Korean J. Math. **28** (2020), No. 2, pp. 275–284
http://dx.doi.org/10.11568/kjm.2020.28.2.275

PSEUDOPARALLEL INVARIANT SUBMANIFOLDS OF $(LCS)_n$-MANIFOLDS

Mehmet Atçeken, Ümit Yildirim, and Süleyman Dirik

Abstract. The aim of this paper is to study the invariant submanifolds of $(LCS)_n$-manifolds. We study pseudo parallel, generalized Ricci-pseudo parallel and 2-pseudo parallel invariant submanifolds of a $(LCS)_n$-manifold and get the necessary and sufficient conditions for an invariant submanifold to be totally geodesic and give some new results contribute to differential geometry.

1. Introduction

In [14], the author defined and studied the notion Lorentzian concircular structure manifolds named $(LCS)_n$-manifolds and constructed example which generalizes the of LP-Sasakian manifolds. After then, Shaikh and Baishya investigated the applications of $(LCS)_n$-manifolds to the general theory of relativity and cosmology [16, 17].

We note that the most interesting fact is that $(LCS)_n$ manifold remains invariant under a D-homothetic transformation which does not hold for an LP-Sasakian manifold. In modern analysis the geometry of submanifolds has become a subject of growing interest for its significant application in applied mathematics and theoretical physics.

Received April 9, 2019. Revised May 26, 2020. Accepted May 27, 2020.
2010 Mathematics Subject Classification: 53C15, 53C44, 53D10.
Key words and phrases: $(LCS)_n$-Manifold, Pseudoparallel submanifold, Generalized Ricci-pseudoparallel and 2-pseudoparallel submanifold.

In this connection, the invariant submanifold is used to discuss properties of non-linear autonomous system [7]. Also, the totally geodesic submanifolds play an important role in the relativity theory for the geodesic of the ambient manifolds remain geodesic in the submanifolds. Even thought totally geodesic submanifolds are simplest, they have very much important roles in physical sciences.

The study of geometry of invariant submanifolds was initiated by Bejancu and Paqaghuic [4]. After, the invariant submanifolds have been studied by many geometers to different extent manifolds type[see references].

Motivated by the above studies, in this paper, we have studied pseudo parallel, generalized Ricci-pseudo parallel and 2-pseudo parallel invariant submanifolds of a $(LCS)_n$-manifold which has not attempted so far.

2. Preliminaries

An n-dimensional Lorentzian manifold M is a smooth connected paracompact housdroff manifold with a Lorentzian metric type-(0,2) such that $g_p : T_M(p) \times T_M(p) \longrightarrow \mathbb{R}$ is a non-degenerate inner product of signature $(-, +, ..., +)$ for each point $p \in M$, where $T_M(P)$ denotes the tangent space of M at p. A non-zero vector $X \in T_M(p)$ is said to be timelike(resp. non-spacelike, null and spacelike) $g_p(X, X) < 0$(resp. $\leq 0, \quad = 0, \quad > 0$).

Let M be the Lorentzian manifold admitting a unit time-like concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g_p(\xi, \xi) = -1. \tag{1}$$

Since ξ is a unit concircular vector field, there is a non-zero 1-form η such that

$$\eta(X) = g(X, \xi), \tag{2}$$

for any vector field X on M. The covariant derivative equation of (2) satisfies

$$(\widetilde{\nabla}_X \eta)Y = \alpha\{g(X, Y) + \eta(X)\eta(Y)\}, \quad \alpha \neq 0 \tag{3}$$

for all vector fields X, Y on M, where $\widetilde{\nabla}$ and α are, respectively, the Levi-Civita connection and non-zero scalar function on M, satisfying

$$\widetilde{\nabla}_X \alpha = X(\alpha) = \rho\eta(X) \tag{4}$$

being a certain scalar function. We put

$$\widetilde{\nabla}_X \xi = \alpha\varphi X, \tag{5}$$

then we have

$$\varphi X = X + \eta(X)\xi, \tag{6}$$

which implies that φ is a symmetric type (1,1)-tensor and it is called the structure tensor of the manifold. The Lorentzian manifold M together with the timelike concircular vector field ξ its associated 1-form η and (1,1)-tensor field φ is said to be a Lorentzian concircular structure (LCS)-manifold [14]. Particularly, if we take $\alpha = 1$, it is called LP-Sasakain manifold of Matsumoto [8].

