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# COMMUTATORS AND ANTI-COMMUTATORS HAVING AUTOMORPHISMS ON LIE IDEALS IN PRIME RINGS

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Abstract. In this manuscript, we discuss the relationship between prime rings and automorphisms satisfying differential identities involving commutators and anti-commutators on Lie ideals. In addition, we provide an example which shows that we cannot expect the same conclusion in case of semiprime rings.

## 1. Motivation

This work is inspired by the work of several algebraist in which they have evaluated certain identities having commutators and anti-commutators with derivations or automorphisms. In the last few decades, there has been a continuing interest pertaining to the relationship between the commutativity of a ring and the existence of certain specific types of mappings viz derivations, automorphisms etc. Herstein [13] has proven that if $\mathcal{R}$ is a prime ring with characteristic different from 2 and $\mathcal{R}$ admits a nonzero derivation $d$ such that $[x^d, y^d] = 0$ for all $x, y \in \mathcal{R}$, then $\mathcal{R}$ is commutative. While in the year 1992, Daif and Bell [9] proved that if $\mathcal{R}$ is a semiprime ring and $d$ is a nonzero derivation of $\mathcal{R}$ such that $[x, y]^d = [x, y]$ for all $x, y \in \mathcal{R}$, then $\mathcal{R}$ is commutative. In another study Ashraf and Rehman [3] proved that if $\mathcal{R}$ is a prime ring, $\mathcal{I}$ is a nonzero ideal of $\mathcal{R}$ and $d$ is a nonzero derivation of $\mathcal{R}$ such that $(x \circ y)^d = x \circ y$

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for all $x, y \in \mathcal{I}$, then $\mathcal{R}$ is commutative. Again in the year 1994, Bell and Daif [6] proved that if $\mathcal{R}$ is a semiprime ring, $\mathcal{I}$ is a nonzero ideal of $\mathcal{R}$ and $\mathcal{R}$ admits a derivation $d$ such that $[x^d, y^d] = [x, y]$ for all $x, y \in \mathcal{I}$, then $\mathcal{I} \subseteq \mathcal{Z}(\mathcal{R})$. Furthermore, when $\mathcal{R}$ is prime, it is considered $\mathcal{R}$ to be commutative. Later on, in an attempt to generalize the theorem proved by Bell and Daif [6], Deng and Ashraf [11] proved that if $\mathcal{R}$ is a semiprime ring and $\mathcal{I}$ a nonzero ideal of $\mathcal{R}$ and $\mathcal{R}$ admits a mapping $f$ and a derivation $d$ such that $[x^f, y^d] = [x, y]$ for all $x, y \in \mathcal{I}$, then $\mathcal{R}$ contains a nonzero central ideal of $\mathcal{R}$. Henceforth in 2002, Ashraf and Rehman [3] replaced commutator by anti-commutator and proved that if $\mathcal{R}$ is a 2-torsion free prime ring, $\mathcal{I}$ a nonzero ideal of $\mathcal{R}$ and $d$ a nonzero derivation of $\mathcal{R}$ such that $x^d \circ y^d = x \circ y$ for all $x, y \in \mathcal{I}$, then $\mathcal{R}$ is commutative.

