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CONFORMAL CHANGE OF THE TENSOR $S_{\lambda\mu}{}^{\nu}$ FOR THE SECOND CATEGORY IN 6-DIMENSIONAL g-UFT

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Abstract. We investigate change of the torsion tensor induced by the conformal change in 6-dimensional g-unified field theory. These topics will be studied for the second class with the second category in 6-dimensional case.

1. Introduction

The conformal change in a generalized 4-dimensional Riemannian space connected by an Einstein's connection was primarily studied by HLAVATÝ([8],1957). CHUNG([6],1968) also investigated the same topic in 4-dimensional ${}^*g$-unified field theory.

The Einstein's connection induced by the conformal change for all classes in 3-dimensional case, for the second and third classes in 5-dimensional case, and for the first class in 5-dimensional case, and for the second class with the first category in 6-dimensional case were investigated by CHO([1],1992, [2],1994, [3],1995).

In the present paper, we investigate change of the torsion tensor $S_{\omega\mu}{}^{\nu}$ induced by the conformal change in 6-dimensional g-unified field theory. These topics will be studied for the second class with the second category in 6-dimensional case.

2. Preliminaries

This chapter is a brief collection of basic concepts, notations, theorems, and results needed in our further considerations. They may be

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reffered to CHUNG([4],1982;[3],1988), CHO([1],1992;[2],1994;[3],1995).

2.1. n-dimensional g-unified field theory

The n-dimensional g-unified field theory (n-g-UFT hereafter) was originally suggested by HLAVATÝ([8],1957) and systematically introduced by CHUNG([7],1963).

Let ${X_n}^1$ be an n-dimensional generalized Riemannian manifold, reffered to a real coordinate system x^ν obeying coordinate transformations $x^\nu \to x^{\nu'}$, for which

$$\text{Det}\left(\left(\frac{\partial x}{\partial x'}\right)\right) \neq 0. \tag{2.1}$$

In the usual Einstein's n-dimensional unified field theory, the manifold X_n is endowed with a general real nonsymmetric tensor g_{λ_μ} which may be split into its symmetric part $h_{\lambda\mu}$ and skew-symmetric part ${k_{\lambda\mu}}^2$:

$$g_{\lambda\mu} = h_{\lambda\mu} + k_{\lambda\mu} \tag{2.2}$$

where

$$\text{Det}((g_{\lambda\mu})) \neq 0 \qquad \text{Det}((h_{\lambda\mu})) \neq 0. \tag{2.3}$$

Therefore we may define a unique tensor $h^{\lambda\nu} = h^{\nu\lambda}$ by

$$h_{\lambda\mu}h^{\lambda\nu} = \delta^\nu_\mu. \tag{2.4}$$

In our n-g-UFT, the tensors $h_{\lambda\mu}$ and $h^{\lambda\nu}$ will serve for raising and/or lowering indices of the tensors in X_n in the usual manner.

The manifold X_n is connected by a general real connection $\Gamma^\nu_{\omega\mu}$ with the following transformation rule :

$$\Gamma^{\nu'}_{\omega'\mu'} = \frac{\partial x^{\nu'}}{\partial x^\alpha}\left(\frac{\partial x^\beta}{\partial x^{\omega'}}\cdot\frac{\partial x^\gamma}{\partial x^{\mu'}}\Gamma^\alpha_{\beta\gamma} + \frac{\partial^2 x^\alpha}{\partial x^{\omega'}\partial x^{\mu'}}\right) \tag{2.5}$$

[1]Throughout the present paper, we assumed that $n \geq 2$.

