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λ^*-CLOSED SETS AND NEW SEPARATION AXIOMS IN ALEXANDROFF SPACES

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Abstract. Here we have studied the ideas of g^*-closed sets, $g\wedge_\tau$-sets and λ^*-closed sets and investigate some of their properties in the spaces of A. D. Alexandroff [1]. We have also studied some separation axioms like $T_{\frac{\omega}{4}}, T_{\frac{3\omega}{8}}, T_\omega$ in Alexandroff spaces and also have introduced a new separation axiom namely $T_{\frac{5\omega}{8}}$ axiom in this space.

1. Introduction

Topological spaces have been generalised in many ways. A. D. Alexandroff (1940) weakened the union requirements where only countable union of open sets were taken to be open.

The idea of generalised closed sets in a topological space was given by Levine [9]. Later many works on generalised closed sets have been done [2], [3], [5], [6] etc. In 2003, P. Das et al obtained a generalisation of closed sets in Alexandroff spaces which was called g^*-closed sets. They investigated various properties on g^*-closed sets and also introduced a new separation axiom namely T_ω axiom in Alexandroff spaces in the same way as that of $T_{\frac{1}{2}}$-spaces introduced by Levine [9] in a topological space. Recently M. S. Sarsak [11] studied the same in a generalised topological

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space where a generalised topology μ on a nonempty set X is a collection of subsets of X such that $\emptyset \in \mu$ and μ is closed under arbitrary unions. Members of μ are called μ-open sets. He also introduced some new separation axioms namely μ-$T_{\frac{1}{4}}$, μ-$T_{\frac{3}{8}}$ and μ-$T_{\frac{1}{2}}$ axioms and studied their properties and relations between the axioms.

Here we have studied the idea of generalised closed sets namely the g^*-closed sets in Alexandroff spaces. We have investigated few more properties of g^*-closed sets which were not studied in [5]. Also we have obtained some separation axioms like $T_{\frac{\omega}{4}}, T_{\frac{3\omega}{8}}, T_\omega$ in Alexandroff spaces in the same way as that of Sarsak [11] and investigate how far several results as valid in [11] are affected in Alexandroff spaces. Also we have introduced a new separation axiom namely $T_{\frac{5\omega}{8}}$ axiom in a Alexandroff space which was not studied before. We have established that $T_{\frac{5\omega}{8}}$-space can be placed in between $T_{\frac{3\omega}{8}}$ and T_ω spaces, $T_{\frac{3\omega}{8}}$ axiom can be placed in between $T_{\frac{\omega}{4}}$ and $T_{\frac{5\omega}{8}}$, and $T_{\frac{\omega}{4}}$ axiom can be placed in between $T_{\frac{3\omega}{8}}$ and T_0. In general these axioms are not equivalent. In [5] we have seen that T_ω axiom is independent with T_1 axiom. However, we have shown that T_0, T_1 axioms and these axioms are equivalent under certain additional conditions.

2. Preliminaries

DEFINITION 2.1. [1] A non empty set X is called a σ-space or simply a space if in it is chosen a system of subsets $\mathcal{F}$ satisfying the following axioms:

(1) The intersection of a countable number of sets in $\mathcal{F}$ is a set in $\mathcal{F}$.
(2) The union of a finite number of sets in $\mathcal{F}$ is a set in $\mathcal{F}$
(3) The void set $\emptyset$ and the whole set X are sets in $\mathcal{F}$.

Sets of $\mathcal{F}$ are called closed sets. Their complementary sets are called open sets. The collection of all such open sets will sometimes be denoted by τ and the space by (X, τ). When there is no confusion, the space (X, τ) will simply be denoted by X.

Note that a topological space is a space but in general τ is not a topology as can be easily seen by taking $X = R$ and τ as the collection of all F_σ-sets in R. Several examples of spaces are seen in [4], [5], [8]. The definition of closure of a set and interior of a set in a space are similar as in the case of a topological space. Note that closure of a set in a space may not be closed in general. Also interior of a set in a space may not be open.

Definition 2.2. [6] Two sets A, B in X are said to be weakly separated if there are two open sets U, V such that $A \subset U, B \subset V$ and $A \cap V = B \cap U = \emptyset$.

Definition 2.3. [1] A space (X, τ) is called T_0 if for any pair of distinct points of X, there is an open set containing one of the points but not the other.

Observe that a space (X, τ) is T_0 if and only if for any pair of distinct points $x, y \in X$, there is a set A containing one of the points such that A is either open or closed.

Clearly it can be easily checked that if a space (X, τ) is T_0 then for every pair of distinct points $p, q \in X$, either $p \notin \overline{\{q\}}$ or $q \notin \overline{\{p\}}$.

Definition 2.4. [1] (X, τ) is said to be T_1 space if for any pair of distinct points $x, y \in X$, there exist open sets U, V such that $x \in U, y \notin U, y \in V, x \notin V$.

Definition 2.5. [7] A point $x \in X$ is said to be a limit point of A in a space (X, τ) if for any open set U containing x, $U \cap (A - \{x\}) \neq \emptyset$. The set of all limit points of A is called derived set of A and is denoted by A'.

Definition 2.6. [10] A space (X, τ) is said to be a R_0-space if every open set contains the closure of each of its singleton.

Definition 2.7. [1] A space (X, τ) is said to be bicompact if every open cover of it has a finite subcover.

Definition 2.8. [9] A set A in a topological space is said to be a generalised closed (g-closed for short) if and only if $\overline{A} \subset U$ whenever $A \subset U$ and U is open.

Definition 2.9. [5] A set A is said to be a g^*-closed set in a space if and only if there is a closed set F containing A such that $F \subset U$ whenever $A \subset U$ and U is open. A set A is called g^*-open set if $X - A$ is g^*-closed.

Clearly every closed set is g^*-closed but Example 1 [5] shows that converse may not hold.

THEOREM 2.10. [5] *For each $x \in X$, $\{x\}$ is either closed or $X - \{x\}$ is g^*-closed.*

THEOREM 2.11. [5] *A set A is g^*-closed if and only if there is a closed set F containing A such that $F - A$ does not contain any non-void closed set.*

THEOREM 2.12. [5] *A set A is g^*-open set if and only if there is an open set V contained in A such that $F \subset V$ whenever F is closed and $F \subset A$.*

Note that open set is g^*-open but the converse may not hold as it can be verified from Example 1 [5]. Also union of two g^*-closed sets is g^*-closed [5] and so intersection of two g^*-open sets is g^*-open.