On other hand, many researchers have studied and made an effort to generalize the results obtained on derivations to automorphisms. In [20], Mayne studied Posner's second theorem on derivations [21] for automorphisms of prime rings. Precisely, he proved that let $\mathcal{R}$ be a prime ring with center $Z(\mathcal{R})$ and $\xi$ be a nontrivial automorphism of $\mathcal{R}$. If $[x^\xi, x] \in Z(\mathcal{R})$ for every $x \in \mathcal{R}$, then $\mathcal{R}$ is a commutative integral domain. In [16], Lee and Lee established that if $char(\mathcal{R}) \neq 2$ and $[x^d, x] \in Z$ for all $x$ in a non-central Lie ideal $\mathscr{L}$ of $\mathcal{R}$, then $\mathcal{R}$ is commutative. An analogous extension for Lie ideals in the automorphism case was obtained by Mayne [18]. He was able to accurately draw a conclusion that let $\mathcal{R}$ be a prime ring of characteristic not equal to 2 and $\xi$ be an automorphism of $\mathcal{R}$. If $\mathscr{L}$ is a Lie ideal of $\mathcal{R}$ such that $\xi$ is nontrivial on $\mathscr{L}$ and $[x^\xi, x]$ is in the center of $\mathcal{R}$ for every $x$ in $\mathscr{L}$, then $\mathscr{L}$ is contained in the center of $\mathcal{R}$. Since then a lot of work has been done in this direction on prime and semipring rings involving automorphisms ( [1, 2, 10, 22–25] and references therein).

Persuaded by the above mentioned works, our aim is to discuss the relationship between prime rings and automorphisms satisfying differential identities having commutators and anti-commutators on Lie ideals.

## 2. Preliminaries

For a given $x, y \in \mathcal{R}$, the commutator (anti-commutator) of $x, y$ is denoted by $[x, y]$ $(x \circ y)$ and defined by $[x, y] = xy - yx$ $(x \circ y = xy + yx)$

respectively. Recall that a ring $\mathcal{R}$ is prime, if for any $a, b \in \mathcal{R}$, $a\mathcal{R}b = (0)$ implies either $a = 0$ or $b = 0$. Throughout, $\mathcal{R}$ is a prime ring with center $Z$ and $Q = Q_{mr}(\mathcal{R})$ is the maximal right ring of quotient of $\mathcal{R}$. To be noted that $Q$ is also a prime ring and the center $C$ of $Q$, which is called the extended centroid of $\mathcal{R}$, is a field. Moreover, $Z \subseteq C$ (further explanation refer to [5]). It is well known that any automorphism of $\mathcal{R}$ can be uniquely extended to an automorphism of $Q$. An automorphism $\xi$ of $\mathcal{R}$ is called $Q$-inner if there exists an invertible element $g \in Q$ such that $x^{\xi} = gxg^{-1}$ for all $x \in \mathcal{R}$. Otherwise, $\xi$ is called $Q$-outer. We symbolize by $G$ the group of all automorphisms of $\mathcal{R}$ and by $A_i$ the group consisting of all $Q$-inner automorphisms of $\mathcal{R}$. Recollect that a subset $\mathfrak{A}$ of $G$ is considered independent (modulo $A_i$) if for any $a_1, a_2 \in \mathfrak{A}$, $a_1a_2^{-1} \in A_i$ implies $a_1 = a_2$. In the same manner, if $a$ is an outer automorphism of $\mathcal{R}$, then 1 and $a$ are independent (modulo $A_i$). Herein, this work we present some well-known facts that will be used in the follow-up.

Fact 2.1. ( [8, Theorem 3]) Suppose that $\mathcal{R}$ is a prime ring and $\mathfrak{A}$ an independent subset of $G$ modulo $A_i$. Let $\phi = \chi(x_i^{a_j}) = 0$ be a generalized identity with automorphisms of $\mathcal{R}$ reduced with respect to $\mathfrak{A}$. If for all $x_i \in X$, $a_j \in \mathfrak{A}$, the $x_i^{a_j}$-degree of $\phi = \chi(x_i^{a_j})$ is strictly less than $char(\mathcal{R})$ when $char(\mathcal{R}) \neq 0$, then $\chi(z_{ij}) = 0$ is also a generalized polynomial identity of $\mathcal{R}$.

