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COMPLEX VALUED DISLOCATED METRIC SPACES

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Abstract. In this paper, we introduce complex valued dislocated metric spaces. We prove Banach contraction principle, Kannan and Chatterjea type fixed point theorems in this new space. Moreover, we give some applications of the results to differential equations and iterated functions.

1. Introduction

Fixed point theory plays a key role in various fields of mathematics such as mathematical analysis, general topology and especially functional analysis. There are important applications of fixed point theory in mathematics, computer science, engineering, image processing (see [9]), etc. Banach [3] proved a well-known fixed point theorem for contraction mapping in metric space and then many researchers have proved a great number of fixed point theorems and have established many generalization of this theorem. Banach contraction principle is the most useful way for solution of existence problems in mathematical analysis since its structure is simple. For some studies using the Banach contraction principle and different type contractions, see [4, 10, 13, 15, 16, 22, 25].

Hitzler and Seda [12] introduced the concept of dislocated metric space in 2000. In dislocated metric space, the self distance of a point need

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not to be zero necessarily. They also generalized the Banach contraction principle in this space. Dislocated metric space has a significant role in topology, logical programming and electronics engineering. Zeyada et al. [26] presented the complete dislocated quasi-metric spaces and generalized the result of Hitzler [11] in dislocated quasi-metric space. In [1], some fixed point theorems in single and pair of mappings in dislocated metric space were established. Jha and Panthi [14] established a common fixed point theorem for two pairs of weakly compatible mappings in dislocated metric space. For some important studies about dislocated metric spaces, see [17–19].

On the other hand, Azam et al. [2] defined the notion of complex valued metric spaces and gave common fixed point result for mappings. In 2012, Sintunavarat and Kumam [23] extended and improved a result of Azam and applied this to the unique common solution of system of Urysohn integral equation. Rao et al. [20] introduced the complex valued b-metric spaces. For other works, see [5–8, 21, 24].

This paper is organized as follows. In the first part, we give the required background about dislocated and complex valued metric spaces. In the next section, we introduce complex valued dislocated metric spaces and prove Banach, Kannan and Chatterjea type fixed point theorems in this space. An application of Banach contraction principle to differential equations is given at the end of the study.

2. Preliminaries

In this section, we give definitions, lemmas and theorems.

Definition 2.1. [26]. Let X be a nonempty set and $d : X \times X \to \mathbb{R}^+$ be a function satisfying the following conditions:

(i) $d(x, y) = d(y, x)$;
(ii) $d(x, y) = d(y, x) = 0$ implies $x = y$;
(iii) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called dislocated metric (or simply d-metric) on X.

It is clear that every metric is a d-metric but the converse is not necessarily true by the following example.

Example 2.2. Let $d : X \times X \to \mathbb{R}^+$ be defined by

$$d(x, y) = \max\{x, y\},$$

where $X = \mathbb{R}^+$. It is easy to check that d is a dislocated metric but not a metric.

Now we recall some definitions from [12].

Definition 2.3. [12]. A sequence $\{x_n\}$ in d-metric space (X, d) is called Cauchy sequence if for $\epsilon > 0$ there exists a positive integer $n_0 \in \mathbb{N}$ such that for $m, n \geq n_0$, we have $d(x_m, x_n) < \epsilon$.

Definition 2.4. [12]. A sequence $\{x_n\}$ is said to be d-convergent in (X, d) if

$$\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x, x_n) = 0.$$

Definition 2.5. [12]. A d-metric space (X, d) is called complete if every Cauchy sequence in X converges to a point of X.

Definition 2.6. [12]. Let (X, d) be a d-metric space. A mapping $T : X \to X$ is said to be contraction if there exists $0 \leq \alpha < 1$ such that

$$d(Tx, Ty) \leq \alpha d(x, y) \quad \text{for all } x, y \in X.$$

Hitzler and Seda [12] showed that limit in a d-metric space is unique and proved an analogue to Banach contraction principle in d-metric spaces.