THEOREM 2.13. Let A be a subset in a space (X, τ), then $X - \overline{X - A} = Int(A)$.

The proof is straightforward and so is omitted.

3. $A^{\wedge}_{\tau}$-sets and $g{\wedge}_{\tau}$-sets in a space

Throughout the paper X stands for a space and sets are always subsets of X. The letters R and Q stand respectively for the set of real numbers and the set of rational numbers.

DEFINITION 3.1. (cf. [11]) Let $A \subset X$. We denote $A^{\wedge}_{\tau} = \cap\{U \in \tau : A \subset U\}$ and $A^{\vee}_{\tau} = \cup\{F : X - F \in \tau : F \subset A\}$. A is called a $\wedge_{\tau}$-set if $A = A^{\wedge}_{\tau}$ or, equivalently, A is the intersection of all open sets containing A. A is called a $\vee_{\tau}$-set if $A = A^{\vee}_{\tau}$ or, equivalently, $X - A$ is a $\wedge_{\tau}$-set (i.e. A is the union of all closed sets contained in A).

LEMMA 3.2. *Let A, B be subsets of X. Then the following hold:*
(1) $\emptyset^{\wedge}_{\tau} = \emptyset, \quad \emptyset^{\vee}_{\tau} = \emptyset, \quad X^{\wedge}_{\tau} = X, \quad X^{\vee}_{\tau} = X$
(2) $A \subset A^{\wedge}_{\tau}, \quad A^{\vee}_{\tau} \subset A$
(3) $(A^{\wedge}_{\tau})^{\wedge}_{\tau} = A^{\wedge}_{\tau}, \quad (A^{\vee}_{\tau})^{\vee}_{\tau} = A^{\vee}_{\tau}$.

(4) $A \subset B \Rightarrow A^{\wedge}_{\tau} \subset B^{\wedge}_{\tau}$.
(5) $A \subset B \Rightarrow A^{\vee}_{\tau} \subset B^{\vee}_{\tau}$.
(6) $(X \backslash A)^{\wedge}_{\tau} = X \backslash A^{\vee}_{\tau}$, $\qquad (X \backslash A)^{\vee}_{\tau} = X \backslash A^{\wedge}_{\tau}$.

The proof is straightforward and so is omitted.

Remark 3.3. It is easy to verify from definition that a set A is g^*-closed if and only if there is a closed set F containing A such that $F \subset A^{\wedge}_{\tau}$.

Theorem 3.4. *Arbitrary union of $\vee_{\tau}$-sets is a $\vee_{\tau}$-set.*

Proof. Suppose that A_i's are $\vee_{\tau}$-sets, $i \in I$ where I is an index set and $A = \cup\{A_i : i \in I\}$. So $A_i \subset A$ for each i. Therefore $Ai^{\vee}_{\tau} \subset A^{\vee}_{\tau}$ for all $i \in I$ and so $\cup\{Ai^{\vee}_{\tau} : i \in I\} \subset A^{\vee}_{\tau}$. So, $\cup\{Ai : i \in I\} = \cup\{Ai^{\vee}_{\tau} : i \in I\} \subset A^{\vee}_{\tau} \subset A$ by Lemma 3.2(2). Therefore $A = A^{\vee}_{\tau}$. □

Corollary 3.5. Arbitrary intersection of $\wedge_{\tau}$-sets is a $\wedge_{\tau}$-set. Proof is simple so is omitted.

Theorem 3.6. *Intersection of two $\vee_{\tau}$-sets is a $\vee_{\tau}$-set.*

Proof. Let A, B be two $\vee_{\tau}$-sets. Then $A = A^{\vee}_{\tau}, B = B^{\vee}_{\tau}$. Now $A \cap B \subset A$ and $A \cap B \subset B$. Therefore $(A \cap B)^{\vee}_{\tau} \subset A^{\vee}_{\tau}$ and $(A \cap B)^{\vee}_{\tau} \subset B^{\vee}_{\tau}$. So $(A \cap B)^{\vee}_{\tau} \subset A^{\vee}_{\tau} \cap B^{\vee}_{\tau} = A \cap B$.

Conversely, suppose that $x \in A^{\vee}_{\tau} \cap B^{\vee}_{\tau}$. Since $x \in A^{\vee}_{\tau}$ and $x \in B^{\vee}_{\tau}$, there exist closed sets F, P such that $x \in F \subset A, x \in P \subset B$. Therefore $x \in F \cap P \subset A \cap B$. This implies that $x \in (A \cap B)^{\vee}_{\tau}$. Therefore $A^{\vee}_{\tau} \cap B^{\vee}_{\tau} \subset (A \cap B)^{\vee}_{\tau}$. So $A \cap B = A^{\vee}_{\tau} \cap B^{\vee}_{\tau} = (A \cap B)^{\vee}_{\tau}$. □

Corollary 3.7. *Clearly collection of all $\vee_{\tau}$-sets in a space (X, τ) forms a topology.*

Theorem 3.8. *In a space (X, τ), a set A is g^*-open if and only if there is an open set V containing A such that $A^{\vee}_{\tau} \subset V$.*

Proof. Let A be a g^*-open set. Then by Theorem 2.12, there exists an open set $V \subset A$ such that $F \subset V$ whenever $F \subset A$ and F is closed. So $\cup\{F : F \subset A, F \text{ is closed}\} \subset V$. This implies that $A^{\vee}_{\tau} \subset V$.

Conversely, let there be an open set $V \subset A$ such that $A^{\vee}_{\tau} \subset V$. This implies taht $A^{\vee}_{\tau} = \cup\{F : F \subset A, F \text{ is closed}\} \subset V$. So there is an open set V such that $F \subset V$ whenever $F \subset A, F$ is closed. Hence A is g^*-open by Theorem 2.12. $\square$

REMARK 3.9. As in [11], following result holds in a space: if A is a $\wedge_\tau$-set (resp. $\vee_\tau$-set), then A is g^*-closed (resp. g^*-open) if and only if A is closed (resp. open). In particular $(A^\wedge_\tau)^\wedge_\tau = A^\wedge_\tau$ (resp. $(A^\vee_\tau)^\vee_\tau = A^\vee_\tau$). So $A^\wedge_\tau$ is a $\wedge_\tau$-set (resp. $A^\vee_\tau$ is a $\vee_\tau$-set). Therefore $A^\wedge_\tau$ is g^*-closed (resp. $A^\vee_\tau$ is g^*-open) if and only if $A^\wedge_\tau$ is closed (resp. $A^\vee_\tau$ is open).