Fact 2.2. Let $\mathcal{R}$ be a prime ring and $\mathscr{L}$ a non-central Lie ideal of $\mathcal{R}$. If $char(\mathcal{R}) \neq 2$, then there exists a nonzero ideal $I$ of $\mathcal{R}$ such that $0 \neq [I, \mathcal{R}] \subseteq \mathscr{L}$. If $char(\mathcal{R}) = 2$ and $dim_C \mathcal{R}C > 4$, then there exists a nonzero ideal $I$ of $\mathcal{R}$ such that $0 \neq [I, \mathcal{R}] \subseteq \mathscr{L}$. Thus if either $char(\mathcal{R}) \neq 2$ or $dim_C \mathcal{R}C > 4$, then we may conclude that there exists a nonzero ideal $I$ of $\mathcal{R}$ such that $[I, I] \subseteq \mathscr{L}$.

Fact 2.3 ( [4, Lemma 7.1 ]). Let $\mathscr{V}_{\mathscr{D}}$ be a vector space over a division ring $\mathscr{D}$ with $dim \mathscr{V}_{\mathscr{D}} \geq 2$ and $\mathscr{S} \in End(\mathscr{V})$. If $s$ and $\mathscr{S}s$ are $\mathscr{D}$-dependent for every $s \in \mathscr{V}$, then there exists $\chi \in \mathscr{D}$ such that $\mathscr{S}s = \chi s$ for every $s \in \mathscr{V}$.

## 3. The results in Prime Rings

We begin with the following results which are indispensable to establish our principle theorem.

Proposition 3.1. *Let $\xi$ be an automorphism of $End(\mathcal{V}_\mathcal{D})$ such that for every $x_1, y_1, x_2, y_2 \in End(\mathcal{V}_\mathcal{D})$, $[x_1, x_2]^\xi \circ [y_1, y_2]^\xi = [[x_1, x_2]^\xi, [y_1, y_2]^\xi]$. If $dim(\mathcal{V}_\mathcal{D}) \geq 2$, then $\xi$ is identity map of $End(\mathcal{V}_\mathcal{D})$.*

*Proof.* By a theorem of Jacobson [14, Isomorphism Theorem, p.79], there exists an invertible semilinear transformation $T : \mathcal{V} \to \mathcal{V}$ such that $x^\xi = PxP^{-1}$ for all $x \in End(\mathcal{V}_\mathcal{D})$. In particular, there exists an automorphism $\zeta$ of $\mathcal{D}$ such that $P(v\gamma) = (Pv)\zeta(\gamma)$ for all $v \in \mathcal{V}$ and $\gamma \in \mathcal{D}$. Using our hypothesis $[x_1, x_2]^\xi \circ [y_1, y_2]^\xi = [[x_1, x_2]^\xi, [y_1, y_2]^\xi]$, we find that

$$P[x_1, x_2]P^{-1} \circ P[y_1, y_2]P^{-1} = [P[x_1, x_2]P^{-1}, P[y_1, y_2]P^{-1}]$$

for all $x, y, z \in End(\mathcal{V}_\mathcal{D})$. We could divide our proof into the following cases:

There exists $v \in \mathcal{V}$ such that $v$ and $P^{-1}v$ are $\mathcal{D}$-independent. Let's first, suppose that $\{v, Pv, P^{-1}v\}$ is $\mathcal{D}$-independent. Let $x, y, z \in End(\mathcal{V}_\mathcal{D})$ such that

$$\begin{array}{lll} x_1v = v, & x_1P^{-1}v = 0, & y_1Pv = P^{-1}v \\ y_1v = 0, & y_1P^{-1}v = P^{-1}v & \\ y_2v = Pv, & x_2P^{-1}v = v & \end{array}$$

Then $[y_1, y_2]v = P^{-1}v$, $[x_1, x_2]P^{-1}v = v$, and hence

$$\begin{aligned} 0 &= (P[x_1, x_2]P^{-1} \circ P[y_1, y_2]P^{-1} - [P[x_1, x_2]P^{-1}, P[y_1, y_2]P^{-1}])v \\ &= 2v, \text{ a contradiction} \end{aligned}$$