[2]Throughout this paper, Greek indices are used for holonomic components of tensors. In X_n all indices take the values $1, \cdots, n$ and follow the summation convention.

and satisfies the system of Einstein's equations

$$D_\omega g_{\lambda\mu} = 2S_{\omega\mu}{}^{\alpha} g_{\lambda\alpha} \tag{2.6}$$

where D_ω denotes the covariant derivative with respect to $\Gamma^{\nu}_{\lambda\mu}$ and

$$S_{\lambda\mu}{}^{\nu} = \Gamma^{\nu}_{[\lambda\mu]} \tag{2.7}$$

is the *torsion tensor* of $\Gamma^{\nu}_{\lambda\mu}$. The connection $\Gamma^{\nu}_{\lambda\mu}$ satisfying (2.6) is called the *Einstein's connection*.

In our further considerations, the following scalars, tensors, abbreviations, and notations for $p = 0, 1, 2, \cdots$ are frequently used :

$$\mathfrak{g} = \mathrm{Det}((g_{\lambda\mu})) \neq 0, \quad \mathfrak{h} = \mathrm{Det}((h_{\lambda\mu})) \neq 0, \quad \mathfrak{k} = \mathrm{Det}((k_{\lambda\mu})), \tag{2.8}a$$

$$g = \frac{\mathfrak{g}}{\mathfrak{h}}, \qquad k = \frac{\mathfrak{k}}{\mathfrak{h}}, \tag{2.8}b$$

$$K_p = k_{[\alpha_1}{}^{\alpha^1} \cdots k_{\alpha_p]}{}^{\alpha_p}, \quad (p = 0, 1, 2, \cdots) \tag{2.8}c$$

$${}^{(0)}k_\lambda{}^{\nu} = \delta^{\nu}_{\lambda}, \qquad {}^{(1)}k_\lambda{}^{\nu} = k_\lambda{}^{\nu}, \qquad {}^{(p)}k_\lambda{}^{\nu} = {}^{(p-1)}k_\lambda{}^{\alpha} k_\alpha{}^{\nu}, \tag{2.8}d$$

$$K_{\omega\mu\nu} = \nabla_\nu k_{\omega\mu} + \nabla_\omega k_{\nu\mu} + \nabla_\mu k_{\omega\nu}, \tag{2.8}e$$

$$\sigma = \begin{cases} 1 & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}. \tag{2.8}f$$

where ∇_ω is the symbolic vector of the convariant derivative with respect to the Christoffel symbols $\left\{ {\nu \atop \lambda\mu} \right\}$ defined by $h_{\lambda\mu}$. The scalars and vectors introduced in (2.8) satisfy

$$K_0 = 1; K_n = k \text{ if } n \text{ is even}; \qquad K_p = 0 \text{ if } p \text{ is odd}, \tag{2.9}a$$

$$(2.9)b \qquad g = 1 + K_2 + \cdots + K_{n-\sigma},$$

$$(2.9)c \qquad {}^{(p)}k_{\lambda\mu} = (-1)^{p(p)}k_{\mu\lambda}, \qquad {}^{(p)}k^{\lambda\nu} = (-1)^{p(p)}k^{\nu\lambda}.$$

Furthermore, we also use the following useful abbrevations, denoting an arbitrary tensor $T_{\omega\mu\nu}$, skew-symmetric in the first two indices, by T :

$$(2.10)a \qquad \overset{pqr}{T} = \overset{pqr}{T}{}_{\omega\mu\nu} = T_{\alpha\beta\gamma}{}^{(p)}k_{\omega}{}^{\alpha(q)}k_{\mu}{}^{\beta(r)}k_{\nu}{}^{\gamma},$$

$$(2.10)b \qquad T = T_{\omega\mu\nu} = \overset{000}{T},$$

$$(2.10)c \qquad 2\overset{pqr}{T}{}_{\omega[\lambda\mu]} = \overset{pqr}{T}{}_{\omega\lambda\mu} - \overset{pqr}{T}{}_{\omega\mu\lambda},$$

$$(2.10)d \qquad 2\overset{(pq)r}{T}{}_{\omega\lambda\mu} = \overset{pqr}{T}{}_{\omega\lambda\mu} + \overset{qpr}{T}{}_{\omega\lambda\mu}.$$