Theorem 2.7. *Let (X, d) be a complete d-metric space and $T : X \to X$ be a contraction. Then T has a unique fixed point.*

The complex metric space was initiated by Azam et al. [2]. Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$$z_1 \precsim z_2 \text{ if and only if } Re(z_1) \leq Re(z_2) \text{ and } Im(z_1) \leq Im(z_2).$$

It follows that $z_1 \precsim z_2$ if one of the following conditions is satisfied:
(C_1) $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$,
(C_2) $Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$,
(C_3) $Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$,
(C_4) $Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.
Particularly, we write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of $(C_2), (C_3)$ and (C_4) is satisfied and we write $z_1 \prec z_2$ if only (C_4) is satisfied. The following statements hold:

(1) If $a, b \in \mathbb{R}$ with $a \leq b$, then $az \prec bz$ for all $z \in \mathbb{C}$.
(2) If $0 \precsim z_1 \precnsim z_2$, then $|z_1| < |z_2|$.
(3) If $z_1 \precsim z_2$ and $z_2 \prec z_3$, then $z_1 \prec z_3$.

3. Main Results

In this section, we introduce the notion of complex valued dislocated metric space.

Definition 3.1. Let X be a nonempty set. Assume that a function $d : X \times X \to \mathbb{C}$ satisfies the following conditions:

(Cd_1) $d(x, y) = d(y, x)$;

(Cd_2) $d(x, y) = d(y, x) = 0$ implies $x = y$;

(Cd_3) $d(x, y) \precsim d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is said to be complex valued dislocated metric on X and (X, d) is called a complex valued dislocated metric space.

Example 3.2. Let $d : X \times X \to \mathbb{C}$ be defined by

$$d(x, y) = \max\{x, y\},$$

where $X = \mathbb{C}$. It is clear that d is a complex valued dislocated metric.

Definition 3.3. Let (X, d) be a complex valued d-metric space, $\{x_n\}$ be a sequence in X and $x \in X$.

(a) The sequence $\{x_n\}$ is said to be complex valued d-convergent in (X, d) and converges to x if for every $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x) \prec \epsilon$ for all $n > n_0$ and is denoted by $x_n \to x$ as $n \to \infty$.

(b) The sequence $\{x_n\}$ is called complex valued Cauchy sequence in (X, d) if $\lim_{n\to\infty} d(x_n, x_{n+p}) = 0$ for all $p > 0$.

(c) (X, d) is said to be a complex valued complete d-metric space if every complex valued Cauchy sequence in X converges to some $x \in X$.

Definition 3.4. Let (X, d) be a complex valued d-metric space. A mapping $T : X \to X$ is called contraction if there exists $0 \leq c < 1$ such that

$$d(Tx, Ty) \precsim cd(x, y) \quad \text{for all } x, y \in X. \tag{1}$$

Since the following two lemmas are the analogues of the lemmas in [2], we state these for complex valued d-metric spaces without their proofs.

Lemma 3.5. *Let (X, d) be a complex valued d-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.*

LEMMA 3.6. *Let (X, d) be a complex valued d-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a complex valued Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$.*

We now prove the Banach contraction principle in complex valued d-metric spaces.

THEOREM 3.7. *Let (X, d) be a complex valued complete d-metric space and $T : X \to X$ be a contraction. Then T has a unique fixed point.*