In a $(LCS)_n$-manifold M, the following relations are satisfied;

$$g(\varphi X, \varphi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{7}$$

$$\eta(\xi) = -1, \quad \varphi\xi = 0, \tag{8}$$

$$\varphi^2 X = X + \eta(X)\xi, \tag{9}$$

$$(\widetilde{\nabla}_X \varphi)Y = \alpha\{g(X, Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\}, \tag{10}$$

$$R(X, Y)\xi = (\alpha^2 - \rho)(\eta(Y)X - \eta(X)Y), \tag{11}$$

$$R(\xi, X)Y = (\alpha^2 - \rho)(g(X, Y)\xi - \eta(Y)X), \tag{12}$$

$$\eta(R(X, Y)Z) = (\alpha^2 - \rho)\{\eta(X)g(Y, Z) - \eta(Y)g(X, Z)\} \tag{13}$$

$$S(X, \xi) = (n-1)(\alpha^2 - \rho)\eta(X), \tag{14}$$

$$S(\varphi X, \varphi Y) = S(X, Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y), \tag{15}$$

where R and S denote the Riemannian curvature tensor and Ricci tensor of M, respectively.

Now let M be a submanifold of a $(LCS)_n$-manifold $\widetilde{M}$ with induced metric tensor g and ∇ is the induced connection on the tangent bundle TM and $\nabla^\perp$ is also induced connection on the normal bundle $T^\perp M$ on M. Then Gauss and Weingarten formulae are given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + h(X, Y), \quad \widetilde{\nabla}_X V = -A_V X + \nabla^\perp_X V, \tag{16}$$

for any $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, where h and A_V are, respectively, called the second fundamental form and the Weingarten map. They are related by

$$g(h(X,Y),V) = g(A_V X, Y). \tag{17}$$

If $h = 0$, then the submanifold M is said to be totally geodesic. Such manifolds are simplest manifolds. But it is of a important role. For the second fundamental form h, the covariant derivative of h is defined by

$$(\widetilde{\nabla}_X h)(Y,Z) = \nabla^{\perp}_X h(Y,Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z), \tag{18}$$

for all $X, Y, Z \in \Gamma(TM)$. If $\widetilde{\nabla} h = 0$, then M is said to be have parallel second fundamental form. For any submanifold M of a Riemannian manifold $\widetilde{M}$, the Gauss equation is given by

$$\begin{aligned} \widetilde{R}(X,Y)Z &= R(X,Y)Z + A_{h(X,Z)}Y - A_{h(Y,Z)}X + (\widetilde{\nabla}_X h)(Y,Z) \\ &- (\widetilde{\nabla}_Y h)(X,Z), \end{aligned} \tag{19}$$

for any $X, Y, Z \in \Gamma(TM)$, where R denote the Riemannian curvature tensor of M.

Definition 2.1. A submanifold M of a $(LCS)_n$-manifold $\widetilde{M}$ is called invariant if the structure vector field ξ is tangent to M at every point of M and φX is tangent to M for any vector field X tangent to M at every point of M, i.e., $\varphi(TM) \subset TM$ at every point of M.

Theorem 2.2. *An invariant submanifold M of a $(LCS)_n$-manifold $\widetilde{M}$ is also (LCS)-manifold [18].*

For a $(0,k)$-tensor field T, $k \geq 1$ and a $(0,2)$-tensor field A on a Riemannian manifold (M,g), $Q(A,T)$-tensor is defined by

$$\begin{aligned} Q(A,T)(X_1, X_2, ..., X_k; X, Y) &= -T((X \wedge_A Y)X_1, X_2, ..., X_k) \\ &- T(X_1, (X \wedge_A Y)X_2, X_3, ...X_k)... \\ &- T(X_1, X_2, ...X_{k-1}, (X \wedge_A Y)X_k), \end{aligned} \tag{20}$$

for all vector fields $X, Y, X_1, ..., X_k$ on M, where $(X \wedge_A Y)$ is an endomorphism given by $(X \wedge_A Y)X_1 = A(Y, X_1)X - A(X, X_1)Y$.

A Riemannian manifold (M,g) is said to be semi-symmetric if its Riemannian curvature tensor satisfies the condition

$$R(X,Y) \cdot R = 0, \tag{21}$$

where $R(X,Y)$ is considered as the derivation of the tensor algebra ar each point of the manifold.