THEOREM 3.10. *If $A^\wedge_\tau$ is g^*-closed, then A is g^*-closed.*

Proof. Let $A \subset X$ and $A^\wedge_\tau$ be g^*-closed. Then by Remark 3.3, there exists a closed set F containing $A^\wedge_\tau$ i.e. $F \supset A^\wedge_\tau \supset A$ such that $F \subset (A^\wedge_\tau)^\wedge_\tau = A^\wedge_\tau$ by Lemma 3.2(3). So by Remark 3.3, we get A is g^*-closed. $\square$

It can be easily proved that if $A^\vee_\tau$ is g^*-open, then A is g^*-open. If $A^\vee_\tau$ is g^*-open then $X - A^\vee_\tau = (X - A)^\wedge_\tau$ is g^*-closed. So $X - A$ is g^*-closed and hence A is g^*-open.

But the converses may not be true in a space as shown in Example 3.11.

EXAMPLE 3.11. Suppose that $X = R - Q$, where G_i runs over all countable subsets of X, each contains $\sqrt{2}$, and $\tau = \{X, \emptyset, G_i, A_i\}$ where A_i's are the cocountable subsets of X each contains $\sqrt{2}$. Then (X, τ) is a space but not a topological space. Let A be a countably infinite subset of X excluding $\sqrt{2}$. Then A is a closed set and so A is g^*-closed. But $A^\wedge_\tau = \cap\{\{\sqrt{2}\} \cup (X - \{\alpha\}), \alpha \in X - A, \alpha \neq \sqrt{2}\} = \{\sqrt{2}\} \cup A$ is an open set which is not closed, since $\sqrt{2} \in A^\wedge_\tau$. Since $A^\wedge_\tau$ is a $\wedge_\tau$-set, by Remark 3.9, $A^\wedge_\tau$ is not g^*-closed. Therefore $X - A$ is g^*-open, but $(X - A)^\vee_\tau = X - A^\wedge_\tau$ is not g^*-open.

LEMMA 3.12. *If A, B are two subsets of X, then $A^\wedge_\tau \cup B^\wedge_\tau = (A \cup B)^\wedge_\tau$.*

Proof. Let A, B be two subsets of X,then $A \subset A \cup B$ implies that $A^\wedge_\tau \subset (A \cup B)^\wedge_\tau$ and $B \subset A \cup B$ implies that $B^\wedge_\tau \subset (A \cup B)^\wedge_\tau$. Therefore $(A^\wedge_\tau \cup B^\wedge_\tau) \subset (A \cup B)^\wedge_\tau$. Again, $A^\wedge_\tau = \cap\{U_i : U_i \supset A, U_i$ is open$\}$ and $B^\wedge_\tau = \cap\{V_j : V_j \supset B, V_j$ is open$\}$. Therefore $A^\wedge_\tau \cup B^\wedge_\tau = \cap\{(U_i \cup V_j) : U_i \supset A, V_j \supset B, U_i, V_j$ are open $\} \supset \cap\{G : G \supset A \cup B; G$ is open $\} = (A \cup B)^\wedge_\tau$. Therefore $A^\wedge_\tau \cup B^\wedge_\tau = (A \cup B)^\wedge_\tau$. $\square$

THEOREM 3.13. *Union of finite number of $\wedge_\tau$-sets is a $\wedge_\tau$set.*
The proof is straightforward by above Lemma 3.12 and so is omitted.

But arbitrary union of $\wedge_\tau$-sets is not a $\wedge_\tau$-set as revealed from the Example 3.14.

EXAMPLE 3.14. Let $X = R - Q$, G_i be the all countable subsets of X and $\tau = \{X, \emptyset, G_i\}$. Therefore (X, τ) is a space but not a topological space. Suppose A is the set of all irrationals in [0, 2]. Here every singleton of A is an open set, so a $\wedge_\tau$-set, but A is not a $\wedge_\tau$-set. Also A is not closed but g^*-closed.

DEFINITION 3.15 (cf. [11]) : A set A is called a generalised $\wedge_\tau$-set denoted as $g\wedge_\tau$-set if $A^\wedge_\tau \subset F$ whenever $F \supset A$ and F is closed. A set A is called a generalised $\vee_\tau$-set denoted as $g\vee_\tau$-set if $X - A$ is $g\wedge_\tau$-set, or equivalently, $A^\vee_\tau \supset U$ whenever $A \supset U$ and U is open.

Clearly if A is a $\wedge_\tau$-set then A is $g\wedge_\tau$-set . But the converse is not true.

EXAMPLE 3.16. Let $X = R$, G_i be the all countable subsets of irrationals in R and $\tau = \{X, \emptyset, G_i\}$. Therefore (X, τ) is a space but not a topological space. Suppose $A = R - Q$. Therefore $A^\wedge_\tau = X$ and $\overline{A} = X$. Therefore A is $g\wedge_\tau$-set , since the only closed set containing A is X. But $A = R - Q \neq X = A^\wedge_\tau$. This implies that A is not $\wedge_\tau$-set.

THEOREM 3.17. *Union of two $g\wedge_\tau$-sets is $g\wedge_\tau$-set.*

Proof. Suppose A, B be two $g\wedge_\tau$-sets of X, then $A^\wedge_\tau \subset F_1$ whenever F_1 is closed and $A \subset F_1$ and $B^\wedge_\tau \subset F_2$ whenever F_2 is closed and $B \subset F_2$. Let $(A \cup B) \subset F, F$ is closed then $A \subset F$ and $B \subset F$ which imply that $A^\wedge_\tau \subset F$ and $B^\wedge_\tau \subset F$. So $(A^\wedge_\tau \cup B^\wedge_\tau) \subset F$. Therefore by Lemma 3.12, $(A \cup B)^\wedge_\tau = (A^\wedge_\tau \cup B^\wedge_\tau) \subset F$. Hence $A \cup B$ is $g\wedge_\tau$-set. □

But union of two $g\vee_\tau$-sets may not be $g\vee_\tau$-set as shown by the Example 3.18.