We then have

$$(2.11) \qquad \overset{pqr}{T}{}_{\omega\lambda\mu} = -\overset{qpr}{T}{}_{\lambda\omega\mu}.$$

If the system (2.6) admits $\Gamma^{\nu}_{\lambda\mu}$, using the above abbreviations it was shown that the connection is of the form

$$(2.12) \qquad \Gamma^{\nu}_{\omega\mu} = \left\{ {\nu \atop \omega\mu} \right\} + S_{\omega\mu}{}^{\nu} + U^{\nu}{}_{\omega\mu}$$

where

$$(2.13) \qquad U_{\nu\omega\mu} = \overset{100}{S}{}_{(\omega\mu)\nu} + \overset{(10)0}{S}{}_{\nu(\omega\mu)}.$$

The above two relations show that *our problem of determining* $\Gamma^{\nu}_{\omega\mu}$ *in terms of* $g_{\lambda\mu}$ *is reduced to that of studying the tensor* $S_{\omega\mu}{}^{\nu}$. On the other hand, it has also been shown that the tensor $S_{\omega\mu}{}^{\nu}$ satisfies

$$(2.14) \qquad S = B - 3\overset{(110)}{S}$$

where

$$(2.15) \qquad 2B_{\omega\mu\nu} = K_{\omega\mu\nu} + 3K_{\alpha[\mu\beta}k_{\omega]}{}^{\alpha}k_{\nu}{}^{\beta}.$$

2.2. Some results for the second class with the second category in 6-g-UFT

In this section, we introduce some results of 6-g-UFT without proof, which are needed in our subsequent considerations.

They may be referred to CHO([4],1993).

DEFINITION 2.1. In 6-g-UFT, the tensor $g_{\lambda\mu}(k_{\lambda\mu})$ is said to be the second class with the second category, if $K_4 \neq 0$, $K_6 = 0$.

THEOREM 2.2. *(Main recurrence relations) For the second class with the second category in 6-UFT, the following recurrence relation hold*

$$(2.16)\qquad {}^{(p+4)}k_{\lambda}{}^{\nu} = -K_2{}^{(p+2)}k_{\lambda}{}^{\nu} - K_4{}^{(p)}k_{\lambda}{}^{\nu}, \qquad (p = 0, 1, 2, \cdots).$$

THEOREM 2.3. *(For the second class with the second category in 6-g-UFT). A necessary and sufficient condition for the existence and uniqueness of the solution of (2.5) is*

$$(2.17)\qquad \begin{aligned}(1 + K_2 + K_4)(1 - K_2 + K_4)(1 - K_4)(1 - 3K_2 + 9K_4)\times\\ \times [(1 - K_2 - 3K_4)^2 - 4K_4((K_2)^2 - 4K_4)] \neq 0.\end{aligned}$$

If the condition (2.17) is satisfied, the unique solution of (2.14) is given by

(2.18)

$$\begin{aligned}&(S - B)(1 + K_2 + K_4)[(1 - K_2 + 5K_4)^2 - 4K_4(2 - K_2)^2]\\ =&4\underset{1}{B}(K_4 - 1) + \underset{2}{B}(1 - K_2 + 5K_4) + 2\underset{3}{B}(1 - 2K_2 + (K_2)^2 - 5K_4)\end{aligned}$$

where

$$\begin{aligned}\underset{1}{B} =&(K_4)^2 B + 2\overset{(12)3}{B} + K_2K_4\overset{002}{B} + (2K_4 - (K_2)^2)\overset{112}{B} -\\ &- 2K_4\overset{(12)1}{B} + K_4(2 + 2K_2 + (K_2)^2)\overset{110}{B} + K_2\overset{222}{B} +\\ &+ 2K_4\overset{(20)2}{B} - 2K_4(1 + K_2)\overset{(10)3}{B} - K_4(1 + K_2)\overset{220}{B} -\\ &- 2K_4(1 + K_2)^2\overset{(10)1}{B}\end{aligned}$$

