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A NOVEL APPROACH TO INTUITIONISTIC FUZZY SETS IN UP-ALGEBRAS

Nattaporn Thongngam and Aiyared Iampan*†

Abstract. The notions of intuitionistic fuzzy UP-subalgebras and intuitionistic fuzzy UP-ideals of UP-algebras were introduced by Kesorn et al. [13]. In this paper, we introduce the notions of intuitionistic fuzzy near UP-filters, intuitionistic fuzzy UP-filters, and intuitionistic fuzzy strong UP-ideals of UP-algebras, prove their generalizations, and investigate their basic properties. Furthermore, we discuss the relations between intuitionistic fuzzy near UP-filters (resp., intuitionistic fuzzy UP-filters, intuitionistic fuzzy strong UP-ideals) and their upper t-(strong) level subsets and lower t-(strong) level subsets in UP-algebras.

1. Introduction and Preliminaries

The fundamental concept of fuzzy sets in a set was first introduced by Zadeh [31] in 1965. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere. The concept of intuitionistic fuzzy sets was first published

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∗ Corresponding author.

by Atanassov in his pioneer papers [3, 4], as generalization of the notion of fuzzy sets. Several researches were conducted on the generalizations of the notion of intuitionistic fuzzy sets and application to many logical algebras. In 2000, Jun and Kim [12] introduced the notion of equivalence relations on the family of all intuitionistic fuzzy ideals of BCK-algebras. In 2004, Zhan and Tan [32] introduced the notion of intuitionistic fuzzy α-ideals of BCI-algebras. In 2006, Kim and Jeong [15] introduced the notion of intuitionistic fuzzy subalgebras of B-algebras. In 2007, Kim [14] introduced the notion of intuitionistic (T, S)-normed fuzzy subalgebras in BCK/BCI-algebras. In 2011, Mostafa et al. [17] introduced the intuitionistic fuzzification of the concept of KU-ideals and the image (preimage) of KU-ideals in KU-algebras. Satyanarayana and Prasad [24] studied the intuitionistic fuzzy implicative ideals, intuitionistic fuzzy positive implicative ideals and intuitionistic fuzzy commutative ideals in BCK-algebras. In 2012, Malik and Touqeer [16] introduced the intuitionistic fuzzification of the concept of BCI-commutative ideals of BCI-algebras. In 2013, Palaniappan et al. [18] introduced the notion of intuitionistic fuzzy n-fold positive implicative ideals of BCI-algebras. In 2014, Sun and Li [29] introduced the notions of intuitionistic fuzzy subalgebras with thresholds (λ, μ) and intuitionistic fuzzy ideals with thresholds (λ, μ) of BCI-algebras. Bhowmik et al. [5] introduced the notion of intuitionistic L-fuzzy closed ideals of BG-algebras and investigated the product of intuitionistic L-fuzzy BG-algebras. In 2015, Kesorn et al. [13] introduced the notions of intuitionistic fuzzy UP-ideals and intuitionistic fuzzy UP-subalgebras of UP-algebras. Senapati et al. [25] introduced the concepts of intuitionistic fuzzy translation to intuitionistic fuzzy subalgebras and ideals in BCK/BCI-algebras. The notion of intuitionistic fuzzy extensions and intuitionistic fuzzy multiplications of intuitionistic fuzzy subalgebras and ideals are introduced and several related properties are investigated. Jana et al. [11] studied intuitionistic fuzzy G-subalgebras of G-algebras. In 2019, Senapati and Shum [26] introduced the notion of interval-valued intuitionistic fuzzy KU-subalgebras of KU-algebras and studied the image and the inverse image of interval-valued intuitionistic fuzzy KU-subalgebras.

In this paper, we introduce the notions of intuitionistic fuzzy near UP-filters, intuitionistic fuzzy UP-filters, and intuitionistic fuzzy strong UP-ideals of UP-algebras, prove their generalizations, and investigate their basic properties. Furthermore, we discuss the relations between intuitionistic fuzzy near UP-filters (resp., intuitionistic fuzzy UP-filters, intuitionistic fuzzy strong UP-ideals) and their upper t-(strong) level subsets and lower t-(strong) level subsets in UP-algebras.

Before we begin our study, we will give the definition of UP-algebras.

Definition 1.1. [8] An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra*, where A is a nonempty set, $\cdot$ is a binary operation on A, and 0 is a fixed element of A (i.e., a nullary operation) if it satisfies the following axioms:

(UP-1): $(\forall x, y, z \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,
(UP-2): $(\forall x \in A)(0 \cdot x = x)$,
(UP-3): $(\forall x \in A)(x \cdot 0 = 0)$, and
(UP-4): $(\forall x, y \in A)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y)$.

From [8], we know that the notion of UP-algebras is a generalization of KU-algebras (see [19]).

Example 1.2. [23] Let X be a universal set and let $\Omega \in \mathcal{P}(X)$, where $\mathcal{P}(X)$ means the power set of X. Let $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. Define a binary operation $\cdot$ on $\mathcal{P}_\Omega(X)$ by putting $A \cdot B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(X)$, where A^C means the complement of a subset A. Then $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 1 with respect to* Ω. Let $\mathcal{P}^\Omega(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$. Define a binary operation $*$ on $\mathcal{P}^\Omega(X)$ by putting $A * B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(X)$. Then $(\mathcal{P}^\Omega(X), *, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 2 with respect to* Ω. In particular, $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra and we shall call it the *power UP-algebra of type 1*, and $(\mathcal{P}(X), *, X)$ is a UP-algebra and we shall call it the *power UP-algebra of type 2*.

Example 1.3. [6] Let $\mathbb{N}$ be the set of all natural numbers with two binary operations $\circ$ and $\bullet$ defined by

$$(\forall x, y \in \mathbb{N}) \left(x \circ y = \begin{cases} y & \text{if } x < y, \\ 0 & \text{otherwise} \end{cases} \right)$$

and

$$(\forall x, y \in \mathbb{N}) \left(x \bullet y = \begin{cases} y & \text{if } x > y \text{ or } x = 0, \\ 0 & \text{otherwise} \end{cases} \right).$$

Then $(\mathbb{N}, \circ, 0)$ and $(\mathbb{N}, \bullet, 0)$ are UP-algebras.

Example 1.4. [28] Let $A = \{0, 1, 2, 3, 4, 5, 6\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	0	0	2	3	2	3	6
2	0	1	0	3	1	5	3
3	0	1	2	0	4	1	2
4	0	0	0	3	0	3	3
5	0	0	2	0	2	0	2
6	0	1	0	0	1	1	0

Then $(A, \cdot, 0)$ is a UP-algebra.

For more examples of UP-algebras, see [1, 2, 9, 22, 23].

In a UP-algebra $A = (A, \cdot, 0)$, the following assertions are valid (see [8, 9]).

(1.1) $(\forall x \in A)(x \cdot x = 0)$,

(1.2) $(\forall x, y, z \in A)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0)$,

(1.3) $(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0)$,

(1.4) $(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0)$,

(1.5) $(\forall x, y \in A)(x \cdot (y \cdot x) = 0)$,

(1.6) $(\forall x, y \in A)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x)$,

(1.7) $(\forall x, y \in A)(x \cdot (y \cdot y) = 0)$,

(1.8) $(\forall a, x, y, z \in A)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0)$,

(1.9) $(\forall a, x, y, z \in A)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0)$,

(1.10) $(\forall x, y, z \in A)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0)$,

(1.11) $(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0)$,

(1.12) $(\forall x, y, z \in A)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0)$, and

(1.13) $(\forall a, x, y, z \in A)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0)$.

From [8], the binary relation $\leq$ on a UP-algebra $A = (A, \cdot, 0)$ defined as follows:

$$(\forall x, y \in A)(x \leq y \Leftrightarrow x \cdot y = 0).$$

Definition 1.5. [7, 8, 10, 27] A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is called

(1) a *UP-subalgebra* of A if $(\forall x, y \in S)(x \cdot y \in S)$.
(2) a *near UP-filter* of A if it satisfies the following properties:
  (i) the constant 0 of A is in S, and
  (ii) $(\forall x, y \in A)(y \in S \Rightarrow x \cdot y \in S)$.
(3) a *UP-filter* of A if it satisfies the following properties:
  (i) the constant 0 of A is in S, and
  (ii) $(\forall x, y \in A)(x \cdot y \in S, x \in S \Rightarrow y \in S)$.
(4) a *UP-ideal* of A if it satisfies the following properties:
  (i) the constant 0 of A is in S, and
  (ii) $(\forall x, y, z \in A)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$.
(5) a *strong UP-ideal* (renamed from a strongly UP-ideal) of A if it satisfies the following properties:
  (i) the constant 0 of A is in S, and
  (ii) $(\forall x, y, z \in A)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S)$.

We know that the notion of UP-subalgebras is a generalization of near UP-filters, the notion of near UP-filters is a generalization of UP-filters, the notion of UP-filters is a generalization of UP-ideals, and the notion of UP-ideals is a generalization of strong UP-ideals. Moreover, they also proved that a UP-algebra A is the only one strong UP-ideal of itself.

Theorem 1.6. *[7,8,20] Let $\mathscr{F}$ be a nonempty family of UP-subalgebras (resp., near UP-filters, UP-filters, UP-ideals, strong UP-ideals) of a UP-algebra $A = (A, \cdot, 0)$. Then $\bigcap \mathscr{F}$ is a UP-subalgebra (resp., near UP-filter, UP-filter, UP-ideal, strong UP-ideal) of A.*

2. Properties of IFSs

In this section, firstly, we recall the definition of fuzzy sets in a nonempty set and the definitions of fuzzy UP-subalgebras, fuzzy near UP-filters, fuzzy UP-filters, fuzzy UP-ideals, and fuzzy strong UP-ideals of a UP-algebra. Secondly, we introduce the notions of intuitionistic fuzzy near UP-filters, intuitionistic fuzzy UP-filters, and intuitionistic fuzzy strong UP-ideals of UP-algebras and study some of their basic properties.