THEOREM 2.3. *A* $(LCS)_n$*-manifold* $\widetilde{M}$ *is semi-symmetric, then it is of constant sectional curvature* $(\alpha^2-\rho)$.

Proof. From (21), we have

$$\begin{aligned} R(X,Y)R(U,V)Z &- R(R(X,Y)U,V)Z - R(U,R(X,Y)V)Z \\ &- R(U,V)R(X,Y)Z = 0, \end{aligned} \tag{22}$$

for all $X,Y,Z,U,V \in \Gamma(T\widetilde{M})$. Taking $Z=\xi$ in (22), we have

$$\begin{aligned} R(X,Y)R(U,V)\xi &- R(R(X,Y)U,V)\xi - R(U,R(X,Y)V)\xi \\ &- R(U,V)R(X,Y)\xi = 0. \end{aligned}$$

Taking into account (11), we obtain

$$\begin{aligned} &R(X,Y)\{(\alpha^2-\rho)(\eta(V)U-\eta(U)V)\} - (\alpha^2-\rho)\{\eta(V)R(X,Y)U \\ -\ &\eta(R(X,Y)U)V\} - (\alpha^2-\rho)\{\eta(R(X,Y)V)U - \eta(U)R(X,Y)V\} \\ -\ &(\alpha^2-\rho)R(X,Y)(\eta(Y)X-\eta(X)Y) = 0. \end{aligned}$$

This implies that

$$\begin{aligned} (\alpha^2-\rho)\{\eta(R(X,Y)U)V &- \eta(R(X,Y)V)U - \eta(Y)R(U,V)X \\ &+ \eta(X)R(U,V)Y\} = 0. \end{aligned} \tag{23}$$

Substituting (13) into (23), we obtain

$$\begin{aligned} (\alpha^2 &- \rho)\{\eta(X)g(Y,U)-\eta(Y)g(U,X)\}V - (\alpha^2-\rho)\{\eta(X)g(Y,V) \\ &- \eta(Y)g(X,V)\}U - \eta(Y)R(U,V)X + \eta(X)R(U,V)Y = 0. \end{aligned} \tag{24}$$

Let $X=\xi$ be in (24), we have

$$R(U,V)Y = (\alpha^2-\rho)\{g(V,Y)U - g(U,Y)V\}, \tag{25}$$

which proves our assertion. □

Similarly, a Riemannian manifold (M,g) is called Ricci-semi symmetric if the condition

$$R(X,Y)\cdot S = 0. \tag{26}$$

We may note that every semi-symmetric a Riemannian manifold is Ricci-semi symmetric but not conversely.

We suppose that $(LCS)_n$-manifold $\widetilde{M}$ is a Ricci-semi symmetric. Then (26) implies

$$(R(X,Y)\cdot S)(U,V) = -S(R(X,Y)U,V) - S(U,R(X,Y)V) = 0, \tag{27}$$

for all $X,Y,V,U \in \Gamma(T\widetilde{M})$. Taking $V=\xi$ in (27) and making use of (14), we obtain

$$\begin{aligned} S(R(X,Y)U,\xi) + S(R(X,Y\xi,U)) &= (n-1)(\alpha^2-\rho)\eta(R(X,Y)U) \\ &+ (\alpha^2-\rho)S(\eta(Y)X-\eta(X)Y,U) = 0. \end{aligned}$$

Since $\alpha^2 - \rho \neq o$, we can derive for $X=\xi$

$$S(Y,U) = (n-1)(\alpha^2-\rho)g(Y,U).$$

Thus we have the following proposition.

Proposition 2.4. *A Ricci-semi symmetric $(LCS)_n$-manifold $\widetilde{M}$ is an Einstein manifold.*

In this connection, a submanifold M of a Riemannian manifold $\widetilde{M}$ is said to be pseudo parallel if its second fundamental form h satisfies the condition

$$\widetilde{R}\cdot h = L_h Q(g,h), \tag{28}$$

where the equation (28) is given by

$$\begin{aligned} R^{\perp}(X,Y)h(U,V) &- h(R(X,Y)U,V) - h(U,R(X,Y)V) \\ &= -L_h\{h((X\wedge_g Y)U,V) + h(U,(X\wedge_g Y)V)\}, \end{aligned} \tag{29}$$

for all vector fields X,Y,U,V tangent to M. If $\widetilde{R}\cdot h = 0$, then the submanifold M is called semi parallel.