Proof. We will seperate the proof into three parts.
(a) Let T satisfy the inequality in (1). For a point $x_0 \in X$ and the sequence $\{x_n\}$ defined by $x_n = T^n x_0$, from (1), we obtain

$$d(x_n, x_{n+1}) \precsim cd(x_{n-1}, x_n). \tag{2}$$

If we apply again (1), we get

$$d(x_{n-1}, x_n) \precsim cd(x_{n-2}, x_{n-1})$$

and from (2), we get

$$d(x_n, x_{n+1}) \precsim c^2 d(x_{n-2}, x_{n-1}).$$

Continuing this process, we have

$$d(x_n, x_{n+1}) \precsim c^n d(x_0, x_1). \tag{3}$$

Let's use (Cd_3) and (3) for all $n, m \in \mathbb{N}$ with $n < m$,

$$\begin{aligned} d(x_n, x_m) &\precsim d(x_n, x_{n+1}) + d(x_{n+1}, x_m) \\ &\precsim d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2} + d(x_{n+2}, x_m) \\ &\vdots \\ &\precsim d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \ldots + d(x_{m-1}, x_m) \\ &\precsim (c^n + c^{n+1} + \ldots + c^{m-1}) d(x_0, x_1) \\ &\precsim c^n [1 + c + c^2 + \ldots + c^{m-n-1}] d(x_0, x_1) \\ &\precsim \frac{c^n - c^m}{1 - c} d(x_0, x_1). \end{aligned}$$

Therefore, we get

$$|d(x_n, x_m)| \leq \frac{c^n - c^m}{1 - c} |d(x_0, x_1)|.$$

Since $c \in [0,1)$, taking limits as $n \to \infty$, then

$$\frac{c^n - c^m}{1-c}|d(x_0, x_1)| \to 0,$$

i.e.,

$$|d(x_n, x_m)| \to 0.$$

From Lemma 3.6, we conclude that $\{x_n\}$ is complex valued Cauchy sequence. So there is an element $w \in X$ such that $\{x_n\}$ is complex valued d-convergent to w since (X, d) is complex valued complete d-metric space.

(b) Let's show that w is a fixed point of T. For any $n \in \mathbb{N}$, using (Cd_3) and the inequality (1), we obtain

$$\begin{aligned} d(w, Tw) &\precsim d(w, x_n) + d(x_n, Tw) \\ &= d(w, x_n) + d(Tx_n, Tw) \\ &\precsim d(w, x_n) + cd(x_n, w). \end{aligned}$$

As a result, we conclude that $d(w, Tw) = 0$ because x_n is complex valued d-convergent to w as $n \to \infty$. By (Cd_2), we have $Tw = w$.

(c) In this part, we need to prove the uniqueness of fixed point. Suppose that $l \neq w$ be another fixed point of T. Using (1),

$$d(w, l) = d(Tw, Tl) \precsim cd(w, l).$$

and

$$|d(w, l)| \leq c|d(w, l)| \quad \Rightarrow \quad (1-c)|d(w, l)| \leq 0.$$

Since $c \in [0,1)$, we get $|d(w, l)| = 0$. This means $w = l$ and so w is a unique fixed point of T. □

Now we give Kannan type fixed point theorem in complex valued dislocated metric spaces.

THEOREM 3.8. *Let (X, d) be a complex valued complete d-metric space and $S : X \to X$ be a map. If there exists a constant $0 \leq \alpha < \frac{1}{2}$ and*

$$d(Sx, Sy) \precsim \alpha[d(x, Sx) + d(y, Sy)] \tag{4}$$

for all $x, y \in X$, then S has a unique fixed point in X.

Proof. Let x be a point in X and consider $x_n = S^n(x)$. Using (4), we obtain the following:

$$\begin{aligned} d(x_n, x_{n+1}) = d(Sx_{n-1}, Sx_n) &\precsim \alpha[d(x_{n-1}, Sx_{n-1}) + d(x_n, Sx_n)] \\ &= \alpha[d(x_{n-1}, Sx_{n-1}) + d(x_n, x_{n+1})] \end{aligned}$$

and

$$d(x_n, x_{n+1}) \precsim \frac{\alpha}{1-\alpha} d(x_{n-1}, x_n).$$

If we continue in the same way, we get

$$\begin{aligned} d(x_n, x_{n+1}) &\precsim \gamma d(x_{n-1}, x_n) \\ &\precsim \gamma^2 d(x_{n-2}, x_{n-1}) \\ &\vdots \\ &\precsim \gamma^n d(x, x_1), \end{aligned}$$