Definition 2.1. [31] A *fuzzy set* (briefly, FS) in a nonempty set X (or a fuzzy subset of X) is an arbitrary function $f \colon X \to [0, 1]$, where $[0, 1]$ is the unit segment of the real line. If $S \subseteq X$, the *characteristic* *function* f_S of X is a function of X into $\{0,1\}$ defined as follows:

$$f_S(x) = \begin{cases} 1 & \text{if } x \in S, \\ 0 & \text{otherwise.} \end{cases}$$

By the definition of characteristic function, f_S is a function of X into $\{0,1\} \subset [0,1]$. Hence, f_S is a FS in X. If f is a FS in X, the FS $\overline{f}$ defined by $\overline{f}(x) = 1 - f(x)$ for all $x \in X$ is called the *complement* of f in X.

Definition 2.2. [7,21,27] A fuzzy set f in a UP-algebra $A = (A, \cdot, 0)$ is called

(1) a *fuzzy UP-subalgebra* of A if $(\forall x, y \in A)(f(x \cdot y) \geq \min\{f(x), f(y)\})$.
(2) a *fuzzy near UP-filter* of A if
  (i) $(\forall x \in A)(f(0) \geq f(x))$, and
  (ii) $(\forall x, y \in A)(f(x \cdot y) \geq f(y))$.
(3) a *fuzzy UP-filter* of A if
  (i) $(\forall x \in A)(f(0) \geq f(x))$, and
  (ii) $(\forall x, y \in A)(f(y) \geq \min\{f(x \cdot y), f(x)\})$.
(4) a *fuzzy UP-ideal* of A if
  (i) $(\forall x \in A)(f(0) \geq f(x))$, and
  (ii) $(\forall x, y, z \in A)(f(x \cdot z) \geq \min\{f(x \cdot (y \cdot z)), f(y)\})$.
(5) a *fuzzy strong UP-ideal* (renamed from a fuzzy strongly UP-ideal) of A if
  (i) $(\forall x \in A)(f(0) \geq f(x))$, and
  (ii) $(\forall x, y, z \in A)(f(x) \geq \min\{f((z \cdot y) \cdot (z \cdot x)), f(y)\})$.

We know that the notion of fuzzy UP-subalgebras is a generalization of fuzzy near UP-filters, the notion of fuzzy near UP-filters is a generalization of fuzzy UP-filters, the notion of fuzzy UP-filters is a generalization of fuzzy UP-ideals, and the notion of fuzzy UP-ideals is a generalization of fuzzy strong UP-ideals. Moreover, fuzzy strong UP-ideals and constant fuzzy sets coincide in UP-algebras.

Theorem 2.3. *[27] A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is a UP-subalgebra (resp., UP-filter, UP-ideal) of A if and only if the FS f_S is a fuzzy UP-subalgebra (resp., fuzzy UP-filter, fuzzy UP-ideal) of A.*

Theorem 2.4. *A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is a near UP-filter of A if and only if the FS f_S is a fuzzy near UP-filter of A.*

Proof. Assume that S is a near UP-filter of A. Since $0 \in S$, we have $f_S(0) = 1 \geq f_S(x)$ for all $x \in A$. Next, let $x, y \in A$.
Case 1: $y \in S$. Since S is a near UP-filter of A, we have $x \cdot y \in S$ and so $f_S(x \cdot y) = 1$ and $f_S(y) = 1$. Thus $f_S(x \cdot y) \geq f_S(y)$.
Case 2: $y \notin S$. Then $f_S(x \cdot y) \geq 0 = f_S(y)$.
Hence, f_S is a fuzzy near UP-filter of A.

Conversely, assume that f_S is a fuzzy near UP-filter of A. Since $f_S(0) \geq f_S(x)$ for all $x \in A$, we have $0 \in S$. Next, let $x, y \in A$ be such that $y \in S$. Then $f_S(x \cdot y) \geq f_S(y) = 1$, so $f_S(x \cdot y) = 1$. Thus $x \cdot y \in S$ and so S is a near UP-filter of A. □

THEOREM 2.5. *A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is a strong UP-ideal of A if and only if the FS f_S is a fuzzy strong UP-ideal of A.*

Proof. Let S be a nonempty subset of A. Then

$$\begin{aligned} S \text{ is a strong UP-ideal of } A &\Leftrightarrow S = A \\ &\Leftrightarrow f_S = f_A \text{ is constant} \\ &\Leftrightarrow f_S \text{ is a fuzzy strong UP-ideal of } A. \end{aligned}$$

□

DEFINITION 2.6. [3,4] An *intuitionistic fuzzy set* (briefly, IFS) in a nonempty set X is an object F having the form

$$F = \{(x, \mu_F(x), \gamma_F(x)) \mid x \in X\},$$

where the FSs $\mu_F \colon X \to [0, 1]$ and $\gamma_F \colon X \to [0, 1]$ denote the degree of membership and the degree of nonmembership, respectively, and

$$(\forall x \in X)(0 \leq \mu_F(x) + \gamma_F(x) \leq 1).$$

An IFS $F = \{(x, \mu_F(x), \gamma_F(x)) \mid x \in X\}$ in X can be identified to an ordered pair (μ_F, γ_F) in $[0, 1]^X \times [0, 1]^X$. For the sake of simplicity, we shall use the symbol $F = (\mu_F, \gamma_F)$ for the IFS $F = \{(x, \mu_F(x), \gamma_F(x)) \mid x \in X\}$.

DEFINITION 2.7. [13] An IFS $F = (\mu_F, \gamma_F)$ in a UP-algebra $A = (A, \cdot, 0)$ is called an *intuitionistic fuzzy UP-subalgebra* of A if it satisfies the following axioms:

(1) $(\forall x, y \in A)(\mu_F(x \cdot y) \geq \min\{\mu_F(x), \mu_F(y)\})$, and
(2) $(\forall x, y \in A)(\gamma_F(x \cdot y) \leq \max\{\gamma_F(x), \gamma_F(y)\})$.

Example 2.8. Let $A = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	0
2	0	1	0	0	4
3	0	1	2	0	4
4	0	4	2	3	0

Let $F = (\mu_F, \gamma_F)$ be an IFS in A defined by

$$\mu_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.3 & 0.1 & 0.2 & 0.5 \end{pmatrix}$$

and

$$\gamma_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.25 & 0.7 & 0.8 & 0.5 \end{pmatrix}.$$

Then $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-subalgebra of A.

Definition 2.9. An IFS $F = (\mu_F, \gamma_F)$ in a UP-algebra $A = (A, \cdot, 0)$ is called an *intuitionistic fuzzy near UP-filter* of A if it satisfies the following axioms:

(1) $(\forall x \in A)(\mu_F(0) \geq \mu_F(x))$,
(2) $(\forall x, y \in A)(\mu_F(x \cdot y) \geq \mu_F(y))$,
(3) $(\forall x \in A)(\gamma_F(0) \leq \gamma_F(x))$, and
(4) $(\forall x, y \in A)(\gamma_F(x \cdot y) \leq \gamma_F(y))$.

Example 2.10. Let $A = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	3	0
3	0	0	2	0	4
4	0	0	1	3	0

Let $F = (\mu_F, \gamma_F)$ be an IFS in A defined by

$$\mu_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.3 & 0.2 & 0.7 & 0.45 \end{pmatrix}$$

and

$$\gamma_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0.2 & 0.8 & 0.2 & 0.4 \end{pmatrix}.$$

Then $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A.

DEFINITION 2.11. An IFS $F = (\mu_F, \gamma_F)$ in a UP-algebra $A = (A, \cdot, 0)$ is called an *intuitionistic fuzzy UP-filter* of A if it satisfies the following axioms:

(1) $(\forall x \in A)(\mu_F(0) \geq \mu_F(x))$,
(2) $(\forall x, y \in A)(\mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\})$,
(3) $(\forall x \in A)(\gamma_F(0) \leq \gamma_F(x))$, and
(4) $(\forall x, y \in A)(\gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\})$.

EXAMPLE 2.12. Let $A = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	2
3	0	1	0	0

Let $F = (\mu_F, \gamma_F)$ be an IFS in A defined by

$$\mu_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.7 & 0.6 & 0.3 & 0.3 \end{pmatrix} \text{ and } \gamma_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix}.$$

Then $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A.

DEFINITION 2.13. [13] An IFS $F = (\mu_F, \gamma_F)$ in a UP-algebra $A = (A, \cdot, 0)$ is called an *intuitionistic fuzzy UP-ideal* of A if it satisfies the following axioms:

(1) $(\forall x \in A)(\mu_F(0) \geq \mu_F(x))$,
(2) $(\forall x, y, z \in A)(\mu_F(x \cdot z) \geq \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\})$,
(3) $(\forall x \in A)(\gamma_F(0) \leq \gamma_F(x))$, and
(4) $(\forall x, y, z \in A)(\gamma_F(x \cdot z) \leq \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\})$.

EXAMPLE 2.14. Let $A = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	2	0

Let $F = (\mu_F, \gamma_F)$ be an IFS in A defined by

$$\mu_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.9 & 0.3 & 0.1 & 0.6 \end{pmatrix} \text{ and } \gamma_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.1 & 0.2 & 0.8 & 0.3 \end{pmatrix}.$$

Then $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-ideal of A.