EXAMPLE 3.18. Let $X = R - Q$, G_i be the nonempty countable subsets of $X - \{\sqrt{3}\}$ and $\tau = \{X, \emptyset, G_i \cup \{\sqrt{3}\}\}$. Therefore (X, τ) is a space but not a topological space. Assume $A = \{\sqrt{3}\}$, then $A^\vee_\tau = \emptyset$. Only open set contained in A is $\emptyset$ and $\emptyset \subset \emptyset = A^\vee_\tau$. So A is a $g\vee_\tau$-set.

Suppose $B = \{\sqrt{5}\}$. So B is a $g\vee_\tau$-set. Let $C = A \cup B = \{\sqrt{5}, \sqrt{3}\}$. Then $C^{\vee}_{\tau} = \emptyset$. Also C is an open set and $C \subset C$ but $C \not\subset C^{\vee}_{\tau}$. Therefore $A \cup B$ is not a $g\vee_\tau$set.

We can easily deduce the following three results in a space (X, τ).

Theorem 3.19. *For each $x \in X$, $\{x\}$ is either open or a $g\vee_\tau$-set.*

Theorem 3.20 . *If A is a $g\vee_\tau$-set and $A^{\vee}_{\tau} \cup (X - A) \subset F, F$ is closed, then $F = X$.*

Corollary 3.21. *If A is a $g\vee_\tau$-set then $A^{\vee}_{\tau} \cup (X - A)$ is closed if and only if A is a $\vee_\tau$-set.*

4. T_ω-Space

Definition 4.1 [5] A space (X, τ) is said to be T_ω-space if and only if g^*-closed set is closed.

In Theorem 16 [5], it is shown that every T_ω-space is T_0 space. But the converse is not true as shown in the Example 6 [5]. Also, in Examples 6 and 7 [5] it has been shown that T_ω and T_1 axioms in a space are independent of each other.

Definition 4.2 [5] For any $E \subset X$ let $\overline{E^*} = \cap\{A : E \subset A, A$ is g^*-closed set in X$\}$, then $\overline{E^*}$ is called g^*-closure of E.

We consider the following sets which will be used frequently in the sequel:
$C = \{A : \overline{(X - A)}$ is closed$\}$ and $C^* = \{A : \overline{(X - A)^*}$ is g^*-closed$\}$.

Theorem 4.3 [5] *A space (X, τ) is T_ω if and only if*
(a) for each $x \in X$, $\{x\}$ is either open or closed and
(b) $C = C^*$.

Theorem 4.4. *The following are equivalent:*
(1) (X, τ) is T_ω-space.
(2) Every $g\wedge_\tau$-set is $\wedge_\tau$-set and $C = C^*$.

(3) Every $g\vee_\tau$-set is $\vee_\tau$-set and $C = C^*$.

Proof. (1) $\Rightarrow$ (2): Let (X, τ) be T_ω-space and let A be a $g\wedge_\tau$-set . We wish to prove $A \supset A^\wedge_\tau$. If not, suppose $x \in A^\wedge_\tau$ and $x \notin A$. By Theorem 4.3, $\{x\}$ is either open or closed. We discuss two cases:

Case (i): Suppose $\{x\}$ is open. So $X - \{x\}$ is closed containing A. Since A is $g\wedge_\tau$-set , we get $A^\wedge_\tau \subset (X - \{x\})$ which implies that $x \notin A^\wedge_\tau$, a contradiction.

Case(ii): Suppose $\{x\}$ is closed. Then $X - \{x\}$ is an open set containing A. But $x \in A^\wedge_\tau = \cap\{U, U \in \tau, U \supset A\} \subset X - \{x\}$, a contradiction. Hence in any case $A \supset A^\wedge_\tau$ and so $A = A^\wedge_\tau$ which implies that A is $\wedge_\tau$-set . Also by Theorem 4.3, $C = C^*$.

(2) $\Rightarrow$ (3): Let A be a $g\vee_\tau$-set and $C = C^*$. Then $X - A$ is $g\wedge_\tau$-set . By supposition $X - A$ is a $\wedge_\tau$-set . So A is a $\vee_\tau$-set .

(3) $\Rightarrow$ (2): Let B be a $g\wedge_\tau$-set and $C = C^*$. So $X - B$ is $g\vee_\tau$-set. By supposition, $X - B$ is $\vee_\tau$-set. So B is $\wedge_\tau$-set.

(2) $\Rightarrow$ (1) : Let $x \in X$ and $C = C^*$. We will prove $\{x\}$ is either open or closed. If $\{x\}$ is not open, $X - \{x\}$ is not closed. So X is the only closed set containing $X - \{x\}$. Also $(X - \{x\})^\wedge_\tau \subset X$ .Therefore by definition $X - \{x\}$ is a $g\wedge_\tau$-set . By supposition, $X - \{x\}$ is $\wedge_\tau$-set i.e. $(X - \{x\})^\wedge_\tau = X - \{x\}$. So $X - \{x\}$ must be open. So $\{x\}$ is a closed set. Therefore the space is T_ω. □

5. λ^*-closed sets and λ^*-open sets in a space

Definition 5.1. A subset A of a space (X, τ) is said to be λ^*-closed if $A = L \cap \overline{F}$ where L is a $\wedge_\tau$-set and F is a subset of X. A is said to be λ^*-open if $X - A$ is λ^*-closed.

Lemma 5.2. *For a subset A of a space (X, τ) the following are equivalent:*

(i) A is λ^*-closed

(ii) $A = A^\wedge_\tau \cap \overline{F}, F \subset X$.

(iii) $A = L \cap \overline{A}$, L is a $\wedge_\tau$-set .

(iv) $A = A^\wedge_\tau \cap \overline{A}$.

The proof is simple, so is omitted.

NOTE 5.3. From Lemma 5.2 (iv) we can say that a subset A is said to be λ^*-closed if A can be expressed as the intersection of all open sets and all closed sets containing it.

REMARK 5.4. Clearly every $\wedge_\tau$-set is λ^*-closed and closed set is λ^*-closed. But the converse is not true as revealed from the Example 3.14, since the set A is λ^*-closed but not $\wedge_\tau$-set and not closed. It is seen that if a subset A is closed then A is g^*-closed and λ^*-closed. But the converse of this result may not hold in a space which is also seen from Example 3.14, although it is true in a Generalized Topological space [11]. However, it is true if the additional condition that $C = C^*$ holds.