$$\begin{aligned}\underset{2}{B} =& -(K_4)^2 B + 2((K_2)^2 - 1 + K_4 + 2K_2K_4)\overset{(10)1}{B} + (2+K_2)\overset{112}{B} - \\ & - \overset{222}{B} - K_4\overset{002}{B} + 2\overset{(20)2}{B} + 2(K_2+2K_4)\overset{(10)3}{B} + 2K_4\overset{(20)0}{B} - \\ & - ((K_2)^2 - 1 + K_4 + 2K_2K_4)\overset{110}{B} + (K_2 - 1 + 2K_4)\overset{220}{B}\end{aligned}$$

$$\begin{aligned}\underset{3}{B} =& 2(K_4)^2 B + 2\overset{(12)3}{B} - K_4\overset{002}{B} + K_2\overset{112}{B} + 2(1+K_2)\overset{(21)1}{B} - \\ & - \overset{222}{B} + 2K_4\overset{(10)3}{B} - (1+K_4)(1+K_2)\overset{110}{B} + (1+K_4)\overset{220}{B} + \\ & + 2K_4(1+K_2)\overset{(10)1}{B} - 2K_4\overset{(20)0}{B}.\end{aligned}$$

3. Conformal change of the 6-dimensional torsion tensor for the second class with the second category

In this final chapter we investigate the change $S_{\lambda\mu}{}^{\nu} \to \overline{S}_{\lambda\mu}{}^{\nu}$ of the torsion tensor induced by the conformal change of the tensor $g_{\lambda\mu}$, using the recurrence relations and theorems introduced in the preceding chapter.

We say that X_n and $\overline{X}_n$ are conformal if and only if

$$\overline{g}\lambda\mu(x) = e^{\Omega} g_{\lambda\mu}(x) \tag{3.1}$$

where $\Omega = \Omega(x)$ is an at least twice differentiable function. This conformal change enforces a change of the torsion tensor $S_{\lambda\mu}{}^{\nu}$. An explicit representation of the change of 6-dimensional torsion tensor $S_{\lambda\mu}{}^{\nu}$ for the second class with the second category will be exhibited in this chapter.

Agreement 3.1. Throughout this section, we agree that, if T is a function of $g_{\lambda\mu}$, then we denote $\overline{T}$ the same function of $\overline{g}_{\lambda\mu}$. In particular, if T is a tensor, so is $\overline{T}$. Furthermore, the indices of T ($\overline{T}$) will be raised and/or lowered by means of $h^{\lambda\nu}(\overline{h}^{\lambda\nu})$ and/or $h_{\lambda\mu}(\overline{h}_{\lambda\mu})$.

The results in the following theorems are needed in our further considerations. They may be referred to CHO([1],1992, [2],1994, [3],1995).

THEOREM 3.2. *In n-g-UFT, the conformal change (3.1) induces the following changes :*

$$(3.2)a \qquad \begin{aligned} {}^{(p)}\overline{k}_{\lambda\mu} &= e^{\Omega(p)}k_{\lambda\mu}, \qquad {}^{(p)}\overline{k}_{\lambda}{}^{\nu} = {}^{(p)}k_{\lambda}{}^{\nu}, \\ {}^{(p)}\overline{k}^{\lambda\nu} &= e^{-\Omega(p)}k^{\lambda\nu} \end{aligned}$$

$$(3.2)b \qquad \overline{g} = g, \qquad \overline{K_p} = K_p, \qquad (p = 1, 2, \cdots).$$

