21} \in C_4$, $c_2 \geq \epsilon_1|c_1|$, $c_2 = k$, $k > 0$, $|c_3| \leq \epsilon_2|(c_1, c_2)|$. By (i) of Lemma 3.1, the image of

$$c_2 = k, \quad |c_1| \leq \frac{1}{\epsilon_1}k, \quad |c_3| \leq \epsilon_2|(c_1, k)|$$

must lie to the right of the line $c_1 = p_1k$ and must cross the positive ϕ_{00} axis in the image space. Thus if $u = c_1\phi_{00} + k\phi_{10} + c_3\phi_{21} + \theta(c_1\phi_{00} + k\phi_{10} + c_3\phi_{21})$, $k > 0$, $|c_1| < \frac{k}{\epsilon_1}$, $|c_3| \leq \epsilon_2|(c_1, k)|$, then u satisfies

$$u_{tt} - u_{xx} + bu^+ - au^- = t\phi_{00} \quad \text{for } t > p_1k, \quad k > 0.$$

If we set

$$\hat{u} = \frac{s}{t}u,$$

then \hat{u} is a solution of $\hat{u}_{tt} - \hat{u}_{xx} + b\hat{u}^+ - a\hat{u}^- = s\phi_{00}$. Thus we obtain a solution \hat{u} in C_4 . Similarly, the image under F of $c_1\phi_{00} + c_2\phi_{10} + c_3\phi_{21} \in C_2$, $|c_2| \geq \epsilon_1|c_1|$, $c_2 = k$, $k < 0$, $|c_3| \leq \epsilon_2|(c_1, c_2)|$. By (i) of Lemma 3.1, the image of

$$c_2 = k, \quad k < 0, \quad |c_1| \leq \frac{1}{\epsilon_1}k, \quad |c_3| \leq \epsilon_2|(c_1, k)|$$

must lie to the right of the line $c_1 = p_1|k|$ and must cross the positive ϕ_{00} axis in the image space. Thus if $u = c_1\phi_{00} + k\phi_{10} + c_3\phi_{21} + \theta(c_1\phi_{00} + k\phi_{10} + c_3\phi_{21})$, $k < 0$, $|c_1| < \frac{k}{\epsilon_1}$, $|c_3| \leq \epsilon_2|(c_1, k)|$, then u satisfies

$$u_{tt} - u_{xx} + bu^+ - au^- = t\phi_{00} \quad \text{for } t > p_1|k|, \quad k < 0.$$

If we set

$$\check{u} = \frac{s}{t}u,$$

then \check{u} is a solution of $\check{u}_{tt} - \check{u}_{xx} + b\check{u}^+ - a\check{u}^- = s\phi_{00}$. Thus we obtain a solution \check{u} in C_2 .

Now we consider the image under F of $c_1\phi_{00} + c_2\phi_{10} + l\phi_{21} \in C_5$, $|c_2| \geq \epsilon_1|c_1|$, $|l| \geq \epsilon_2|(c_1, c_2)|$, $l > 0$. By (ii) of Lemma 3.1, the image of

$$c_3 = l, \quad |c_2| \geq \epsilon_1|c_1|, \quad |l| \geq \epsilon_2|(c_1, c_2)|$$

must lie to the right of the line $c_1 = p_2|l|$ and must cross the positive ϕ_{00} axis in the image space. Thus if $u = c_1\phi_{00} + c_2\phi_{10} + l\phi_{21} + \theta(c_1\phi_{00} + c_2\phi_{10} + l\phi_{21})$, $l > 0$, $|c_2| \geq \epsilon_1|c_1|$, $|l| \geq \epsilon_2|(c_1, c_2)|$, then u satisfies

$$u_{tt} - u_{xx} + bu^+ - au^- = t\phi_{00} \quad \text{for } t > p_2l, \quad l > 0.$$

If we set

$$\bar{u} = \frac{s}{t}u,$$

then \bar{u} is a solution of $\bar{u}_{tt} - \bar{u}_{xx} + b\bar{u}^+ - a\bar{u}^- = s\phi_{00}$. Thus we obtain a solution \bar{u} in C_5 for given $s > 0$.

Now we consider the image under F of $c_1\phi_{00} + c_2\phi_{10} + l\phi_{21} \in C_6$, $|c_2| \geq \epsilon_1|c_1|$, $|l| \geq \epsilon_2|(c_1, c_2)|$, $l < 0$. By (ii) of Lemma 3.1, the image of

$$c_3 = l, \quad |c_2| \geq \epsilon_1|c_1|, \quad |l| \geq \epsilon_2|(c_1, c_2)|$$

must lie to the right of the line $c_1 = p_2|l|$ and must cross the positive ϕ_{00} axis in the image space. Thus if $u = c_1\phi_{00} + c_2\phi_{10} + l\phi_{21} + \theta(c_1\phi_{00} + c_2\phi_{10} + l\phi_{21})$, $l < 0$, $|c_2| \geq \epsilon_1|c_1|$, $|l| \geq \epsilon_2|(c_1, c_2)|$, then u satisfies

$$u_{tt} - u_{xx} + bu^+ - au^- = t\phi_{00} \quad \text{for } t > p_2|l|, \quad l < 0.$$

If we set

$$\tilde{u} = \frac{s}{t}u,$$

then \tilde{u} is a solution of $\tilde{u}_{tt} - \tilde{u}_{xx} + b\tilde{u}^+ - a\tilde{u}^- = s\phi_{00}$. Thus we also have a solution \tilde{u} in C_6 for given $s > 0$.

For given $s > 0$, there exist six solutions, one in each of the six regions. There exist a positive solution $\frac{s\phi_{00}}{b+1}$ in C_1 , a negative solution $\frac{s\phi_{00}}{a+1}$ in C_3 ,

a solution \hat{u} in C_4 , a solution \check{u} in C_2 , a solution \bar{u} in C_5 , a solution \tilde{u} in C_6 of (1.2). Thus we complete the proof of Theorem 1.1.

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