On the other hand, in [3], the authors defined submanifolds satisfying condition

$$\widetilde{R}\cdot h = L_S Q(S,h). \tag{30}$$

This kind of submanifold is called Ricci-generalized pseudo parallel, where the equation (30) is defined by

$$\begin{aligned} R^{\perp}(X,Y)h(U,V) &- h(R(X,Y)U,V) - h(U,R(X,Y)V) \\ &= -L_S\{h((X\wedge_S Y)U,V) + h(U,(X\wedge_S Y)V)\}. \end{aligned} \tag{31}$$

Also $\widetilde{R}\cdot\widetilde{\nabla}h$ is defined by

$$\begin{aligned}(\widetilde{R}(X,Y)\cdot\widetilde{\nabla}h)(U,V,Z) &= R^{\perp}(X,Y)(\widetilde{\nabla}_U h)(V,Z)\\ &- (\widetilde{\nabla}_{R(X,Y)U}h)(V,Z)-(\widetilde{\nabla}_U h)(R(X,Y)V,Z)\\ &- (\widetilde{\nabla}_U h)(V,R(X,Y)Z),\end{aligned}\tag{32}$$

for all $X,Y,Z,U,V\in\Gamma(TM)$.

2-semi parallel and 2-pseudo-parallel submanifolds are defined and studied K. Arslan and M. Cengizhan. They called it as following:

If $\widetilde{R}\cdot\widetilde{\nabla}h=0$, then submanifold M is said to be 2-semi parallel [2].

Furthermore, the submanifolds satisfying the condition

$$\widetilde{R}\cdot\widetilde{\nabla}h = L_{\widetilde{\nabla}h}Q(g,\widetilde{\nabla}h)\tag{33}$$

are called 2-pseudo parallel [13].

3. On Invariant Submanifolds of $(LCS)_n$-Manifolds

Theorem 3.1. *Let M be pseudo parallel invariant submanifold of a $(LCS)_n$-manifold $\widetilde{M}$. Then M is either totally geodesic or $L_h=\alpha^2-\rho$.*

Proof. Let us assume M is a pseudo parallel invariant submanifold of a $(LCS)_n$-manifold $\widetilde{M}$. Then from (28), we have

$$\begin{aligned}R^{\perp}(X,Y)h(Z,U) &- h(R(X,Y)Z,U)-h(Z,R(X,Y)U)\\ &= -L_h\{h((X\wedge_g Y)Z,U)+h(Z,(X\wedge_g Y)U)\}\\ &= -L_h\{g(Y,Z)h(X,U)-g(X,Z)h(Y,U)\\ &+ g(Y,U)h(X,Z)-g(X,U)h(Z,Y)\},\end{aligned}\tag{34}$$

for all $X,Y,Z,U\in\Gamma(TM)$. If we put $Z=\xi$ in (34), we get

$$\begin{aligned}R^{\perp}(X,Y)h(\xi,U) &- h(R(X,Y)\xi,U)-h(\xi,R(X,Y)U)\\ &= -L_h\{\eta(Y)h(X,U)-\eta(X)h(Y,U)\\ &+ g(Y,U)h(X,\xi)-g(X,U)h(Y,\xi)\}.\end{aligned}$$

Since M is invariant submanifold and ξ is tangent to M, we note that $h(X,\xi)=0$. Also by using (11), we can infer

$$(\alpha^2-\rho)h(\eta(Y)X-\eta(X)Y,U)=L_h\{\eta(Y)h(X,U)-\eta(X)h(Y,U)\}.$$

This implies that $L_h = \alpha^2 - \rho$ or $h = 0$, which proves our assertion. □

PROPOSITION 3.2. *Let M be an invariant submanifold of a $(LCS)_n$-manifold $\widetilde{M}$ such that $\alpha^2 - \rho \neq 0$. Then M is a semi parallel if and only if M is a totally geodesic.*

THEOREM 3.3. *Let M be a generalized Ricci-pseudo parallel invariant submanifold of a $(LCS)_n$-manifold $\widetilde{M}$ such that $\alpha^2 - \rho \neq 0$. Then M is either totally geodesic submanifold or $L_S = \frac{1}{n-1}$.*