THEOREM 5.5. *If A is g^*-closed and λ^*-closed satisfies $C = C^*$ then A is closed.*

Proof. Suppose A is g^*-closed and λ^*-closed satisfying the condition $C = C^*$. Since A is g^*-closed, there is a closed set F containing A such that $F \subset A^\wedge_\tau$. Since $A \subset F, \overline{A} \subset \overline{F}$ and so $\overline{A} \subset F \subset A^\wedge_\tau$. Again since A is λ^*-closed, $A = A^\wedge_\tau \cap \overline{A} = \overline{A}$...............(1).

Now A is g^*-closed, so $\overline{A^*} = A$, therefore $(X-A) \in C^*$. Since $C = C^*$, $(X - A) \in C$ which implies that $\overline{A}$ is closed. Therefore, by (1), we have A is a closed set. □

THEOREM 5.6. *If A is λ^*-closed set and $g\wedge_\tau$-set with $\overline{A}$ is closed, then A is a $\wedge_\tau$-set.*

Proof. Suppose the conditions hold. Since $\overline{A}$ contains A and A is $g\wedge_\tau$-set, then $A^\wedge_\tau \subset \overline{A}$. Since A is λ^*-closed, $A = A^\wedge_\tau \cap \overline{A}$ which implies $A = A^\wedge_\tau$. Hence A is $\wedge_\tau$-set. □

THEOREM 5.7. *A set A of X is λ^*-open if and only if $A = M \cup Int(V)$ where M is a $\vee_\tau$-set and V is a subset of X.*

Proof. Let A be a λ^*-open set. Then $X - A$ is a λ^*-closed set. So $X - A = L \cap \overline{F}$, where L is a $\wedge_\tau$-set and $F \subset X$. Therefore $X - L \subset A$ and $X - \overline{F} \subset A$. So by Theorem 2.13, $A = L^c \cup (\overline{F})^c = M \cup Int(X-F) = M \cup Int(V)$ where $M = X - L$, a $\vee_\tau$-set and $V = X - F \subset X$.

Conversely, Let $A = M \cup Int(V), M$ is a $\vee_\tau$-set and $V \subset X$. So $A^c = M^c \cap (Int(V))^c$ where M^c is a $\wedge_\tau$-set and $(Int(V))^c = \overline{X - V}$ by Theorem 2.13. So $X - A = L \cap \overline{F}$, where $L = M^c$ and $F = X - V$. Hence $X - A$ is λ^*-closed and hence A is λ^*-open. □

Corollary 5.8. *A is λ^*-open if and only if $A = A^{\vee}_{\tau} \cup Int(A)$.*

Proof. Let A is λ^*-open. Then $A = M \cup Int(V)$ where M is a $\vee_\tau$-set and $V \subset X$. Since $M \subset A, M^{\vee}_{\tau} \subset A^{\vee}_{\tau}$ and since $Int(V) \subset A, Int(Int(V)) \subset Int(A)$. So, $A = M \cup Int(V) = M^{\vee}_{\tau} \cup Int(V) \subset A^{\vee}_{\tau} \cup Int(V) = A^{\vee}_{\tau} \cup Int(Int(V)) \subset A^{\vee}_{\tau} \cup Int(A)$. Again since $A^{\vee}_{\tau} \subset A$ and $Int(A) \subset A, A^{\vee}_{\tau} \cup Int(A) \subset A$. Therefore we get $A = A^{\vee}_{\tau} \cup Int(A)$.

Conversely, let $A = A^{\vee}_{\tau} \cup Int(A)$. Since $A^{\vee}_{\tau}$ is a $\vee_\tau$-set and $A \subset X$, A is λ^*-open. □

REMARK 5.9. Clearly $\vee_\tau$-sets are λ^*-open sets and open sets are λ^*-open sets. On the other hand if a set A is open and $g\vee_\tau$-set then A is a $\vee_\tau$-set and if A is g^*-open and λ^*-open satisfies the condition $C = C^*$, then A is open.

In view of above Note 5.3 and by Theorem 18 [5] we have the following Theorem 5.10. However we are also giving a separate proof of the Theorem 5.10.

THEOREM 5.10. *A space (X, τ) is T_ω if and only if every subset of (X, τ) is λ^*-closed and $C = C^*$.*

Proof. Suppose every subset of (X, τ) is λ^*-closed and $C = C^*$ and $x \in$ X. We shall show that $\{x\}$ is either open or closed. Suppose $\{x\}$ is not open, then $X - \{x\}$ is not closed. Since $X - \{x\}$ is also a λ^*-closed set then $X - \{x\} = (X - \{x\})^{\wedge}_{\tau} \cap (\overline{X - \{x\}}) = (X - \{x\})^{\wedge}_{\tau} \cap X = (X - \{x\})^{\wedge}_{\tau}$. Therefore $X - \{x\}$ is a $\wedge_\tau$-set. So $X - \{x\}$ is an open set which implies $\{x\}$ is closed. Then by Theorem 4.3, (X, τ) is T_ω-space.

Conversely, suppose that (X, τ) is T_ω-space and $A \subset X$. Then by Theorem 4.3, every singleton is either open or closed and $C = C^*$. So each $x \in X - A$, either $\{x\} \in \tau$ or $(X - \{x\}) \in \tau$. Let $A_1 = \{x : x \in X - A, \{x\} \in \tau\}$, $A_2 = \{x : x \in X - A, X - \{x\} \in \tau\}$, $L = \cap[X - \{x\} : x \in A_2] = X - A_2$ and $F = \cap[X - \{x\} : x \in A_1] = X - A_1$. Note that L is a $\wedge_\tau$-set i.e. $L = L^{\wedge}_{\tau}$ and $F = \overline{F}$. Now, $L \cap \overline{F} = (X - A_2) \cap (X - A_1) = X - (A_1 \cup A_2) = X - (X - A) = A$. Thus A is λ^*-closed. □

Following Example shows that union of two λ^*-closed sets may not be λ^*-closed.