DEFINITION 2.15. An IFS $F = (\mu_F, \gamma_F)$ in a UP-algebra $A = (A, \cdot, 0)$ is called an *intuitionistic fuzzy strong UP-ideal* of A if it satisfies the following axioms:

(1) $(\forall x \in A)(\mu_F(0) \geq \mu_F(x))$,
(2) $(\forall x, y, z \in A)(\mu_F(x) \geq \min\{\mu_F((z \cdot y) \cdot (z \cdot x)), \mu_F(y)\})$,
(3) $(\forall x \in A)(\gamma_F(0) \leq \gamma_F(x))$, and
(4) $(\forall x, y, z \in A)(\gamma_F(x) \leq \max\{\gamma_F((z \cdot y) \cdot (z \cdot x)), \gamma_F(y)\})$.

EXAMPLE 2.16. From Example 1.4, let $F = (\mu_F, \gamma_F)$ be an IFS in A defined by

$$(\forall x \in A)(\mu_F(x) = 0.3) \text{ and } (\forall x \in A)(\gamma_F(x) = 0.1).$$

Then $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong UP-ideal of A.

THEOREM 2.17. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong UP-ideal of a UP-algebra $A = (A, \cdot, 0)$ if and only if the FSs μ_F and γ_F are constant.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong UP-ideal of A. Then $\mu_F(0) \geq \mu_F(x)$ and $\gamma_F(0) \leq \gamma_F(x)$ for all $x \in A$. Let $x \in A$. Then

Definition 2.15 (2)	$\mu_F(x) \geq \min\{\mu_F((x \cdot 0) \cdot (x \cdot x)), \mu_F(0)\}$
(UP-3)	$= \min\{\mu_F(0 \cdot (x \cdot x)), \mu_F(0)\}$
(UP-2)	$= \min\{\mu_F(x \cdot x), \mu_F(0)\}$
(1.1)	$= \min\{\mu_F(0), \mu_F(0)\}$
	$= \mu_F(0)$

and

Definition 2.15 (4)	$\gamma_F(x) \leq \max\{\gamma_F((x \cdot 0) \cdot (x \cdot x)), \gamma_F(0)\}$
(UP-3)	$= \max\{\gamma_F(0 \cdot (x \cdot x)), \gamma_F(0)\}$
(UP-2)	$= \max\{\gamma_F(x \cdot x), \gamma_F(0)\}$
(1.1)	$= \max\{\gamma_F(0), \gamma_F(0)\}$
	$= \gamma_F(0)$.

Thus $\mu_F(x) = \mu_F(0)$ and $\gamma_F(x) = \gamma_F(0)$ for all $x \in A$, that is, μ_F and γ_F are constant.

Conversely, assume that μ_F and γ_F are constant. Then obviously $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong UP-ideal of A. □

THEOREM 2.18. *Every intuitionistic fuzzy strong UP-ideal of a UP-algebra $A = (A, \cdot, 0)$ is an intuitionistic fuzzy UP-ideal.*

Proof. Let $F = (\mu_F, \gamma_F)$ be an intuitionistic fuzzy strong UP-ideal of A and let $x, y, z \in A$. Then

Definition 2.15 (2)	$\mu_F(x \cdot z) \geq \min\{\mu_F((z \cdot y) \cdot (z \cdot (x \cdot z))), \mu_F(y)\}$
(1.5)	$= \min\{\mu_F((z \cdot y) \cdot 0), \mu_F(y)\}$
(UP-3)	$= \min\{\mu_F(0), \mu_F(y)\}$
Definition 2.15 (1)	$= \mu_F(y)$
	$\geq \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\}$

and

Definition 2.15 (4)	$\gamma_F(x \cdot z) \leq \max\{\gamma_F((z \cdot y) \cdot (z \cdot (x \cdot z))), \gamma_F(y)\}$
(1.5)	$= \max\{\gamma_F((z \cdot y) \cdot 0), \gamma_F(y)\}$
(UP-3)	$= \max\{\gamma_F(0), \gamma_F(y)\}$
Definition 2.15 (3)	$= \gamma_F(y)$
	$\leq \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\}$.

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-ideal of A. □

The following example show that the converse of Theorem 2.18 is not true in general.

EXAMPLE 2.19. From Example 2.14, we have $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-ideal of A. By Theorem 2.17, we have $F = (\mu_F, \gamma_F)$ is not an intuitionistic fuzzy strong UP-ideal of A.

Theorem 2.20. *Every intuitionistic fuzzy UP-ideal of a UP-algebra* $A = (A, \cdot, 0)$ *is an intuitionistic fuzzy UP-filter.*

Proof. Let $F = (\mu_F, \gamma_F)$ be an intuitionistic fuzzy UP-ideal of A and let $x, y \in A$. Then

(UP-2)	$\mu_F(y) = \mu_F(0 \cdot y)$
Definition 2.13 (3)	$\geq \min\{\mu_F(0 \cdot (x \cdot y)), \mu_F(x)\}$
(UP-2)	$= \min\{\mu_F(x \cdot y), \mu_F(x)\}$

and

(UP-2)	$\gamma_F(y) = \gamma_F(0 \cdot y)$
Definition 2.13 (4)	$\leq \max\{\gamma_F(0 \cdot (x \cdot y)), \gamma_F(x)\}$
(UP-2)	$= \max\{\gamma_F(x \cdot y), \gamma_F(x)\}$.

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. □

The following example show that the converse of Theorem 2.20 is not true in general.

Example 2.21. From Example 2.12, we have $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. Since $\mu_F(2 \cdot 3) = 0.3 < 0.6 = \min\{0.7, 0.6\} = \min\{\mu_F(2 \cdot (1 \cdot 3)), \mu_F(1)\}$, we have $F = (\mu_F, \gamma_F)$ is not an intuitionistic fuzzy UP-ideal of A.

Theorem 2.22. *Every intuitionistic fuzzy UP-filter of a UP-algebra* $A = (A, \cdot, 0)$ *is an intuitionistic fuzzy near UP-filter.*

Proof. Let $F = (\mu_F, \gamma_F)$ be an intuitionistic fuzzy UP-filter of A and let $x, y \in A$. Then

Definition 2.11 (2)	$\mu_F(x \cdot y) \geq \min\{\mu_F(y \cdot (x \cdot y)), \mu_F(y)\}$
(1.5)	$= \min\{\mu_F(0), \mu_F(y)\}$
Definition 2.11 (1)	$= \mu_F(y)$

and

Definition 2.11 (4)	$\gamma_F(x \cdot y) \leq \max\{\gamma_F(y \cdot (x \cdot y)), \gamma_F(y)\}$
(1.5)	$= \max\{\gamma_F(0), \gamma_F(y)\}$
Definition 2.11 (3)	$= \gamma_F(y)$.

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A. □

The following example show that the converse of Theorem 2.22 is not true in general.

EXAMPLE 2.23. From Example 2.10, we have $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A. Since $\gamma_F(2) = 0.8 > 0.4 = \max\{0.2, 0.4\} = \max\{\gamma_F(4 \cdot 2), \gamma_F(4)\}$, we have $F = (\mu_F, \gamma_F)$ is not an intuitionistic fuzzy UP-filter of A.

THEOREM 2.24. *Every intuitionistic fuzzy near UP-filter of a UP-algebra $A = (A, \cdot, 0)$ is an intuitionistic fuzzy UP-subalgebra.*

Proof. Let $F = (\mu_F, \gamma_F)$ be an intuitionistic fuzzy near UP-filter of A and let $x, y \in A$. Then

Definition 2.9 (2) $\qquad \mu_F(x \cdot y) \geq \mu_F(y) \geq \min\{\mu_F(x), \mu_F(y)\}$

and

Definition 2.9 (4) $\qquad \gamma_F(x \cdot y) \leq \gamma_F(y) \leq \max\{\gamma_F(x), \gamma_F(y)\}.$

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-subalgebra of A. $\square$

The following example show that the converse of Theorem 2.24 is not true in general.

EXAMPLE 2.25. From Example 2.8, we have $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-subalgebra of A. Since $\gamma_F(4 \cdot 1) = \gamma_F(4) = 0.5 > 0.25 = \gamma_F(1)$, we have $F = (\mu_F, \gamma_F)$ is not an intuitionistic fuzzy near UP-filter of A.

The following lemmas and corollaries are the interesting properties of each type of IFSs in UP-algebras.

LEMMA 2.26. *[13] Every intuitionistic fuzzy UP-subalgebra $F = (\mu_F, \gamma_F)$ of a UP-algebra $A = (A, \cdot, 0)$ satisfies the following axioms:*

(1) $(\forall x \in A)(\mu_F(0) \geq \mu_F(x))$, *and*
(2) $(\forall x \in A)(\gamma_F(0) \leq \gamma_F(x))$.

LEMMA 2.27. *Every intuitionistic fuzzy UP-filter $F = (\mu_F, \gamma_F)$ of a UP-algebra $A = (A, \cdot, 0)$ satisfies the following axioms:*

(1) $(\forall x, y, z \in A)(x \leq y \cdot z \Rightarrow \mu_F(z) \geq \min\{\mu_F(x), \mu_F(y)\})$, *and*
(2) $(\forall x, y, z \in A)(x \leq y \cdot z \Rightarrow \gamma_F(z) \leq \max\{\gamma_F(x), \gamma_F(y)\})$.