Suppose next that $\{v, Pv, P^{-1}v\}$ is $\mathcal{D}$-dependent. Then there exist $\mu, \chi \in \mathcal{D}$ such that $Pv = v\mu + P^{-1}v\chi$. Moreover, we claim that $\chi \neq 0$. Indeed, if $\chi = 0$, then $Tv = v\mu$ and $v = P^{-1}v\mu$, a contradiction. Let $x, y, z \in End(\mathcal{V}_\mathcal{D})$ such that

$$\begin{array}{ll} x_1v = v, & x_1P^{-1}v = 0 \\ y_1v = 0, & y_1P^{-1}v = P^{-1}v \\ y_2v = v\mu + P^{-1}v\chi, & x_2P^{-1}v = v \end{array}$$

We can easily see that

$$\begin{aligned} 0 &= (P[x_1, x_2]P^{-1} \circ P[y_1, y_2]P^{-1} - [P[x_1, x_2]P^{-1}, P[y_1, y_2]P^{-1}])v \\ &= 2v\chi, \text{ a contradiction} \end{aligned}$$

We have that $v$ and $P^{-1}v$ are $\mathcal{D}$-dependent for every $v \in \mathcal{V}$. By Fact 2.3 $P^{-1}v = v\alpha$ for all $v \in \mathcal{V}$, where $\alpha \in \mathcal{D}$. Therefore $P^{-1}(xv) = xv\alpha$ for all $x \in End(\mathcal{V}_{\mathcal{D}})$, the same for $xv = P(xv\alpha) = P(x(v\alpha)) = PxP^{-1}(v) = x^{\xi}v$ for all $x \in End(\mathcal{V}_{\mathcal{D}})$ and $v \in \mathcal{V}$. In particular, $(x^{\xi} - x)V = 0$ for all $x \in End(\mathcal{V}_{\mathcal{D}})$. Thus $x^{\xi} = x$ for all $x \in End(\mathcal{V}_{\mathcal{D}})$. This implies $\xi$ is the identity map of $End(\mathcal{V}_{\mathcal{D}})$, proving the proposition. □

THEOREM 3.1. *Let $\mathcal{R}$ be a prime ring of characteristic different from 2 and $\xi$ be an automorphism of $\mathcal{R}$ such that $x^{\xi} \circ y^{\xi} = [x^{\xi}, y^{\xi}]$ for all $x, y \in \mathscr{L}$, a nonzero Lie ideal of $\mathcal{R}$. Then $\mathscr{L}$ contained in the center of $\mathcal{R}$.*

*Proof.* On the contrary suppose that $\mathscr{L}$ is non-central. By given hypothesis and Fact 2.2, there exists a nonzero ideal $I$ of $\mathcal{R}$ such that $0 \neq [I, I] \subseteq \mathscr{L}$. Also, $\mathcal{R}$ is non-commutative as $\mathscr{L}$ is non-central Lie ideal of $\mathcal{R}$. Accordingly, we have

$$[x_1, x_2]^{\xi} \circ [y_1, y_2]^{\xi} = [[x_1, x_2]^{\xi}, [y_1, y_2]^{\xi}] \tag{3.1}$$

for all $x_1, y_1, x_2, y_2 \in I$. Firstly, we suppose that $\xi$ is an identity automorphism and hence we can easily observe that $2[y_1, y_2][x_1, x_2] = 0$ for all $x_1, y_1, x_2, y_2 \in I$. This gives,

$$[y_1, y_2][x_1, x_2] = 0$$

for all $x_1, y_1, x_2, y_2 \in I$. Replace $x_1$ by $x_1 r$, where $x_1 \in I$ and $r \in \mathcal{R}$, we have

$$[y_1, y_2]x_1[r, x_2] + [y_1, y_2][x_1, x_2]r = 0$$

and hence

$$[y_1, y_2]x_1[r, x_2] = (0)$$

for all $x_1, y_1, x_2, y_2 \in I$ and $r \in \mathcal{R}$. This implies

$$[y_1, y_2]I[r, x_2] = (0).$$

Thus

$$[y_1, y_2]I\mathcal{R}[r, x_2]I = (0).$$

Therefore either $[y_1, y_2] = 0$ or $[r, x_2] = 0$, which gives in each case $[I, \mathcal{R}] = (0)$. This leads to a contradiction that $\mathcal{R}$ is commutative [19].