Example 5.11. Let $X = R - Q$, $\tau = \{X, \emptyset, G_i \cup \{\sqrt{3}\}\}$ where G_i be all countable subsets of $X - \{\sqrt{2}\}$. Then (X, τ) is a space but not a topological space. Suppose $A = \{\sqrt{2}\}$, then $A^{\wedge}_{\tau} =$ X and $\overline{A} = A$ and so $A = A^{\wedge}_{\tau} \cap \overline{A}$. This implies that A is λ^*-closed. Again suppose $B = \{\sqrt{3}\}$, then B is also λ^*-closed, since B $= B^{\wedge}_{\tau}$ and $\overline{B} = X$. It can be verified that every singleton is λ^*-closed. Now let $C = \{\sqrt{2}, \sqrt{3}\}$. Then $C^{\wedge}_{\tau} = X$ and $\overline{C} = X$. So $C^{\wedge}_{\tau} \cap \overline{C} = X \neq C$ which implies that C is not λ^*-closed.

Remark 5.12. It is shown in [5] that in a bicompact space (Alexandroff space), g^*-closed sets may not be bicompact. Likewise, λ^*-closed sets in a bicompact space may not be bicompact as shown in the following Example 5.13.

Example 5.13. Let $X = R - Q$, where G_i be the all countable subsets of $X - \{\sqrt{2}\}$, A_i be the all cofinite subsets of X and $\tau = \{X, \emptyset, G_i, A_i\}$. Then (X, τ) is a space but not a topological space. Clearly (X, τ) is a bicompact space. Suppose A is the set of all irrationals in $(0, 1)$. Then there is no open set of G_i type containing A and so $A^{\wedge}_{\tau} = \cap\{X - \{\alpha\} : \alpha \in X - A\} = A$. Therefore A is a $\wedge_{\tau}$-set which implies A is a λ^*-closed set. But A is not bicompact since $\{\{r\} : r \in A\}$ forms an open cover for A which has no finite subcover.

In [11] it is seen that the collection of all λ_{μ}-open sets forms a generalised topology μ on X, but unlikely the collecion of λ^*-open sets does not form a space structure σ on X since intersection of two λ^*-open sets is not λ^*-open as can be easily verified from the Example 5.11.

6. $T_{\frac{\omega}{4}}$-space, $T_{\frac{3\omega}{8}}$-space, $T_{\frac{5\omega}{8}}$-space

Definition 6.1. A space (X, τ) is called $T_{\frac{\omega}{4}}$-space if for every finite subset P of X and for every $y \in X - P$, there exists a set A_y containing P and disjoint from $\{y\}$ such that A_y is either open or closed.

Theorem 6.2. *Every $T_{\frac{\omega}{4}}$-space is T_0-space.*

Proof. Suppose that (X, τ) is $T_{\frac{\omega}{4}}$ and x, y are two distinct points in X. Since the space is $T_{\frac{\omega}{4}}$, then for every $y \in X - \{x\}$ there exists a set

A_y such that for every $\{x\} \subset A_y$ and $\{y\} \not\subset A_y$ where A_y is either open or closed. This implies $T_{\frac{\omega}{4}}$-space is T_0-space. □

THEOREM 6.3. *A space (X, τ) is $T_{\frac{\omega}{4}}$ if and only if every finite subset of X is λ^*-closed*

Proof. Suppose (X, τ) is $T_{\frac{\omega}{4}}$-space and P is a finite subset of X. So for every $y \in X - P$ there is a set A_y containing P and disjoint from $\{y\}$ such that A_y is either open or closed. Let L be the intersection of all open sets A_y and F be the intersection of all closed sets A_y. Then $L = L^{\wedge}_{\tau}$ and $\overline{F} = F$. Therefore $P = L \cap F = L \cap \overline{F}$. So P is λ^*-closed.

Conversely, let P be a finite set. So by the condition it is λ^*-closed. Then by Lemma 5.2 (iii) $P = L \cap \overline{P}$, where L is a $\wedge_\tau$-set . Let $y \in X - P$. If $y \notin \overline{P}$ then there exists a closed set $F = A_y$ containing P such that $\{y\} \not\subset A_y$. Again if $y \in \overline{P} - P$, then $y \notin L$ and so $y \notin U$ for some open set $U = A_y$ containing L. So $P \subset U$. Hence (X, τ) is $T_{\frac{\omega}{4}}$-space. □

THEOREM 6.4. *A space (X, τ) is T_0 if and only if every singleton of X is λ^*-closed.*

Proof is similar to that of the Theorem 6.3.

Note that converse of Theorem 6.2 may not be true as revealed from the Example 5.11.

DEFINITION 6.5. A space(X, τ) is called $T_{\frac{3\omega}{8}}$-space if for every countable subset P of X and for every $y \in X - P$, there exists a set A_y containing P and disjoint from $\{y\}$ such that A_y is either open or closed.

THEOREM 6.6. *A space(X, τ) is $T_{\frac{3\omega}{8}}$-space if and only if every countable subset of X is λ^*-closed.*

Proof is similar to the proof of Theorem 6.3, so is omitted.

DEFINITION 6.7. A space(X, τ) is called $T_{\frac{5\omega}{8}}$-space if for any subset P of X and for every $y \in X - P$, there exists a set A_y containing P and disjoint from $\{y\}$ such that A_y is either open or closed.

THEOREM 6.8 . *A space(X, τ) is $T_{\frac{5\omega}{8}}$-space if and only if for every subset E of X is λ^*-closed.* Proof is similar to the proof of Theorem

6.3, so is omitted.

Note that $T_{\frac{5\omega}{8}}$ axiom does not imply $C = C^*$.

Remark 6.9. It follows from Theorem 5.10, Theorem 6.8, Theorem 6.6, Theorem 6.3 that every T_ω-space is $T_{\frac{5\omega}{8}}$-space and $T_{\frac{5\omega}{8}}$-space is $T_{\frac{3\omega}{8}}$-space and $T_{\frac{3\omega}{8}}$-space is $T_{\frac{\omega}{4}}$-space.

However, the converse of each implication may not be true as shown in the undermentioned Examples 6.10, 6.11, 6.12.

Example 6.10. Example of a $T_{\frac{5\omega}{8}}$-space which is not T_ω-space.

This can be easily verified from the Example 3.14.

Example 6.11. Example of a $T_{\frac{3\omega}{8}}$-space which is not $T_{\frac{5\omega}{8}}$-space.