Proof. (1) Let $x, y, z \in A$ be such that $x \leq y \cdot z$. Then $x \cdot (y \cdot z) = 0$. Thus

$$
\begin{aligned}
&\text{Definition 2.11 (2)} \quad && \mu_F(z) \geq \min\{\mu_F(y \cdot z), \mu_F(y)\} \\
&\text{Definition 2.11 (2)} && \geq \min\{\min\{\mu_F(x \cdot (y \cdot z)), \mu_F(x)\}, \mu_F(y)\} \\
& && = \min\{\min\{\mu_F(0), \mu_F(x)\}, \mu_F(y)\} \\
&\text{Definition 2.11 (1)} && = \min\{\mu_F(x), \mu_F(y)\}.
\end{aligned}
$$

(2) Let $x, y, z \in A$ be such that $x \leq y \cdot z$. Then $x \cdot (y \cdot z) = 0$. Thus

$$
\begin{aligned}
&\text{Definition 2.11 (4)} \quad && \gamma_F(z) \leq \max\{\gamma_F(y \cdot z), \gamma_F(y)\} \\
&\text{Definition 2.11 (4)} && \leq \max\{\max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(x)\}, \gamma_F(y)\} \\
& && = \max\{\max\{\gamma_F(0), \gamma_F(x)\}, \gamma_F(y)\} \\
&\text{Definition 2.11 (3)} && = \max\{\gamma_F(x), \gamma_F(y)\}.
\end{aligned}
$$

□

COROLLARY 2.28. *Every intuitionistic fuzzy UP-filter $F = (\mu_F, \gamma_F)$ of a UP-algebra $A = (A, \cdot, 0)$ satisfies the following axioms:*

(1) $(\forall x, y \in A)(x \leq y \Rightarrow \mu_F(x) \leq \mu_F(y))$, *and*
(2) $(\forall x, y \in A)(x \leq y \Rightarrow \gamma_F(x) \geq \gamma_F(y))$.

Proof. Replacing y by 0 and z by y in Lemma 2.27. □

LEMMA 2.29. *[13] Every intuitionistic fuzzy UP-ideal $F = (\mu_F, \gamma_F)$ of a UP-algebra $A = (A, \cdot, 0)$ satisfies the following axioms:*

(1) $(\forall w, x, y, z \in A)(x \leq w \cdot (y \cdot z) \Rightarrow \mu_F(x \cdot z) \geq \min\{\mu_F(w), \mu_F(y)\})$, *and*
(2) $(\forall w, x, y, z \in A)(x \leq w \cdot (y \cdot z) \Rightarrow \gamma_F(x \cdot z) \leq \max\{\gamma_F(w), \gamma_F(y)\})$.

COROLLARY 2.30. *[13] Every intuitionistic fuzzy UP-ideal $F = (\mu_F, \gamma_F)$ of a UP-algebra $A = (A, \cdot, 0)$ satisfies the following axioms:*

(1) $(\forall x, y, z \in A)(x \leq y \cdot z \Rightarrow \mu_F(x \cdot z) \geq \mu_F(y))$, *and*
(2) $(\forall x, y, z \in A)(x \leq y \cdot z \Rightarrow \gamma_F(x \cdot z) \leq \gamma_F(y))$.

LEMMA 2.31. *[30] Let $a, b, c \in \mathbb{R}$. Then the following statements hold:*

(1) $a - \min\{b, c\} = \max\{a - b, a - c\}$, *and*
(2) $a - \max\{b, c\} = \min\{a - b, a - c\}$.

Now, we discuss the relations between IFSs and FSs in UP-algebras.

THEOREM 2.32. *If an IFS* $F = (\mu_F, \gamma_F)$ *is an intuitionistic fuzzy strong UP-ideal of a UP-algebra* $A = (A, \cdot, 0)$, *then the FSs* $\mu_F, \gamma_F, \overline{\mu}_F$, *and* $\overline{\gamma}_F$ *are fuzzy strong UP-ideals of* A.

Proof. Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong UP-ideal of A. By Theorem 2.17, we have μ_F and γ_F are constant and so $\overline{\mu}_F$ and $\overline{\gamma}_F$ are constant. Hence, $\mu_F, \gamma_F, \overline{\mu}_F$, and $\overline{\gamma}_F$ are fuzzy strong UP-ideals of A. □

THEOREM 2.33. *[13] An IFS* $F = (\mu_F, \gamma_F)$ *is an intuitionistic fuzzy UP-ideal of a UP-algebra* $A = (A, \cdot, 0)$ *if and only if the FSs* μ_F *and* $\overline{\gamma}_F$ *are fuzzy UP-ideals of* A.

THEOREM 2.34. *An IFS* $F = (\mu_F, \gamma_F)$ *is an intuitionistic fuzzy UP-filter of a UP-algebra* $A = (A, \cdot, 0)$ *if and only if the FSs* μ_F *and* $\overline{\gamma}_F$ *are fuzzy UP-filters of* A.

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. Then for any $x, y \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\}.$$

Hence, μ_F is a fuzzy UP-filter of A. Now, for any $x, y \in A$, we have

$$\gamma_F(0) \leq \gamma_F(x) \text{ and } \gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\}.$$

Thus $\overline{\gamma}_F(0) = 1 - \gamma_F(0) \geq 1 - \gamma_F(x) = \overline{\gamma}_F(x)$ and

$$\begin{aligned} \overline{\gamma}_F(y) &= 1 - \gamma_F(y) \\ &\geq 1 - \max\{\gamma_F(x \cdot y), \gamma_F(x)\} \\ \text{Lemma 2.31 (2)} \quad &= \min\{1 - \gamma_F(x \cdot y), 1 - \gamma_F(x)\} \\ &= \min\{\overline{\gamma}_F(x \cdot y), \overline{\gamma}_F(x)\}. \end{aligned}$$

Hence, $\overline{\gamma}_F$ is a fuzzy UP-filter of A.

Conversely, assume that the FSs μ_F and $\overline{\gamma}_F$ are fuzzy UP-filters of A. Then for any $x, y \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\}.$$

Now, for any $x, y \in A$, we have

$$\overline{\gamma}_F(0) \geq \overline{\gamma}_F(x) \text{ and } \overline{\gamma}_F(y) \geq \min\{\overline{\gamma}_F(x \cdot y), \overline{\gamma}_F(x)\}.$$

Thus $1 - \gamma_F(0) \geq 1 - \gamma_F(x)$, so $\gamma_F(0) \leq \gamma_F(x)$. Now,

Lemma 2.31 (2)

$1 - \gamma_F(y) \geq \min\{1 - \gamma_F(x \cdot y), 1 - \gamma_F(x)\} = 1 - \max\{\gamma_F(x \cdot y), \gamma_F(x)\}$, so $\gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\}$. Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. □

THEOREM 2.35. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of a UP-algebra $A = (A, \cdot, 0)$ if and only if the FSs μ_F and $\overline{\gamma}_F$ are fuzzy near UP-filters of A.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic near fuzzy UP-filter of A. Then for any $x, y \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(x \cdot y) \geq \mu_F(y).$$

Hence, μ_F is a fuzzy near UP-filter of A. Now, for any $x, y \in A$, we have

$$\gamma_F(0) \leq \gamma_F(x) \text{ and } \gamma_F(x \cdot y) \leq \gamma_F(y).$$

Thus $\overline{\gamma}_F(0) = 1 - \gamma_F(0) \geq 1 - \gamma_F(x) = \overline{\gamma}_F(x)$ and

$$\overline{\gamma}_F(x \cdot y) = 1 - \gamma_F(x \cdot y) \geq 1 - \gamma_F(y) = \overline{\gamma}_F(y).$$

Hence, $\overline{\gamma}_F$ is a fuzzy near UP-filter of A.

Conversely, assume that the FSs μ_F and $\overline{\gamma}_F$ are fuzzy near UP-filters of A. Then for any $x, y \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(x \cdot y) \geq \mu_F(y).$$

Now, for any $x, y \in A$, we have

$$\overline{\gamma}_F(0) \geq \overline{\gamma}_F(x) \text{ and } \overline{\gamma}_F(x \cdot y) \geq \overline{\gamma}_F(y).$$

Thus $1 - \gamma_F(0) \geq 1 - \gamma_F(x)$, so $\gamma_F(0) \leq \gamma_F(x)$. Now,

$$1 - \gamma_F(x \cdot y) \geq 1 - \gamma_F(y),$$

so $\gamma_F(x \cdot y) \leq \gamma_F(y)$. Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A. □

THEOREM 2.36. [13] *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-subalgebra of a UP-algebra $A = (A, \cdot, 0)$ if and only if the FSs μ_F and $\overline{\gamma}_F$ are fuzzy UP-subalgebras of A.*

For a subset S of a nonempty set X, we denote the IFS in X with the degree of membership f_S and the degree of nonmembership $\overline{f}_S$ by F_S.