Proof. Since M is a generalized Ricci-pseudo parallel invariant submanifold, from (29), we obtain

$$\begin{aligned} R^{\perp}(X,Y)h(Z,U) &- h(R(X,Y)Z,U) - h(Z,R(X,Y)U) \\ &= -L_S\{h((X \wedge_S Y)Z,U) + h(Z,(X \wedge_S Y)U)\} \\ &= -L_S\{S(Y,Z)h(X,U) - S(X,Z)h(Y,U) \\ &+ S(Y,U)h(Z,X) - S(X,U)h(Y,Z)\}, \end{aligned} \tag{35}$$

for all $X, Y, Z, U \in \Gamma(TM)$. In (35), taking $U = \xi$ and making use of (14), we obtain

$$h(R(X,Y)\xi, Z) = L_S\{S(Y,\xi)h(X,Z) - S(X,\xi)h(Y,Z)\},$$

from which

$$\begin{aligned} (\alpha^2 - \rho)h(\eta(Y)X - \eta(X)Y, Z) &= (\alpha^2 - \rho)(n-1)L_S\{\eta(Y)h(X,Z) \\ &- \eta(X)h(Y,Z)\}. \end{aligned}$$

Since $\alpha^2 - \rho \neq 0$, this implies that $h = 0$ or $L_S = \frac{1}{n-1}$. This completes of the proof. □

THEOREM 3.4. *Let M be a 2-pseudo parallel invariant submanifold of a $(LCS)_n$-manifold $\widetilde{M}$. Then M is either totally geodesic submanifold or $L_{\widetilde{\nabla}h} = \alpha^2 - \rho$.*

Proof. If M is a 2-pseudo parallel invariant submanifold of a $(LCS)_n$-manifold $\widetilde{M}$, from (34), we have

$$\begin{aligned} R^{\perp}(X,Y)(\widetilde{\nabla}_Z h)(U,V) &- (\widetilde{\nabla}_{R(X,Y)Z} h)(U,V) - (\widetilde{\nabla}_Z h)(R(X,Y)U,V) \\ &- (\widetilde{\nabla}_Z h)(U,R(X,Y)V) = -L_{\widetilde{\nabla}h}\{(\widetilde{\nabla}_{(X \wedge_g Y)Z} h)(U,V) \\ &+ (\widetilde{\nabla}_Z h)((X \wedge_g Y)U,V) + (\widetilde{\nabla}_Z h)(U,(X \wedge_g Y)Z,V)\}, \end{aligned} \tag{36}$$

for all $X, Y, Z, U, V \in \Gamma(TM)$. On the other hand, from (18), we know that

$$(\widetilde{\nabla}_X h)(Y,\xi) = \nabla^{\perp}_X h(Y,\xi) - h(\nabla_X Y,\xi) - h(Y,\nabla_X \xi) \\ = -h(\alpha\varphi X, Y) = -\alpha h(\varphi X, Y). \tag{37}$$

Taking $U = V = \xi$ in (36) and by using (37) we obtain

$$(\widetilde{\nabla}_Z h)(R(X,Y)\xi,\xi) + (\widetilde{\nabla}_Z h)(\xi, R(X,Y)\xi) \\ = L_{\widetilde{\nabla} h}\{(\widetilde{\nabla}_Z h)((X\wedge_g Y)\xi,\xi) + (\widetilde{\nabla}_Z h)((X\wedge_g Y)\xi,\xi)\}.$$

Consequently, we arrive at

$$(\widetilde{\nabla}_Z h)((\alpha^2-\rho)(\eta(Y)X - \eta(X)Y,\xi)) = L_{\widetilde{\nabla} h}\{(\widetilde{\nabla}_Z h)(\eta(Y)X - \eta(X)Y,\xi)\}.$$

Since M is a $(LCS)_n$-manifold, we know that $\alpha \neq 0$. Thus we obtain

$$\alpha(\alpha^2-\rho)h(\eta(Y)X - \eta(X)Y,\varphi Z) = L_{\widetilde{\nabla} h}\{\alpha h(\eta(Y)X - \eta(X)Y,\varphi Z)\}.$$

This implies that $L_{\widetilde{\nabla} h} = \alpha^2 - \rho$ or $h = 0$. This completes of the proof. □

PROPOSITION 3.5. *Let M be an invariant submanifold of a $(LCS)_n$-manifold $\widetilde{M}$. Then M is a 2-semi parallel if and only is M is a totally geodesic.*

References

[1] JB. S. Anitha and C.S. Bagewadi, *Invariant Submanifolds of a Sasakian Manifold*, Diff. Integral Equ. **16** (10) (2003), 1249–1280.