Let $X = R - Q$, G_i be the countable subsets of X containing $\sqrt{2}$ and $\tau = \{X, \emptyset, G_i\}$. So (X, τ) is a space but not a topological space. Take any countable subset $A \subset X$. Then if $\sqrt{2} \in A$, A is an open set. Therefore $A = A^\wedge_\tau$ which implies that A is λ^*-closed. If $\sqrt{2} \notin A$, $A^\wedge_\tau = \{\sqrt{2}\} \cup A$ and $\overline{A} = A$. This implies that A is λ^*-closed. So (X, τ) is a $T_{\frac{3\omega}{8}}$-space. Now let B be an uncountably infinite subset of X containing the point $\sqrt{2}$. Therefore $B^\wedge_\tau = X$ and $\overline{B} = X$. Therefore $B^\wedge_\tau \cap \overline{B} = X \neq B$ which implies that B is not λ^*-closed by Lemma 5.2 (iv). So (X, τ) is not $T_{\frac{5\omega}{8}}$-space by Theorem 6.8.

Example 6.12. Example of a $T_{\frac{\omega}{4}}$-space which is not $T_{\frac{3\omega}{8}}$-space.

Suppose $X = R - Q, X^* = \{2\} \cup X$. Let $\tau = \{\emptyset, X^*, \{2\} \cup (X - A); A \subset X\}$ where A's are the finite subsets of X. Therefore (X^*, τ) is a topological space so a space also. Take any finite subset $E \subset X^*$, we get the following observations:

(i) if $2 \in E, E^\wedge_\tau = \cap\{(X - \{\alpha\}) \cup \{2\}, \alpha \in X - E\} = \{2\} \cup E = E$ which implies that E is a $\wedge_\tau$-set. Therefore E is a λ^*-closed set .

(ii) if $2 \notin E, E$ is a closed set which implies that E is λ^*-closed.

So (X^*, τ) is a $T_{\frac{\omega}{4}}$-space.

Now suppose Y is a countably infinite subset of X, so $2 \notin Y$. Here closed sets are finite. Therefore $\overline{Y} = X^*$ and $Y^\wedge_\tau = \{2\} \cup Y$. Thus $\overline{Y} \cap Y^\wedge_\tau = \{2\} \cup Y \neq Y$. Therefore Y is not λ^*-closed. Hence (X, τ) is not $T_{\frac{3\omega}{8}}$-space.

THEOREM 6.13. *A space (X, τ) is T_1 if and only if it is T_0 and R_0.*

Proof. Let (X, τ) be T_1-space. Obviously then it is T_0. Let $A \subset X$, A be open, $x \in A$. Since the space is T_1, for $x, y \in X$ and $x \neq y$ there are open sets U, V such that $x \in U$ and $y \notin U$ and $y \in V$ and $x \notin V$. Hence y cannot be a limit point of $\{x\}$. Therefore no point lying outside $\{x\}$ can be a limit point of $\{x\}$, so $\{x\}' \subset \{x\}$. Hence $\overline{\{x\}} = \{x\} \subset A$. So (X, τ) is R_0-space.

Conversely, let (X, τ) be T_0 and R_0. So for $x, y \in X$ and $x \neq y$, either $x \notin \overline{\{y\}}$ or $y \notin \overline{\{x\}}$. Suppose $x \notin \overline{\{y\}}$. Then there exists a closed set F containing y such that $x \notin F$. Therefore $x \in X - F$, an open set and $y \notin X - F$. Since the space is also R_0, $\overline{\{x\}} \subset X - F$. So $\overline{\{x\}} \cap F = \emptyset$ which implies that $\overline{\{x\}} \cap \{y\} = \emptyset$. Hence $y \notin \overline{\{x\}}$, so y is not a limiting point of $\{x\}$. Therefore there exists an open set V containing y such that $x \notin V$. Since $x, y \in X$ and $x \neq y$, we get two open sets $X - F$ and V containing x, y respectively and $y \notin X - F$ and $x \notin V$. Thus (X, τ) is T_1-space. $\square$

THEOREM 6.14. *A space (X, τ) is T_1 if and only if every singleton is $\wedge_\tau$-set.*

Proof. Let (X, τ) be T_1. So by Theorem 6.13, (X, τ) is T_0 and R_0. Since (X, τ) is T_0, every singleton is λ^*-closed , by Theorem 6.4. Suppose $x \in X$, then $\{x\}$ is λ^*-closed. So $\{x\} = \{x\}^{\wedge}_{\tau} \cap \overline{\{x\}}$, by Lemma 5.2 (iv). We claim that $\{x\} = \{x\}^{\wedge}_{\tau}$. If not, there exists $y \in \{x\}^{\wedge}_{\tau} - \{x\}$. So $y \notin \overline{\{x\}}$, hence there is a closed set F, $F \supset \{x\}$ such that $y \notin F$. Therefore $y \in X - F$, an open set. Again since (X, τ) is R_0, $\overline{\{y\}} \subset X - F$. Thus $\overline{\{y\}} \cap F = \emptyset$. Since $x \in F$, $x \notin \overline{\{y\}}$. Therefore there exists an open set V containing x but $y \notin V$. This implies that $y \notin \{x\}^{\wedge}_{\tau}$, a contradiction. Hence $\{x\}$ is a $\wedge_\tau$-set.

Conversely, let $x, y \in X$ and $x \neq y$. So $y \notin \{x\}$. By supposition $\{x\}$ and $\{y\}$ are $\wedge_\tau$-sets i.e. $\{x\} = \{x\}^{\wedge}_{\tau}$ and $y \notin \{x\}^{\wedge}_{\tau}$. Therefore there exists an open set V' such that $x \in V'$, but $y \notin V'$. Similarly, since $\{y\} = \{y\}^{\wedge}_{\tau}$, there exists an open set U' such that $y \in U'$ and $x \notin U'$. Hence x, y are weakly separated by open sets V' and U' and (X, τ) is T_1-space. $\square$

Definition 6.15. A space (X,τ) is said to be Weak R_0-space if every λ^*-closed singleton is a $\wedge_\tau$-set.