THEOREM 2.37. *A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is a strong UP-ideal of A if and only if the IFS $F_S = (f_S, \overline{f}_S)$ is an intuitionistic fuzzy strong UP-ideal of A.*

Proof. It is straightforward from Theorems 2.5 and 2.17. □

THEOREM 2.38. *A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is a UP-ideal of A if and only if the IFS $F_S = (f_S, \overline{f}_S)$ is an intuitionistic fuzzy UP-ideal of A.*

Proof. It is straightforward from Theorems 2.3 and 2.33. $\square$

THEOREM 2.39. *A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is a UP-filter of A if and only if the IFS $F_S = (f_S, \overline{f}_S)$ is an intuitionistic fuzzy UP-filter of A.*

Proof. It is straightforward from Theorems 2.3 and 2.34. $\square$

THEOREM 2.40. *A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is a near UP-filter of A if and only if the IFS $F_S = (f_S, \overline{f}_S)$ is an intuitionistic fuzzy near UP-filter of A.*

Proof. It is straightforward from Theorems 2.4 and 2.35. $\square$

THEOREM 2.41. *A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is a UP-subalgebra of A if and only if the IFS $F_S = (f_S, \overline{f}_S)$ is an intuitionistic fuzzy UP-subalgebra of A.*

Proof. It is straightforward from Theorems 2.3 and 2.36. $\square$

THEOREM 2.42. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong UP-ideal of a UP-algebra $A = (A, \cdot, 0)$ if and only if the IFSs $\Box F = (\mu_F, \overline{\mu}_F)$ and $\Diamond F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy strong UP-ideals of A.*

Proof. It is straightforward from Theorem 2.17. $\square$

THEOREM 2.43. [13] *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-ideal of a UP-algebra $A = (A, \cdot, 0)$ if and only if the IFSs $\Box F = (\mu_F, \overline{\mu}_F)$ and $\Diamond F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy UP-ideals of A.*

THEOREM 2.44. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of a UP-algebra $A = (A, \cdot, 0)$ if and only if the IFSs $\Box F = (\mu_F, \overline{\mu}_F)$ and $\Diamond F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy UP-filters of A.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. Then for any $x, y, z \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\}.$$

Thus for any $x, y, z \in A$, we have $\overline{\mu}_F(0) = 1-\mu_F(0) \leq 1-\mu_F(x) = \overline{\mu}_F(x)$ and

$$\begin{aligned} \overline{\mu}_F(y) &= 1 - \mu_F(y) \\ &\leq 1 - \min\{\mu_F(x \cdot y), \mu_F(x)\} \\ \text{Lemma 2.31 (1)} \quad &= \max\{1 - \mu_F(x \cdot y), 1 - \mu_F(x)\} \\ &= \max\{\overline{\mu}_F(x \cdot y), \overline{\mu}_F(x)\}. \end{aligned}$$

Hence, $\Box F = (\mu_F, \overline{\mu}_F)$ is an intuitionistic fuzzy UP-filter of A. Now, for any $x, y, z \in A$, we have

$$\gamma_F(0) \leq \gamma_F(x) \text{ and } \gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\}.$$

Thus for any $x, y, z \in A$, we have $\overline{\gamma}_F(0) = 1-\gamma_F(0) \geq 1-\gamma_F(x) = \overline{\gamma}_F(x)$ and

$$\begin{aligned} \overline{\gamma}_F(y) &= 1 - \gamma_F(y) \\ &\geq 1 - \max\{\gamma_F(x \cdot y), \gamma_F(x)\} \\ \text{Lemma 2.31 (2)} \quad &= \min\{1 - \gamma_F(x \cdot y), 1 - \gamma_F(x)\} \\ &= \min\{\overline{\gamma}_F(x \cdot y), \overline{\gamma}_F(x)\}. \end{aligned}$$

Hence, $\Diamond F = (\overline{\gamma}_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A.

Conversely, assume that the IFSs $\Box F = (\mu_F, \overline{\mu}_F)$ and $\Diamond F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy UP-filters of A. Then for any $x, y, z \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\},$$

and

$$\gamma_F(0) \leq \gamma_F(x) \text{ and } \gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\}.$$

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. □

THEOREM 2.45. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of a UP-algebra $A = (A, \cdot, 0)$ if and only if the IFSs $\Box F = (\mu_F, \overline{\mu}_F)$ and $\Diamond F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy near UP-filters of A.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A. Then for any $x, y \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(x \cdot y) \geq \mu_F(y).$$

Thus for any $x, y \in A$, we have $\overline{\mu}_F(0) = 1 - \mu_F(0) \leq 1 - \mu_F(x) = \overline{\mu}_F(x)$ and

$$\overline{\mu}_F(x \cdot y) = 1 - \mu_F(x \cdot y) \leq 1 - \mu_F(y) = \overline{\mu}_F(y).$$

Hence, $\square F = (\mu_F, \overline{\mu}_F)$ is an intuitionistic fuzzy near UP-filter of A. Now, for any $x, y \in A$, we have

$$\gamma_F(0) \leq \gamma_F(x) \text{ and } \gamma_F(x \cdot y) \leq \gamma_F(y).$$

Thus for any $x, y \in A$, we have $\overline{\gamma}_F(0) = 1 - \gamma_F(0) \geq 1 - \gamma_F(x) = \overline{\gamma}_F(x)$ and

$$\overline{\gamma}_F(x \cdot y) = 1 - \gamma_F(x \cdot y) \geq 1 - \gamma_F(y) = \overline{\gamma}_F(y).$$

Hence, $\lozenge F = (\overline{\gamma}_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A.

Conversely, assume that the IFSs $\square F = (\mu_F, \overline{\mu}_F)$ and $\lozenge F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy near UP-filters of A. Then for any $x, y \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(x \cdot y) \geq \mu_F(y),$$

and

$$\gamma_F(0) \leq \gamma_F(x) \text{ and } \gamma_F(x \cdot y) \leq \gamma_F(y).$$

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic near fuzzy UP-filter of A. □

THEOREM 2.46. [13] *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-subalgebra of a UP-algebra $A = (A, \cdot, 0)$ if and only if the IFSs $\square F = (\mu_F, \overline{\mu}_F)$ and $\lozenge F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy UP-subalgebras of A.*

THEOREM 2.47. *Let $F = (\mu_F, \gamma_F)$ be an IFS in a UP-algebra $A = (A, \cdot, 0)$ with $\mu_F = \overline{\gamma}_F$. Then $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong UP-ideal of A if and only if the IFS $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy strong UP-ideal of A.*

Proof. It is straightforward from Theorem 2.17. □

THEOREM 2.48. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-ideal of a UP-algebra $A = (A, \cdot, 0)$ if and only if the IFS $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy UP-ideal of A.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-ideal of A. Then for any $x, y, z \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(x \cdot z) \geq \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\}.$$

Thus for any $x, y, z \in A$, we have $\overline{\mu}_F(0) = 1-\mu_F(0) \leq 1-\mu_F(x) = \overline{\mu}_F(x)$ and

$$\begin{aligned} \overline{\mu}_F(x \cdot z) &= 1 - \mu_F(x \cdot z) \\ &\leq 1 - \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\} \\ \text{Lemma 2.31 (1)} \quad &= \max\{1 - \mu_F(x \cdot (y \cdot z)), 1 - \mu_F(y)\} \\ &= \max\{\overline{\mu}_F(x \cdot (y \cdot z)), \overline{\mu}_F(y)\}. \end{aligned}$$

Now, for any $x, y, z \in A$, we have

$$\gamma_F(0) \leq \gamma_F(x) \text{ and } \gamma_F(x \cdot z) \leq \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\}.$$

Thus for any $x, y, z \in A$, we have $\overline{\gamma}_F(0) = 1-\gamma_F(0) \geq 1-\gamma_F(x) = \overline{\gamma}_F(x)$ and

$$\begin{aligned} \overline{\gamma}_F(x \cdot z) &= 1 - \gamma_F(x \cdot z) \\ &\geq 1 - \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\} \\ \text{Lemma 2.31 (2)} \quad &= \min\{1 - \gamma_F(x \cdot (y \cdot z)), 1 - \gamma_F(y)\} \\ &= \min\{\overline{\gamma}_F(x \cdot (y \cdot z)), \overline{\gamma}_F(y)\}. \end{aligned}$$

Hence, $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy UP-ideal of A.

Conversely, assume that the IFS $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy UP-ideal of A. Then for any $x, y, z \in A$, we have

$$\overline{\gamma}_F(0) \geq \overline{\gamma}_F(x) \text{ and } \overline{\gamma}_F(x \cdot z) \geq \min\{\overline{\gamma}_F(x \cdot (y \cdot z)), \overline{\gamma}_F(y)\}.$$

Thus for any $x, y, z \in A$, we have $1 - \gamma_F(0) \geq 1 - \gamma_F(x)$ and

$$\text{Lemma 2.31 (2)} \qquad 1 - \gamma_F(x \cdot z) \geq 1 - \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\},$$

so $\gamma_F(0) \leq \gamma_F(x)$ and $\gamma_F(x \cdot z) \leq \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\}$. Now, for any $x, y, z \in A$, we have

$$\overline{\mu}_F(0) \leq \overline{\mu}_F(x) \text{ and } \overline{\mu}_F(x \cdot z) \leq \max\{\overline{\mu}_F(x \cdot (y \cdot z)), \overline{\mu}_F(y)\}.$$

Thus for any $x, y, z \in A$, we have $1 - \mu_F(0) \leq 1 - \mu_F(x)$ and

$$\text{Lemma 2.31 (1)} \qquad 1 - \mu_F(x \cdot z) \leq 1 - \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\},$$

so $\mu_F(0) \geq \mu_F(x)$ and $\mu_F(x \cdot z) \geq \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\}$. Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-ideal of A. □

Theorem 2.49. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of a UP-algebra $A = (A, \cdot, 0)$ if and only if the IFS $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy UP-filter of A.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. Then for any $x, y \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\}.$$

Thus for any $x, y \in A$, we have $\overline{\mu}_F(0) = 1 - \mu_F(0) \leq 1 - \mu_F(x) = \overline{\mu}_F(x)$ and

$$\begin{aligned} \overline{\mu}_F(y) &= 1 - \mu_F(y) \\ &\leq 1 - \min\{\mu_F(x \cdot y), \mu_F(x)\} \\ \text{Lemma 2.31 (1)} \quad &= \max\{1 - \mu_F(x \cdot y), 1 - \mu_F(x)\} \\ &= \max\{\overline{\mu}_F(x \cdot y), \overline{\mu}_F(x)\}. \end{aligned}$$

Now, for any $x, y \in A$, we have

$$\gamma_F(0) \leq \gamma_F(x) \text{ and } \gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\}.$$

Thus for any $x, y \in A$, we have $\overline{\gamma}_F(0) = 1 - \gamma_F(0) \geq 1 - \gamma_F(x) = \overline{\gamma}_F(x)$ and

$$\begin{aligned} \overline{\gamma}_F(y) &= 1 - \gamma_F(y) \\ &\geq 1 - \max\{\gamma_F(x \cdot y), \gamma_F(x)\} \\ \text{Lemma 2.31 (2)} \quad &= \min\{1 - \gamma_F(x \cdot y), 1 - \gamma_F(x)\} \\ &= \min\{\overline{\gamma}_F(x \cdot y), \overline{\gamma}_F(x)\}. \end{aligned}$$

Hence, $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy UP-filter of A.