[2] K. Arslan, Ü. Lumiste, C. Murathan and C. Özgür, *2-Semiparallel Surfaces in Space Forms*, Proc. Estonian Acad. Sci. Phys. Math. **49** (2000), 139–148.

[3] A. C. Asperti, G. A. Lobos and F. Mercuri, *Pseudo-Parallel Immersions in Space Forms*, Math. Contemp. **17** (1999), 59–70.

[4] A. Bejancu and N. Papaqhuic, *Semi-Invariant Submanifolds of a Sasakian Manifold*, An. Sti. Univ. "AL ICUZA" Iasi **27** (1981), 163–170.

[5] V. Deepmala, K. Drachal and V. N. Mishra, *Some algebra-geometric aspects of spacetime c-boundary*, Mathematica Aeterna. **6** (4) (2016), 561–572.

[6] V. Deepmala, L. N. Mishra, *Differential operators over models and rings as a path to the generalized differential geometry*, Facta Universitatis(NIŠ). Ser. Math. Inform. **30** (5) (2015), 753–764.

[7] Z. Guojing and W. Jiangu, *Invariant Submanifolds and Modes of non Linear Autunomous System*, Appl. Math. Mech. **19** (1998), 687–693.

[8] K. Matsumoto, *On Lorentzian Almost Paracontact Manifold*, Yamagata Univ. Nat. Sci **12** (1989), 151–156.

[9] C. Murathan, K. Arslan and R. Ezentas, *Ricci Generalized Paseudo-Parallel Immersions*, Diff. Geom. and its Appl. Matfy2press Praque (2005), 99–108.

[10] L. Pişcoran and V. N. Mishra, *Projectively flatness of a new class of* (α,β)*-metrics*, Georgian Math. Journal. 2017, DOI:10.1515 gmj-2017-0034.

[11] L. Pişcoran and V. N. Mishra, *S-curvature for a new class of* (α,β)*-metrics*, Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas (RACSAM), **111** (4) (2017), 1187–1200. DOI:10.1007/s13398-016-0358-3.

[12] L. Pişcoran and V. N. Mishra, *The variational problem in Lagrange spaces endowed with a special type of* (α,β)*-metrics*, Filomat. **32** (2) (2018), 643–652.

[13] C. Özgür and C. Murathan, *On 2-Pseudo parallel Invariant Submanifolds of a Sasakain Manfifold*, [preprint].

[14] A.A. Shaikh, *On Lorentzian Almost Paracontact Manifolds with a Structure of the Concircular Type*, Kyungpook Math. J. **43** (2003), 305–314.

[15] A. A. Shaikh, *Some Results on* $(LCS)_n$*-Manifolds*, J. Korean Math. Soc. **46** (2009), 449–461.

[16] A.A, Shaikh and K.K, Baishya, *On Concircular Structure Spacetimes*, J. Math. Stat, (2005), 129-132.

[17] A.A, Shaikh and K.K, Baishya, *On Concircular Structure Spacetimes II*, Am. J. Appl. Sci. **3** (4) (2006), 1790–1794.

[18] A. A. Shaikh, Y. Matsuyama and S. K. Hui, *On Invariant Submanifolds of* $(LCS)_n$*-Manifolds*, Journal of the Egyptian Mathematical Society, **24** (2016), 263–269.

Mehmet Atçeken
Department of Mathematics
University of Gaziosmanpasa, 60100, Tokat, TURKEY
E-mail: `mehmet.atceken382@gmail.com`

Ümit Yildirim
Department of Mathematics
University of Gaziosmanpasa, 60100, Tokat, TURKEY
E-mail: `umit.yildirim@gop.edu.tr`

Süleyman Dirik
Department of Statistics
University of Amasya, 05100, Amasya, TURKEY
E-mail: `suleyman.dirik@amasya.edu.tr`