Theorem 6.16. *Every R_0-space is a Weak R_0-space.*

Proof. Suppose $x \in X$ and $\{x\}$ is λ^*-closed, then $\{x\} = \{x\}^\wedge_\tau \cap \overline{\{x\}}$ by Lemma 5.2(iv). We claim that $\{x\}$ is a $\wedge_\tau$-set. If not, then $\{x\} \neq \{x\}^\wedge_\tau$ and so let $y \in \{x\}^\wedge_\tau - \{x\}$. Then $y \notin \overline{\{x\}}$. So there is a closed set $F, F \supset \{x\}$ such that $y \notin F$. This implies that $y \in X - F$, an open set. Since(X,τ) is R_0-space, $\overline{\{y\}} \subset X - F$. Therefore $\overline{\{y\}} \cap F = \emptyset$. Since $x \in F$, $x \notin \overline{\{y\}} = \{y\} \cup \{y\}'$ where $\{y\}'$ denotes the set of limit points of $\{y\}$. Therefore there exists an open set $V \supset \{x\}$ such that $y \notin V$, since $x \neq y$ and x is not also the limit point of $\{y\}$. This implies that $y \notin \{x\}^\wedge_\tau$, a contradiction. Hence (X,τ) is weak R_0-space. □

But following Example shows that the converse of the Theoem 6.16 may not be true.

Example 6.17. Let $X = R$ and G_i be the all countable subsets of $X - Q - \{\sqrt{2}\}$, $\tau = \{X, \emptyset, G_i\}$. Then (X,τ) is a space but not a topological space. We will easily check that rational singletons including $\sqrt{2}$ are not λ^*-closed. For any irrational r except $\sqrt{2}, \{r\}$ is λ^*-closed and $\{r\} = \{r\}^\wedge_\tau$. So (X,τ) is a weak R_0-space. If $B = \{\sqrt{5}\}$ then $\overline{B} = B \cup Q \cup \{\sqrt{2}\}$ and B is an open set but B does not contain $\overline{B}$, hence the space is not R_0.

Lemma 6.18. *If every subset of X is $\wedge_\tau$-set, then (X,τ) is T_1-space. The proof is simple.*

Note 6.19. By Theorem 6.14, converse of Lemma 6.18 is not true as revealed from the Example 3.14. But the converse is true by imposing additional conditions as given in the undermentioned Lemma 6.21. Also note that the converse part is true in a μ-space [11].

Definition 6.20 [5] A space (X,τ) is said to be strongly symmetric if $\{x\}$ is g^*-closed for each $x \in X$.

Lemma 6.21. *If (X,τ) is a strongly symmetric T_1-space and satisfies the condition $C = C^*$, then every subset of X is a $\wedge_\tau$-set.*

Proof. Let (X,τ) be a strongly symmetric T_1-space satisfying the condition $C = C^*$ and $A \subset X$, $x \in X$ and $x \notin A$. Then, by definition, $\{x\}$ is g^*-closed. Since (X,τ) is T_1-space, $\{x\}$ is a $\wedge_\tau$-set by Theorem 6.14 and so a λ^*-closed set. Therefore $\{x\}$ is a closed set by Theorem 5.5. Therefore $X-\{x\}$ is an open set containing A. So $A = \cap\{X-\{x\}, x \in X-A\}$ which implies A is a $\wedge_\tau$-set. □

REMARK 6.22. If the space (X,τ) is a strongly symmetric T_1-space and satisfies the condition $C = C^*$, then union and intersection of two λ^*-closed sets are λ^*-closed sets.

THEOREM 6.23 (cf. [11]) *For a space (X,τ) , the following statements are equivalent:*

(1) (X,τ) is T_1

(2) (X,τ) is T_0 and R_0

(3) (X,τ) is T_0 and weak R_0.

Proof. (1) $\Rightarrow$ (2): It follows from Theorem 6.13.

(2) $\Rightarrow$ (3): It follows from Theorem 6.16.

For (3) $\Rightarrow$ (1): Let (X,τ) be T_0 and weak R_0 and $\{x\} \subset X$. So by Theorem 6.4, $\{x\}$ is λ^*-closed . Again (X,τ) is weak R_0, $\{x\}$ is $\wedge_\tau$-set. By Theorem 6.14, (X,τ) is T_1. □

THEOREM 6.24. *If (X,τ) is a strongly symmetric T_1-space and satisfies the condition $C = C^*$, then it is $T\omega$-space.*

Proof. Let (X,τ) be strongly symmetric T_1-space satisfying the condition $C = C^*$ and let A be a g^*-closed set. Then $\overline{A^*} = A$ and so $\overline{A^*}$ is g^*-closed. Therefore $A^c \in C^* = C$ which implies that $\overline{A}$ is closed. Now let $x \in \overline{A} - A$. Since (X,τ) is T_1 and Strongly symmetric space, $\{x\}$ is g^*-closed and λ^*-closed and since $C = C^*$, $\{x\}$ is a closed set by Theorem 5.5. But by Theorem 2.11, $\{x\} \not\subset \overline{A} - A$. Therefore $x \in A$ and so $\overline{A} = A$ which implies that A is a closed set and hence the space (X,τ) is T_ω. □

THEOREM 6.25. *If the space (X,τ) is strongly symmetric, Weak R_0, and satisfies the condition $C = C^*$, then the following are equivalent:*

(1) (X,τ) is T_0

(2) (X,τ) is T_1

(3) (X,τ) is T_ω

(4) (X,τ) is $T_{\frac{5\omega}{8}}$

(5) (X, τ) *is* $T_{\frac{3\omega}{8}}$
(6) (X, τ) *is* $T_{\frac{\omega}{4}}$.

7. Conclusion

In this paper we have characterised g^*-closed sets in terms of $A^{\wedge}_{\tau}$-sets, and also T_ω-space in terms of generalized $\wedge_\tau$ (in short $g\wedge_\tau$)-sets and λ^*-closed sets. Besides, introducing few new separation axioms viz. $T_{\frac{\omega}{4}}, T_{\frac{3\omega}{8}}$ and $T_{\frac{5\omega}{8}}$ in Alexandroff spaces we have shown that a T_ω space is $T_{\frac{5\omega}{8}}$, a $T_{\frac{5\omega}{8}}$ space is $T_{\frac{3\omega}{8}}$, a $T_{\frac{3\omega}{8}}$ space is $T_{\frac{\omega}{4}}$ and a $T_{\frac{\omega}{4}}$ space is T_0 but the reverse implications may not hold which are substantiated by suitable examples. We have seen in [5] that T_ω and T_1 axioms in a space are independent of each other. But we have obtained a result showing that T_0, T_1 and all the above axioms are equivalent under additional three conditions viz. the space is (i) strongly symmetric (ii) weak R_0 and (iii) it satisfies the condition $C = C^*$.

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