where $\gamma = \frac{\alpha}{1-\alpha}$. On the other hand, from the triangle inequality,

$$\begin{aligned} d(x_n, x_{n+k}) &\precsim d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \ldots + d(x_{n+k-1}, x_{n+k}) \\ &\precsim (\gamma^n + \gamma^{n+1} + \ldots + \gamma^{n+k-1}) d(x, x_1) \\ &= \frac{\gamma^n}{1-\gamma} d(x, x_1). \end{aligned}$$

Thus, we have

$$|d(x_n, x_{n+k})| \leq \frac{\gamma^n}{1-\gamma} |d(x, x_1)|.$$

From the fact that $0 \leq \gamma < 1$, taking limits as $n \to \infty$, then $|d(x_n, x_{n+k})| \to 0$. By Lemma 3.6, (x_n) is a complex valued Cauchy sequence. There is a point $w \in X$ such that

$$\lim_{n\to\infty} d(x_n, w) = 0$$

because of the completeness of (X, d).

We need to show that w is a fixed point of S. For this purpose, we use (4) as follows:

$$\begin{aligned} d(w,Sw) &\precsim d(w,x_n)+d(x_n,Sw) \\ &= d(w,x_n)+d(Sx_{n-1},Sw) \\ &\precsim d(w,x_n)+\alpha[d(x_{n-1},x_n)+d(w,Sw)] \\ &\precsim d(w,x_n)+\alpha d(w,Sw)+\alpha\gamma^{n-1}d(x,x_1) \\ &\precsim \frac{1}{1-\alpha}d(w,x_n)+\gamma^n d(x,x_1). \end{aligned}$$

We obtain $d(w,Sw)=0$ for $n\to\infty$. On the other hand,

$$\begin{aligned} d(Sw,w) &\precsim d(Sw,x_n)+d(x_n,w) \\ &= d(Sw,Sx_{n-1})+d(x_n,w) \\ &\precsim \alpha[d(w,Sw)+d(x_{n-1},x_n)]+d(x_n,w) \\ &\precsim \alpha d(x_{n-1},x_n)+d(x_n,w) \end{aligned}$$

since $d(w,Sw)=0$. Taking limit as $n\to\infty$, $|d(Sw,w)|=0$, i.e., $d(Sw,w)=0$. As a result,

$$d(w,Sw)=d(Sw,w)=0 \quad \Rightarrow \quad Sw=w.$$

Now we show the uniqueness. Let z be a fixed point of S. From (4),

$$\begin{aligned} d(z,z)=d(Sz,Sz) &\precsim \alpha[d(z,Sz)+d(z,Sz)] \\ &= \alpha[d(z,z)+d(z,z)] \\ &= 2\alpha d(z,z) \\ (1-2\alpha)d(z,z) &\precsim 0. \end{aligned}$$

From the last inequality, we have $(1-2\alpha)|d(z,z)|=0$, i.e., $d(z,z)=0$ because $\alpha\in[0,\frac{1}{2})$.

If a_1,a_2 are fixed points of S, then

$$\begin{aligned} d(a_1,a_2)=d(Sa_1,Sa_2) &\precsim \alpha[d(a_1,Sa_1)+d(a_2,Sa_2)] \\ &= \alpha[d(a_1,a_1)+d(a_2,a_2)] \\ &= 0. \end{aligned}$$

Thus, $d(a_1,a_2)=d(a_2,a_1)=0$ implies that $a_1=a_2$. This completes the proof. □

Remark 3.9. In general, every continuous Kannan mapping S on a complete metric space has a unique fixed point but in Theorem 3.8, the assumption of continuity of S is neglected.

The next result is Chatterjea fixed point theorem in complex valued dislocated metric spaces.