Next, we suppose that $\xi$ is a non-identity automorphism. Suppose that $\xi$ is a $Q$-inner automorphism. In this case, there exists an invertible element $p \in Q$ such that $x^{\xi} = pxp^{-1}$ for all $x \in \mathcal{R}$. Then $I$ satisfies

$$p[x_1, x_2]p^{-1} \circ p[y_1, y_2]p^{-1} = [p[x_1, x_2]p^{-1}, p[y_1, y_2]p^{-1}] \tag{3.2}$$

By a theorem of Chuang [7], $I$ and $Q$ satisfy the same generalized polynomial identities. Thus $Q$ satisfies

$$p[x_1,x_2]p^{-1} \circ p[y_1,y_2]p^{-1} = [p[x_1,x_2]p^{-1}, p[y_1,y_2]p^{-1}] \tag{3.3}$$

Thus this is a nontrivial generalized polynomial identity on $Q$ as $p \notin C$. Denote by $F$ the algebraic closure of $C$ if $C$ is infinite and set $F = C$ for $C$ finite. Then $Q \otimes_C F$ is a prime ring with extended centroid $F$ [12, Theorem 3.5]. Clearly $Q \cong Q \otimes_C C \subseteq Q \otimes_C F$. So we may regards $Q$ as a subring $Q \otimes_C F$ and hence (3.3) is also a nontrivial generalized polynomial identity of $Q \otimes_C F$. Let $\mathcal{Q} = Q_{mr}(Q \otimes_C F)$, the maximal right ring of quotients of $Q \otimes_C F$. By [5, Theorem 6.4.4], (3.3) is also a nontrivial generalized polynomial identity on $\mathcal{Q}$. By Martindale's theorem [17], $\mathcal{Q} \cong End(\mathcal{V}_{\mathcal{D}})$, where $\mathcal{V}$ is a vector space over a division ring $\mathcal{D}$ and $\mathcal{D}$ is finite dimension over its center $F$. Recall that $F$ is either algebraically closed or finite. From the finite dimensionality of $D$ over $F$, it follows that $\mathcal{D} = F$. Hence $\mathcal{Q} \cong End(\mathcal{V}_F)$. By Proposition 3.1, we get a contradiction.

We now assume that $\xi$ is a $Q$-outer automorphism. By Chuang [7, Main Theorem] $I$ and $Q$ satisfies the same polynomial identities and hence $\mathcal{R}$ does as well. Thus,

$$[x_1,x_2]^{\xi} \circ [y_1,y_2]^{\xi} = [[x_1,x_2]^{\xi}, [y_1,y_2]^{\xi}]$$

for all $x_1, y_1, x_2, y_2 \in \mathcal{R}$. Since $x_i, y_i$-degree is less that $char(\mathcal{R})$, therefore by Fact 2.1, $\mathcal{R}$ satisfies

$$[x_1', x_2'] \circ [y_1', y_2'] = [[x_1', x_2'], [y_1', y_2']]$$

for all $x_1', y_1', x_2', y_2' \in \mathcal{R}$. Note that this is a polynomial identities and thus there exists a field $\mathbb{F}$ such that $\mathcal{R} \subseteq M_k(\mathbb{F})$, the ring of $k \times k$ matrices over a field $\mathbb{F}$, where $k > 1$. Moreover, $\mathcal{R}$ and $M_k(\mathbb{F})$ satisfy the same polynomial identities [15, Lemma 1], that is