Conversely, assume that the IFS $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy UP-filter of A. Then for any $x, y \in A$, we have

$$\overline{\gamma}_F(0) \geq \overline{\gamma}_F(x) \text{ and } \overline{\gamma}_F(y) \geq \min\{\overline{\gamma}_F(x \cdot y), \overline{\gamma}_F(x)\}.$$

Thus for any $x, y \in A$, we have $1 - \gamma_F(0) \geq 1 - \gamma_F(x)$ and

$$\text{Lemma 2.31 (2)} \qquad 1 - \gamma_F(y) \geq 1 - \max\{\gamma_F(x \cdot y), \gamma_F(x)\},$$

so $\gamma_F(0) \leq \gamma_F(x)$ and $\gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\}$. Now, for any $x, y \in A$, we have

$$\overline{\mu}_F(0) \leq \overline{\mu}_F(x) \text{ and } \overline{\mu}_F(y) \leq \max\{\overline{\mu}_F(x \cdot y), \overline{\mu}_F(x)\}.$$

Thus for any $x, y \in A$, we have $1 - \mu_F(0) \leq 1 - \mu_F(x)$ and

$$\text{Lemma 2.31 (1)} \qquad 1 - \mu_F(y) \leq 1 - \min\{\mu_F(x \cdot y), \mu_F(x)\},$$

so $\mu_F(0) \geq \mu_F(x)$ and $\mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\}$. Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. $\square$

Theorem 2.50. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of a UP-algebra $A = (A, \cdot, 0)$ if and only if the IFS $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy near UP-filter of A.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A. Then for any $x, y \in A$, we have

$$\mu_F(0) \geq \mu_F(x) \text{ and } \mu_F(x \cdot y) \geq \mu_F(y).$$

Thus for any $x, y \in A$, we have $\overline{\mu}_F(0) = 1 - \mu_F(0) \leq 1 - \mu_F(x) = \overline{\mu}_F(x)$ and

$$\overline{\mu}_F(x \cdot y) = 1 - \mu_F(x \cdot y) \leq 1 - \mu_F(y) = \overline{\mu}_F(y).$$

Now, for any $x, y \in A$, we have

$$\gamma_F(0) \leq \gamma_F(x) \text{ and } \gamma_F(x \cdot y) \leq \gamma_F(y).$$

Thus for any $x, y \in A$, we have $\overline{\gamma}_F(0) = 1 - \gamma_F(0) \geq 1 - \gamma_F(x) = \overline{\gamma}_F(x)$ and

$$\overline{\gamma}_F(x \cdot y) = 1 - \gamma_F(x \cdot y) \geq 1 - \gamma_F(y) = \overline{\gamma}_F(y).$$

Hence, $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy near UP-filter of A.

Conversely, assume that the IFS $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic near fuzzy UP-filter of A. Then for any $x, y \in A$, we have

$$\overline{\gamma}_F(0) \geq \overline{\gamma}_F(x) \text{ and } \overline{\gamma}_F(x \cdot y) \geq \overline{\gamma}_F(y).$$

Thus for any $x, y \in A$, we have $1 - \gamma_F(0) \geq 1 - \gamma_F(x)$ and $1 - \gamma_F(x \cdot y) \geq 1 - \gamma_F(y)$, so $\gamma_F(0) \leq \gamma_F(x)$ and $\gamma_F(x \cdot y) \leq \gamma_F(y)$. Now, for any $x, y \in A$, we have

$$\overline{\mu}_F(0) \leq \overline{\mu}_F(x) \text{ and } \overline{\mu}_F(x \cdot y) \leq \overline{\mu}_F(y).$$

Thus for any $x, y \in A$, we have $1 - \mu_F(0) \leq 1 - \mu_F(x)$ and $1 - \mu_F(x \cdot y) \leq 1 - \mu_F(y)$, so $\mu_F(0) \geq \mu_F(x)$ and $\mu_F(x \cdot y) \geq \mu_F(y)$. Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic near fuzzy UP-filter of A. □

Theorem 2.51. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-subalgebra of a UP-algebra $A = (A, \cdot, 0)$ if and only if the IFS $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy UP-subalgebra of A.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-subalgebra of A. Then for any $x, y \in A$, we have

$$\mu_F(x \cdot y) \geq \min\{\mu_F(x), \mu_F(y)\}.$$

Thus for any $x, y \in A$, we have

$$\begin{aligned} \overline{\mu}_F(x \cdot y) &= 1 - \mu_F(x \cdot y) \\ &\leq 1 - \min\{\mu_F(x), \mu_F(y)\} \\ \text{Lemma 2.31 (1)} \quad &= \max\{1 - \mu_F(x), 1 - \mu_F(y)\} \\ &= \max\{\overline{\mu}_F(x), \overline{\mu}_F(y)\}. \end{aligned}$$

Now, for any $x, y \in A$, we have

$$\gamma_F(x \cdot y) \leq \max\{\gamma_F(x), \gamma_F(y)\}.$$

Thus for any $x, y \in A$, we have

$$\begin{aligned} \overline{\gamma}_F(x \cdot y) &= 1 - \gamma_F(x \cdot y) \\ &\geq 1 - \max\{\gamma_F(x), \gamma_F(y)\} \\ \text{Lemma 2.31 (2)} \quad &= \min\{1 - \gamma_F(x), 1 - \gamma_F(y)\} \\ &= \min\{\overline{\gamma}_F(x), \overline{\gamma}_F(y)\}. \end{aligned}$$

Hence, $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy UP-subalgebra of A.

Conversely, assume that the IFS $\overline{F} = (\overline{\gamma}_F, \overline{\mu}_F)$ is an intuitionistic fuzzy UP-subalgebra of A. Then for any $x, y \in A$, we have

$$\overline{\gamma}_F(x \cdot y) \geq \min\{\overline{\gamma}_F(x), \overline{\gamma}_F(y)\}.$$

Thus for any $x, y \in A$, we have

$$\text{Lemma 2.31 (2)} \qquad 1 - \gamma_F(x \cdot y) \geq 1 - \max\{\gamma_F(x), \gamma_F(y)\},$$

so $\gamma_F(x \cdot y) \leq \max\{\gamma_F(x), \gamma_F(y)\}$. Now, for any $x, y \in A$, we have

$$\overline{\mu}_F(x \cdot y) \leq \max\{\overline{\mu}_F(x), \overline{\mu}_F(y)\}.$$

Thus for any $x, y \in A$, we have

$$\text{Lemma 2.31 (1)} \qquad 1 - \mu_F(x \cdot y) \leq 1 - \min\{\mu_F(x), \mu_F(y)\},$$

so $\mu_F(x \cdot y) \geq \min\{\mu_F(x), \mu_F(y)\}$. Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-subalgebra of A. □

3. Level Subsets and IFSs

In this section, we discuss the relations between intuitionistic fuzzy near UP-filters (resp., intuitionistic fuzzy UP-filters, intuitionistic fuzzy strong UP-ideals) and their upper t-(strong) level subsets and lower t-(strong) level subsets in UP-algebras.

Definition 3.1. [27] Let f and g be FSs in a nonempty set X. For any $t, s \in [0,1]$, the sets

$$U(f;t) = \{x \in X \mid f(x) \geq t\} \text{ and } U^{+}(f;t) = \{x \in X \mid f(x) > t\}$$

are called an *upper t-level subset* and an *upper t-strong level subset* of f, respectively. The sets

$$L(f;t) = \{x \in X \mid f(x) \leq t\} \text{ and } L^{-}(f;t) = \{x \in X \mid f(x) < t\}$$

are called a *lower t-level subset* and a *lower t-strong level subset* of f, respectively. The set

$$E(f;t) = \{x \in X \mid f(x) = t\}$$

is called an *equal t-level subset* of f. The set

$$C(f,g;t,s) = U(f;t) \cap L(g;s)$$

is called the *(t,s)-cut* of f and g.

Theorem 3.2. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong UP-ideal of a UP-algebra $A = (A, \cdot, 0)$ if and only if $E(\mu_F; \mu_F(0)) = A$ and $E(\gamma_F; \gamma_F(0)) = A$.*

Proof. It is straightforward from Theorem 2.17. □

Theorem 3.3. [13] *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-ideal of a UP-algebra $A = (A, \cdot, 0)$ if and only if for all $s, t \in [0,1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or UP-ideals of A.*

Corollary 3.4. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-ideal of a UP-algebra $A = (A, \cdot, 0)$ if and only if for all $s, t \in [0,1]$, the set $C(\mu_F, \gamma_F; t, s)$ is either empty or a UP-ideal of A.*

Proof. The necessary is straightforward from Theorems 1.6 and 3.3.