Theorem 3.10. *If* (X,d) *is a complex valued complete d-metric space and* $T : X \to X$ *is a continuous map satisfying*

$$d(Tx,Ty) \precsim k[d(x,Ty)+d(y,Tx)] \tag{5}$$

where $0 \le k < \frac{1}{4}$ *and for all* $x,y \in X$*, then* T *has a unique fixed point in* X.

Proof. Let $x_0 \in X$. Consider a sequence (x_n) in X defined by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$. Using (5) and the triangle inequality, we obtain

$$\begin{aligned} d(x_n,x_{n+1}) = d(Tx_{n-1},Tx_n) &\precsim k[d(x_{n-1},Tx_n)+d(x_n,Tx_{n-1})] \\ &= k[d(x_{n-1},x_{n+1})+d(x_n,x_n)] \\ &\precsim k[d(x_{n-1},x_n)+d(x_n,x_{n+1}) \\ &\quad + d(x_n,x_{n-1})+d(x_{n-1},x_n)] \\ &= kd(x_n,x_{n+1})+3kd(x_{n-1},x_n) \end{aligned}$$

and so

$$d(x_n,x_{n+1}) \precsim \frac{3k}{1-k} d(x_{n-1},x_n).$$

Applying this procedure consecutively, we get

$$\begin{aligned} d(x_n,x_{n+1}) &\precsim \theta d(x_{n-1},x_n) \\ &\precsim \theta^2 d(x_{n-2},x_{n-1}) \\ &\vdots \\ &\precsim \theta^n d(x_0,x_1), \end{aligned}$$

where $\theta = \frac{3k}{1-k}$. The triangle inequality implies that

$$\begin{aligned} d(x_n,x_{n+k}) &\precsim d(x_n,x_{n+1})+d(x_{n+1},x_{n+2})+\ldots+d(x_{n+k-1},x_{n+k}) \\ &\precsim (\theta^n+\theta^{n+1}+\ldots+\theta^{n+k-1})d(x_0,x_1) \\ &= \frac{\theta^n}{1-\theta} d(x_0,x_1). \end{aligned}$$

Thus, we have

$$|d(x_n, x_{n+k})| \leq \frac{\theta^n}{1-\theta}|d(x_0, x_1)|.$$

Since $\theta \in [0, 1)$, $|d(x_n, x_{n+k})| \to 0$ when $n \to \infty$, i.e., (x_n) is a complex valued Cauchy sequence. By the completeness of (X, d), there is a point $w \in X$ such that

$$\lim_{n\to\infty} x_n = w.$$

Since T is a continuous map, we have

$$T(\lim_{n\to\infty} x_n) = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} x_{n+1} = w \quad \Rightarrow \quad w = Tw.$$

As a result, w is a fixed point of T.
Uniqueness: Let u, w be two different fixed points of T. From (5), we get

$$d(u, w) = d(Tu, Tw) \precsim k[d(u, Tw) + d(w, Tu)]$$

and

$$d(w, u) = d(Tw, Tu) \precsim k[d(w, Tu) + d(u, Tw)].$$

Since

$$|d(u, w)-d(w, u)| \leq |k-k|.|d(u, w)-d(w, u)| \quad \Rightarrow \quad |d(u, w)-d(w, u)| = 0,$$

we have $d(u, w) = d(w, u)$. If $d(u, w) = d(w, u) = 0$, then $u = w$. Therefore, $u = w$ and T has a unique fixed point. □

4. Applications to Differential Equations

In this section we first give an application of Theorem 3.7 to the existence and uniqueness of the ordinary differential equation:

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0 \tag{6}$$

THEOREM 4.1. *Let $f(x, y)$ be a continuous function on an area*

$$A = \{(x, y) : a \leq x \leq b, \; c \leq y \leq d\}$$

and satisfy the following condition:

$$|f(a, b) - f(a, b')| \leq k|b - b'| \quad \text{for all } b, b' \in [c, d] \tag{7}$$

Then the differential equation (6) has a unique solution.