$$2[y_1', y_2'][x_1', x_2'] = 0$$

for all $x_1', y_1', x_2', y_2' \in M_k(\mathbb{F})$. Let $e_{ij}$ be a matrix unit with 1 in the $(i,j)$-entry and zero elsewhere. By choosing $x_1 = e_{12}, x_2 = e_{21}$, $y_1 = e_{11}, y_2 = e_{12}$, we get $0 = 2[y_1,y_2][x_1,x_2] = 2[e_{11},e_{12}][e_{12},e_{21}] = -2e_{12}$, a contradiction and hence proof is completed. □

As a result above theorem, we can easily get the following corollary

Corollary 3.1. *Let $\mathcal{R}$ be a prime ring of characteristic different from 2 and $\xi$ be an automorphism of $\mathcal{R}$ such that $x^\xi \circ y^\xi = [x^\xi, y^\xi]$ for all $x, y \in [\mathcal{R}, \mathcal{R}]$. Then $\mathcal{R}$ is commutative.*

Now, we are ready to prove our principle theorem.

Theorem 3.2. *Let $\mathcal{R}$ be a prime ring of characteristic different from 2 and $\xi$ be an automorphism of $\mathcal{R}$ such that $(x^n)^\xi \circ (y^n)^\xi = [(x^n)^\xi, (y^n)^\xi]$ for all $x, y \in \mathcal{R}$, where $n$ is a fixed positive integer. Then $\mathcal{R}$ is commutative.*

*Proof.* We are given that $(x^n)^\xi \circ (y^n)^\xi - [(x^n)^\xi, (y^n)^\xi] = 0$ for all $x, y \in \mathcal{R}$. Let $S$ be the additive subgroup generated by the subset $\{r^n | r \in \mathcal{R}\}$. It is easy to see that $x^\xi \circ y^\xi - [x^\xi, y^\xi] = 0$ for all $x, y \in S$. By main theorem of [7], and since $char(\mathcal{R}) \neq 2$, we have either $S$ contains a non-central Lie ideal $\mathscr{L}$ of $\mathcal{R}$ or $r^m \in Z(\mathcal{R})$ for all $r \in \mathcal{R}$. It is well known that the latter case forces $\mathcal{R}$ to be commutative. Moreover, by Fact 2.2, there exist $I$ nonzero two-sided ideals of $\mathcal{R}$ such that $0 \neq [I, \mathcal{R}] \subseteq \mathscr{L}$. Therefore $x^\xi \circ y^\xi - [x^\xi, y^\xi] = 0$ for all $x, y \in [I, I]$. Since $I$ and $\mathcal{R}$ satisfy the same differential identities (see [15, Theorem 3]), so we have $x^\xi \circ y^\xi - [x^\xi, y^\xi] = 0$ for all $x, y \in [\mathcal{R}, \mathcal{R}]$. Applying Corollary 3.1, we are done. □

The following example shows the assumption that $\mathcal{R}$ should necessarily be prime in Theorem 3.1.

Example 3.1. Let $\mathcal{S} = \mathscr{M}_2(\mathbb{F})$ denote the ring of $2 \times 2$ matrices over a field $\mathbb{F}$. Let $\mathcal{R} = \mathscr{M}_2(\mathbb{F}) \oplus \mathbb{M}_2(\mathbb{F})$ and $\mathscr{L} = \mathscr{M}_2(\mathbb{F}) \oplus 0$. Then $\mathcal{R}$ is a semiprime ring and $\mathscr{L}$ is a nonzero Lie ideal of $\mathcal{R}$. We define $\xi : \mathcal{R} \to \mathcal{R}$ as follows $(x_1, x_2)^\xi = (x_2, x_1)$. It can be easily seen that $\xi$ is an automorphism which satisfying $x^\xi \circ y^\xi = [x^\xi, y^\xi]$ for all $x, y \in \mathscr{L}$.

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