Conversely, assume that for all $s, t \in [0,1]$, the set $C(\mu_F, \gamma_F; t, s)$ is either empty or a UP-ideal of A. Let $t \in [0,1]$ be such that $U(\mu_F; t) \neq \emptyset$. Then

$$\emptyset \neq U(\mu_F;t) = U(\mu_F;t) \cap A = U(\mu_F;t) \cap L(\gamma_F;1) = C(\mu_F, \gamma_F; t, 1).$$

By assumption, we have $U(\mu_F; t) = C(\mu_F, \gamma_F; t, 1)$ is a UP-ideal of A. Let $s \in [0,1]$ be such that $L(\gamma_F; s) \neq \emptyset$. Then

$$\emptyset \neq L(\gamma_F;s) = A \cap L(\gamma_F;s) = U(\mu_F;0) \cap L(\gamma_F;s) = C(\mu_F, \gamma_F; 0, s).$$

By assumption, we have $L(\gamma_F; s) = C(\mu_F, \gamma_F; 0, s)$ is a UP-ideal of A. Hence, by Theorem 3.3, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-ideal of A. □

THEOREM 3.5. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of a UP-algebra $A = (A, \cdot, 0)$ if and only if for all $s, t \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or UP-filters of A.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. Let $s, t \in [0, 1]$ be such that $U(\mu_F; t)$ and $L(\gamma_F; s)$ are nonempty subsets of A. Then there exist $a \in U(\mu_F; t)$ and $b \in L(\gamma_F; s)$, that is, $\mu_F(a) \geq t$ and $\gamma_F(b) \leq s$. Since $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A, we have $\mu_F(0) \geq \mu_F(x)$ and $\gamma_F(0) \leq \gamma_F(x)$ for all $x \in A$. Thus $\mu_F(0) \geq \mu_F(a) \geq t$ and $\gamma_F(0) \leq \gamma_F(b) \leq s$, so $0 \in U(\mu_F; t)$ and $0 \in L(\gamma_F; s)$. Let $x, y \in A$ be such that $x \cdot y \in U(\mu_F; t)$ and $x \in U(\mu_F; t)$. Then $\mu_F(x \cdot y) \geq t$ and $\mu_F(x) \geq t$. Thus

Definition 2.11 (2) $\qquad \mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\} \geq t,$

so $y \in U(\mu_F; t)$. Hence, $U(\mu_F; t)$ is a UP-filter of A. Finally, let $x, y \in A$ be such that $x \cdot y \in L(\gamma_F; s)$ and $x \in L(\gamma_F; s)$. Then $\gamma_F(x \cdot y) \leq s$ and $\gamma_F(x) \leq s$. Thus

Definition 2.11 (4) $\qquad \gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\} \leq s,$

so $y \in L(\gamma_F; s)$. Hence, $L(\gamma_F; s)$ is a UP-filter of A.

Conversely, assume that for all $s, t \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or UP-filters of A. For any $x \in A$, let $\mu_F(x) = t$ and $\gamma_F(x) = s$. Then $x \in U(\mu_F; t) \neq \emptyset$ and $x \in L(\gamma_F; s) \neq \emptyset$. By assumption, we have $U(\mu_F; t)$ and $L(\gamma_F; s)$ are UP-filters of A. Thus $0 \in U(\mu_F; t)$ and $0 \in L(\gamma_F; s)$, so $\mu_F(0) \geq t = \mu_F(x)$ and $\gamma_F(0) \leq s = \gamma_F(x)$ for all $x \in A$. Suppose that there exist $x, y \in A$ such that $\mu_F(y) < \min\{\mu_F(x \cdot y), \mu_F(x)\}$. Put

$$t_0 = \frac{1}{2}[\mu_F(y) + \min\{\mu_F(x \cdot y), \mu_F(x)\}].$$

Thus $t_0 \in [0, 1]$ and $\mu_F(y) < t_0 < \min\{\mu_F(x \cdot y), \mu_F(x)\}$. This implies that $y \notin U(\mu_F; t_0)$ but $x \cdot y, x \in U(\mu_F; t_0)$. Thus $U(\mu_F; t_0)$ is not a UP-filter of A. Now, suppose that there exist $a, b \in A$ such that $\gamma_F(b) > \max\{\gamma_F(a \cdot b), \gamma_F(a)\}$. Put

$$s_0 = \frac{1}{2}[\gamma_F(b) + \max\{\gamma_F(a \cdot b), \gamma_F(a)\}].$$

Thus $s_0 \in [0, 1]$ and $\max\{\gamma_F(a \cdot b), \gamma_F(a)\} < s_0 < \gamma_F(b)$. This implies that $b \notin L(\gamma_F; s_0)$ but $a \cdot b, a \in L(\gamma_F; s_0)$. Thus $L(\gamma_F; s_0)$ is not a UP-filter of A. By assumption, we have $U(\mu_F; t_0)$ and $L(\gamma_F; s_0)$ are empty. This is a contradiction to the fact that $x \in U(\mu_F; t_0) \neq \emptyset$ and $a \in L(\gamma_F; s_0) \neq \emptyset$. Hence, $\mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\}$ and $\gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\}$ for all $x, y \in A$. Therefore, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. □

COROLLARY 3.6. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of a UP-algebra $A = (A, \cdot, 0)$ if and only if for all $s, t \in [0, 1]$, the set $C(\mu_F, \gamma_F; t, s)$ is either empty or a UP-filter of A.*

Proof. The necessary is straightforward from Theorems 1.6 and 3.5.

Conversely, assume that for all $s, t \in [0, 1]$, the set $C(\mu_F, \gamma_F; t, s)$ is either empty or a UP-filter of A. Let $t \in [0, 1]$ be such that $U(\mu_F; t) \neq \emptyset$. Then

$$\emptyset \neq U(\mu_F; t) = U(\mu_F; t) \cap A = U(\mu_F; t) \cap L(\gamma_F; 1) = C(\mu_F, \gamma_F; t, 1).$$

By assumption, we have $U(\mu_F; t) = C(\mu_F, \gamma_F; t, 1)$ is an UP-filter of A. Let $s \in [0, 1]$ be such that $L(\gamma_F; s) \neq \emptyset$. Then

$$\emptyset \neq L(\gamma_F; s) = A \cap L(\gamma_F; s) = U(\mu_F; 0) \cap L(\gamma_F; s) = C(\mu_F, \gamma_F; 0, s).$$

By assumption, we have $L(\gamma_F; s) = C(\mu_F, \gamma_F; 0, s)$ is an UP-filter of A. Hence, by Theorem 3.5, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. □

THEOREM 3.7. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of a UP-algebra $A = (A, \cdot, 0)$ if and only if for all $s, t \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or near UP-filters of A.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A. Let $s, t \in [0, 1]$ be such that $U(\mu_F; t)$ and $L(\gamma_F; s)$ are nonempty subsets of A. Then there exist $a \in U(\mu_F; t)$ and $b \in L(\gamma_F; s)$, that is, $\mu_F(a) \geq t$ and $\gamma_F(b) \leq s$. Since $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A, we have $\mu_F(0) \geq \mu_F(x)$ and $\gamma_F(0) \leq \gamma_F(x)$ for all $x \in A$. Thus $\mu_F(0) \geq \mu_F(a) \geq t$ and $\gamma_F(0) \leq \gamma_F(b) \leq s$, so $0 \in U(\mu_F; t)$ and $0 \in L(\gamma_F; s)$. Let $x, y \in A$ be such that $y \in U(\mu_F; t)$. Then $\mu_F(x) \geq t$. Thus

Definition 2.9 (2) $$\mu_F(x \cdot y) \geq \mu_F(y) \geq t,$$

so $x \cdot y \in U(\mu_F; t)$. Hence, $U(\mu_F; t)$ is a near UP-filter of A. Finally, let $x, y \in A$ be such that $y \in L(\gamma_F; s)$. Then $\gamma_F(y) \leq s$. Thus

Definition 2.9 (4) $$\gamma_F(x \cdot y) \leq \gamma_F(y) \leq s,$$

so $x \cdot y \in L(\gamma_F; s)$. Hence, $L(\gamma_F; s)$ is a near UP-filter of A.

Conversely, assume that for all $s, t \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or near UP-filters of A. For any $x \in A$, let $\mu_F(x) = t$ and $\gamma_F(x) = s$. Then $x \in U(\mu_F; t) \neq \emptyset$ and $x \in L(\gamma_F; s) \neq \emptyset$. By assumption, we have $U(\mu_F; t)$ and $L(\gamma_F; s)$ are near UP-filters of A. Thus $0 \in U(\mu_F; t)$ and $0 \in L(\gamma_F; s)$, so $\mu_F(0) \geq t = \mu_F(x)$ and $\gamma_F(0) \leq s = \gamma_F(x)$ for all $x \in A$. Suppose that there exist $x, y \in A$ such that $\mu_F(x \cdot y) < \mu_F(y)$. Put

$$t_0 = \frac{1}{2}[\mu_F(x \cdot y) + \mu_F(y)].$$

Thus $t_0 \in [0, 1]$ and $\mu_F(x \cdot y) < t_0 < \mu_F(y)$. This implies that $x \cdot y \notin U(\mu_F; t_0)$ but $y \in U(\mu_F; t_0)$. Thus $U(\mu_F; t_0)$ is not a near UP-filter of A. Now, suppose that there exist $a, b \in A$ such that $\gamma_F(a \cdot b) > \gamma_F(b)$. Put

$$s_0 = \frac{1}{2}[\gamma_F(a \cdot b) + \gamma_F(b)].$$

Thus $s_0 \in [0, 1]$ and $\gamma_F(b) < s_0 < \gamma_F(a \cdot b)$. This implies that $a \cdot b \notin L(\gamma_F; s_0)$ but $b \in L(\gamma_F; s_0)$. Thus $L(\gamma_F; s_0)$ is not a near UP-filter of A. By assumption, we have $U(\mu_F; t_0)$ and $L(\gamma_F; s_0)$ are empty. This is a contradiction to the fact that $y \in U(\mu_F; t_0) \neq \emptyset$ and $b \in L(\gamma_F; s_0) \neq \emptyset$. Hence, $\mu_F(x \cdot y) \geq \mu_F(y)$ and $\gamma_F(x \cdot y) \leq \gamma_F(y)$ for all $x, y \in A$. Therefore, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A. □

Corollary 3.8. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of a UP-algebra $A = (A, \cdot, 0)$ if and only if for all $s, t \in [0, 1]$, the set $C(\mu_F, \gamma_F; t, s)$ is either empty or a near UP-filter of A.*

Proof. The necessary is straightforward from Theorems 1.6 and 3.7.