Proof. If $y = h(x)$ satisfies (6) and $h(x_0) = y_0$, then we get

$$h(x) = y_0 + \int_{x_0}^{x} f(t, h(t))dt \tag{8}$$

Observe that a unique solution of (6) is equivalent to a unique solution of (8). We use the Theorem 3.7 to obtain the solution of (8).

Let X be the set of all complex valued continuous functions $y = h(x)$ defined on $[-p + x_0, p + x_0]$ such that $d(h(x), y_0) \precsim kp$. It is clear that (X, d) is complex valued complete d-metric space where d is sup metric.

Assume that $T : X \to X$ is defined by

$$T(h) = g$$

where $g(x) = y_0 + \int_{x_0}^{x} f(t, h(t))dt$. We need to prove that the mapping T is a contraction. For all $h, h_1 \in X$, since

$$\begin{aligned} d(T(h), T(h_1)) = d(g, g_1) = \sup &\left\| \int_{x_0}^{x} [f(t, h(t)) - f(t, h_1(t))]dt \right\| \\ &\precsim \int_{x_0}^{x} \sup |f(t, h(t)) - f(t, h_1(t))|dt \\ &\precsim u. \int_{x_0}^{x} |h(t) - h_1(t)|dt \\ &\precsim u.pd(h, h_1) \\ &\precsim kd(h, h_1), \end{aligned}$$

where $0 \leq k = up < 1$, we conclude that T is a contraction mapping. By Theorem 3.7, T has a unique fixed point $h^* \in X$. As a result, it is the unique solution of the differential equation (6). □

Let's give another application of Theorem 3.7.

THEOREM 4.2. *Let (X, d) be a complex valued complete d-metric space and $f : X \to X$ be a function. If f^q satisfies the inequality* (1) *for some $q \in \mathbb{N}$, then f has a unique fixed point.*

Proof. If f^q satisfies (1), then we get that f^q has a unique fixed point $x \in X$, i.e., $f^q(x) = x$ by the Theorem 3.7. Since

$$f(x) = f(f^q(x)) = f^q(f(x)),$$

$f(x)$ is a fixed point of f^q. The uniqueness of fixed point of f^q implies that $x = f(x)$.

Now we prove the uniqueness. If we assume that y is another fixed point of f, then we obtain

$$\begin{aligned} d(x,y) = d(f^q(x), f^q(y)) &\precsim cd(f^{q-1}(x), f^{q-1}(y)) \\ &\precsim c^2 d(f^{q-2}(x), f^{q-2}(y)) \\ &\vdots \\ &\precsim c^{q-1} d(f(x), f(y)) \\ &= c^{q-1} d(x,y) \end{aligned}$$

using (1) consecutively where $0 \leq c < 1$. Thus, we have

$$(1 - c^{q-1}) d(x,y) \precsim 0$$

and so $(1 - c^{q-1})|d(x,y)| \leq 0$. Since $1 - c^{q-1} < 1$, we conclude that $d(x,y) = 0$ implies that $x = y$. □

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References

[1] C.T. Aage and J.N. Salunke, *The results on fixed point theorems in dislocated and dislocated quasi-metric space*, Appl. Math. Sci. **2** (2008), 2941–2948.

[2] A. Azam, B. Fisher and M. Khan, *Common fixed point theorems in complex valued metric spaces*, Number. Funct. Anal. Optim. **32** (2011), 243–253.

[3] S. Banach, *Sur les operations dans les ensembles abstraits et leurs applications aux equations integrales*, Fund. Math. **3** (1922), 133–181.

[4] A. Batool, T. Kamran, S.Y. Jang, C. Park, *Generalized ϕ-weak contractive fuzzy mappings and related fixed point results on complete metric space*, J. Comput. Anal. Appl. **21** (4) (2016), 729–737.