Conversely, assume that for all $s, t \in [0, 1]$, the set $C(\mu_F, \gamma_F; t, s)$ is either empty or a near UP-filter of A. Let $t \in [0, 1]$ be such that $U(\mu_F; t) \neq \emptyset$. Then

$$\emptyset \neq U(\mu_F; t) = U(\mu_F; t) \cap A = U(\mu_F; t) \cap L(\gamma_F; 1) = C(\mu_F, \gamma_F; t, 1).$$

By assumption, we have $U(\mu_F; t) = C(\mu_F, \gamma_F; t, 1)$ is a near UP-filter of A. Let $s \in [0, 1]$ be such that $L(\gamma_F; s) \neq \emptyset$. Then

$$\emptyset \neq L(\gamma_F; s) = A \cap L(\gamma_F; s) = U(\mu_F; 0) \cap L(\gamma_F; s) = C(\mu_F, \gamma_F; 0, s).$$

By assumption, we have $L(\gamma_F; s) = C(\mu_F, \gamma_F; 0, s)$ is a near UP-filter of A. Hence, by Theorem 3.7, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A. □

Theorem 3.9. [13] *An IFS* $F = (\mu_F, \gamma_F)$ *is an intuitionistic fuzzy UP-subalgebra of a UP-algebra* $A = (A, \cdot, 0)$ *if and only if for all* $s, t \in [0, 1]$*, the sets* $U(\mu_F; t)$ *and* $L(\gamma_F; s)$ *are either empty or UP-subalgebras of* A*.*

Corollary 3.10. *an IFS* $F = (\mu_F, \gamma_F)$ *is an intuitionistic fuzzy UP-subalgebra of a UP-algebra* $A = (A, \cdot, 0)$ *if and only if for all* $s, t \in [0, 1]$*, the set* $C(\mu_F, \gamma_F; t, s)$ *are either empty or a UP-subalgebra of* A*.*

Proof. The necessary is straightforward from Theorems 1.6 and 3.9.

Conversely, assume that for all $s, t \in [0, 1]$, the set $C(\mu_F, \gamma_F; t, s)$ are either empty or a UP-subalgebra of A. Let $t \in [0, 1]$ be such that $U(\mu_F; t) \neq \emptyset$. Then

$$\emptyset \neq U(\mu_F; t) = U(\mu_F; t) \cap A = U(\mu_F; t) \cap L(\gamma_F; 1) = C(\mu_F, \gamma_F; t, 1).$$

By assumption, we have $U(\mu_F; t) = C(\mu_F, \gamma_F; t, 1)$ is a UP-subalgebra of A. Let $s \in [0, 1]$ be such that $L(\gamma_F; s) \neq \emptyset$. Then

$$\emptyset \neq L(\gamma_F; s) = A \cap L(\gamma_F; s) = U(\mu_F; 0) \cap L(\gamma_F; s) = C(\mu_F, \gamma_F; 0, s).$$

By assumption, we have $L(\gamma_F; s) = C(\mu_F, \gamma_F; 0, s)$ is a UP-subalgebra of A. Hence, by Theorem 3.9, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-subalgebra of A. □

Theorem 3.11. [13] *If an IFS* $F = (\mu_F, \gamma_F)$ *is an intuitionistic fuzzy UP-ideal of a UP-algebra* $A = (A, \cdot, 0)$*, then for all* $s, t \in [0, 1]$*, the sets* $U^+(\mu_F; t)$ *and* $L^-(\gamma_F; s)$ *are either empty or UP-ideals of* A*.*

Theorem 3.12. *If an IFS* $F = (\mu_F, \gamma_F)$ *is an intuitionistic fuzzy UP-filter of a UP-algebra* $A = (A, \cdot, 0)$*, then for all* $s, t \in [0, 1]$*, the sets* $U^+(\mu_F; t)$ *and* $L^-(\gamma_F; s)$ *are either empty or UP-filters of* A*.*

Proof. Assume that an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A. Let $s, t \in [0, 1]$ be such that $U^+(\mu_F; t)$ and $L^-(\gamma_F; s)$ are nonempty subsets of A. Then there exist $a \in U^+(\mu_F; t)$ and $b \in L^-(\gamma_F; s)$, that is, $\mu_F(a) > t$ and $\gamma_F(b) < s$. Since $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-filter of A, we have $\mu_F(0) \geq \mu_F(x)$ and $\gamma_F(0) \leq \gamma_F(x)$ for all $x \in A$. Thus $\mu_F(0) \geq \mu_F(a) > t$ and $\gamma_F(0) \leq \gamma_F(b) < s$, so $0 \in U^+(\mu_F; t)$ and $0 \in L^-(\gamma_F; s)$. Let $x, y \in A$ be such that $x \cdot y \in U^+(\mu_F; t)$ and $x \in U^+(\mu_F; t)$. Then $\mu_F(x \cdot y) > t$ and $\mu_F(x) > t$. Thus

Definition 2.11 (2) $\qquad \mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\} > t,$

so $y \in U^+(\mu_F;t)$. Hence, $U^+(\mu_F;t)$ is a UP-filter of A. Finally, let $x, y \in A$ be such that $x \cdot y \in L^-(\gamma_F;s)$ and $x \in L^-(\gamma_F;s)$. Then $\gamma_F(x \cdot y) < s$ and $\gamma_F(x) < s$. Thus

Definition 2.11 (4) $$\gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\} < s,$$

so $y \in L^-(\gamma_F;s)$. Hence, $L^-(\gamma_F;s)$ is a UP-filter of A. □

THEOREM 3.13. *If an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of a UP-algebra $A = (A, \cdot, 0)$, then for all $s, t \in [0, 1]$, the sets $U^+(\mu_F;t)$ and $L^-(\gamma_F;s)$ are either empty or near UP-filters of A.*

Proof. Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A. Let $s, t \in [0, 1]$ be such that $U^+(\mu_F;t)$ and $L^-(\gamma_F;s)$ are nonempty subsets of A. Then there exist $a \in U^+(\mu_F;t)$ and $b \in L^-(\gamma_F;s)$, that is, $\mu_F(a) > t$ and $\gamma_F(b) < s$. Since $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy near UP-filter of A, we have $\mu_F(0) \geq \mu_F(x)$ and $\gamma_F(0) \leq \gamma_F(x)$ for all $x \in A$. Thus $\mu_F(0) \geq \mu_F(a) > t$ and $\gamma_F(0) \leq \gamma_F(b) < s$, so $0 \in U^+(\mu_F;t)$ and $0 \in L^-(\gamma_F;s)$. Let $x, y \in A$ be such that $y \in U^+(\mu_F;t)$. Then $\mu_F(y) > t$. Thus

Definition 2.9 (2) $$\mu_F(x \cdot y) \geq \mu_F(y) > t,$$

so $x \cdot y \in U^+(\mu_F;t)$. Hence, $U^+(\mu_F;t)$ is a near UP-filter of A. Finally, let $x, y \in A$ be such that $y \in L^-(\gamma_F;s)$. Then $\gamma_F(y) < s$. Thus

Definition 2.9 (4) $$\gamma_F(x \cdot y) \leq \gamma_F(y) < s,$$

so $x \cdot y \in L^-(\gamma_F;s)$. Hence, $L^-(\gamma_F;s)$ is a near UP-filter of A. □

THEOREM 3.14. *[13] If an IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy UP-subalgebra of a UP-algebra $A = (A, \cdot, 0)$, then for all $s, t \in [0, 1]$, the sets $U^+(\mu_F;t)$ and $L^-(\gamma_F;s)$ are either empty or UP-subalgebras of A.*

4. Conclusions and Future Work

In this paper, we have introduced the notions intuitionistic fuzzy near UP-filters, intuitionistic fuzzy UP-filters, and intuitionistic fuzzy strong UP-ideals of UP-algebras, and proved their generalizations and investigated some of their important properties. Then, we get the diagram of generalization of IFSs in UP-algebras as shown in Figure 1.

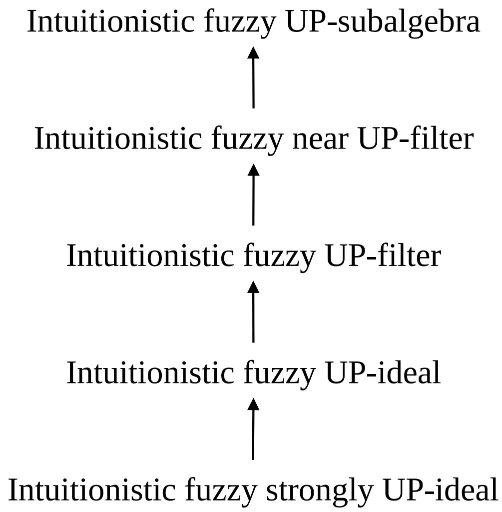

Figure 1. IFSs in UP-algebras

In our future study, we will apply this notion/results to other type of IFSs in UP-algebras. Also, we will study the soft set theory of intuitionistic fuzzy UP-subalgebras, intuitionistic fuzzy near UP-filters, intuitionistic fuzzy UP-filters, intuitionistic fuzzy UP-ideals, and intuitionistic fuzzy strong UP-ideals.

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Nattaporn Thongngam
Department of Mathematics and Statistics
Faculty of Science, Prince of Songkla University
Songkla 90110, Thailand
E-mail: snt22092540@hotmail.com

Aiyared Iampan
Department of Mathematics
School of Science, University of Phayao
Phayao 56000, Thailand
E-mail: aiyared.ia@up.ac.th