[5] S. Bhatt, S. Chaukiyal and R.C. Dimri, *Common fixed point of mappings satisfying rational inequality in complex valued metric space*, Int. J. Pure Appl. Math. **73(2)** (2011), 159–164.

[6] O. Ege, *Complex valued rectangular b-metric spaces and an application to linear equations*, J. Nonlinear Sci. Appl. **8** (6) (2015), 1014–1021.

[7] O. Ege, *Complex valued G_b-metric spaces*, J. Comput. Anal. Appl. **21** (2) (2016), 363–368.

[8] O. Ege, *Some fixed point theorems in complex valued G_b-metric spaces*, J. Nonlinear Convex Anal. **18** (11) (2017), 1997–2005.

[9] O. Ege and I. Karaca, *Banach fixed point theorem for digital images*, J. Nonlinear Sci. Appl. **8** (3) (2015), 237–245.

[10] M. Eshaghi Gordji, S. Pirbavafa, M. Ramezani and C. Park, *Presic-Kannan-Rus fixed point theorem on partially ordered metric spaces*, Fixed Point Theory **15** (2) (2014), 463–474.

[11] P. Hitzler, *Generalized metrics and topology in logic programming semantics*, Ph.D Thesis, National University of Ireland, University College Cork, 2001.

[12] P. Hitzler and A.K. Seda, *Dislocated topologies*, J. Electr. Eng. **51** (2000), 3–7.

[13] Sh. Jain, Sh. Jain and L.B. Jain, *On Banach contraction principle in a cone metric space*, J. Nonlinear Sci. Appl. **5** (2012), 252–258.

[14] K. Jha and D. Panthi, *A common fixed point theorem in dislocated metric space*, Appl. Math. Sci. **6** (2012), 4497–4503.

[15] M. Jleli and B. Samet, *A new generalization of the Banach contraction principle*, J. Inequal. Appl. **2014:38** (2014).

[16] M. Nazam, M. Arshad and C. Park, *Fixed point theorems for improved α-Geraghty contractions in partial metric spaces*, J. Nonlinear Sci. Appl. **9(6)** (2016), 4436–4449.

[17] L. Pasicki, *Dislocated metric and fixed point theorems*, Fixed Point Theory Appl. **2015:82** (2015).

[18] M.U. Rahman and M. Sarwar, *Fixed point theorems for expanding mappings in dislocated metric space*, Math. Sci. Lett. **4** (2015), 69–73.

[19] K.P.R. Rao and P. Rangaswamy, *A coincidence point theorem for four mappings in dislocated metric spaces*, Int. J. Contemp. Math. Sci. **6** (2011), 1675–1680.

[20] K.P.R. Rao, P.R. Swamy and J.R. Prasad, *A common fixed point theorem in complex valued b-metric spaces*, Bull. Math. Stat. Res. **1** (1) (2013), 1–8.

[21] F. Rouzkard and M. Imdad, *Some common fixed point theorems on complex valued metric spaces*, Comput. Math. Appl. **64** (6) (2012), 1866–1874.

[22] W. Shatanawi and H.K. Nashine, *A generalization of Banach's contraction principle for nonlinear contraction in a partial metric space*, J. Nonlinear Sci. Appl. **5** (2012), 37–43.

[23] W. Sintunavarat and P. Kumam, *Generalized common fixed point theorems in complex valued metric spaces and applications*, J. Inequal. Appl. **2012:84** (2012).

[24] K. Sitthikul and S. Saejung, *Some fixed point theorems in complex valued metric spaces*, Fixed Point Theory Appl. **2012:189** (2012).

[25] J. Sun and X. Han, *The Banach fixed point theorem application to Hopf bifurcation of a generalized Boussinesq system*, Adv. Difference Equ. **2015:43** (2015).

[26] F.M. Zeyada, G.H. Hassan and M.A. Ahmad, *A generalization of fixed point theorem due to Hitzler and Seda in dislocated quasi-metric space*, Arabian J. Sci. Eng. **31** (2005), 111–114.

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