THEOREM 3.3. *(For all classes in 6-g-UFT). The change of the tensor $B_{\omega\mu\nu}$ induced by the conformal change (3.1) may be given by*

$$(3.3) \qquad \begin{aligned} \overline{B}_{\omega\mu\nu} =& e^{\Omega}(B_{\omega\mu\nu} + k_{\nu[\omega}\Omega_{\mu]} - k_{\omega\mu}\Omega_{\nu} \\ & - h_{\nu[\omega}k_{\mu]}{}^{\delta}\Omega_{\delta} + 2^{(2)}k_{\nu[\omega}k_{\mu]}{}^{\delta}\Omega_{\delta} + k_{\omega\mu}{}^{(2)}k_{\nu}{}^{\delta}\Omega_{\delta}). \end{aligned}$$

Now, we are ready to derive representations of the changes $S_{\omega\mu}{}^{\nu} \to \overline{S}_{\omega\mu}{}^{\nu}$ in 6-g-UFT for the second class with the second category induced by the conformal change (3.1).

THEOREM 3.4. *The conformal change (3.1) induces the following change :*

$$(3.4) \qquad \begin{aligned} 2\overline{\overset{(10)1}{B}}_{\omega\mu\nu} =& e^{\Omega}[2\overset{(10)1}{B}_{\omega\mu\nu} + (-2^{(4)}k_{\nu[\omega}k_{\mu]}{}^{\delta} \\ & + 2^{(2)}k_{\nu[\omega}k_{\mu]}{}^{\delta} - k_{\nu[\omega}{}^{(2)}k_{\mu]}{}^{\delta})\Omega_{\delta} - {}^{(3)}k_{\nu[\omega}\Omega_{\mu]}]. \end{aligned}$$

THEOREM 3.5. *The conformal change (3.1) induces the following change :*

$$(3.5) \qquad \begin{aligned} \overline{\overset{ppq}{B}}_{\omega\mu\nu} =& e^{\Omega}[\overset{ppq}{B}_{\omega\mu\nu} + (-1)^{p}\{2^{(p+q+2)}k_{\nu[\omega}{}^{(p+1)}k_{\mu]}{}^{\delta} \\ & + {}^{(2p+1)}k_{\omega\mu}{}^{(2+q)}k_{\nu}{}^{\delta} - {}^{(2p+1)}k_{\omega\mu}{}^{(q)}k_{\nu}{}^{\delta} \\ & + {}^{(p+q+1)}k_{\nu[\omega}{}^{(p)}k_{\mu]}{}^{\delta} - {}^{(p+q)}k_{\nu[\omega}{}^{(p+1)}k_{\mu]}{}^{\delta}\}\Omega_{\delta}]. \\ & \qquad \begin{pmatrix} p = 0, 1, 2, 3, 4, \cdots \\ q = 0, 1, 2, 3, 4, \cdots \end{pmatrix} \end{aligned}$$

THEOREM 3.6. *The change* $S_{\omega\mu}{}^{\nu} \to \overline{S}_{\omega\mu}{}^{\nu}$ *induced by conformal change (3.1) may be represented by*

$$
\begin{aligned}
\overline{S}_{\omega\mu}{}^{\nu} =& S_{\omega\mu}{}^{\nu} + \frac{1}{C}[a_1 k_{\omega\mu}\Omega^{\nu} + a_2 k^{\nu}{}_{[\omega}\Omega_{\mu]} \\
&+ a_3 h^{\nu}{}_{[\omega}k_{\mu]}{}^{\delta}\Omega_{\delta} + a_4\delta^{\nu}{}_{[\omega}k_{\mu]} \\
&+ a_5 k^{\nu}{}_{[\omega}{}^{(2)}k_{\mu]}{}^{\delta}\Omega_{\delta} + a_6{}^{(2)}k^{\nu}{}_{[\omega}k_{\mu]}{}^{\delta}\Omega_{\delta} \\
&+ a_7 k_{\omega\mu}{}^{(2)}k^{\nu\delta}\Omega_{\delta} + a_8{}^{(3)}k_{\omega\mu}\Omega^{\nu} \\
&+ a_9{}^{(3)}k^{\nu}{}_{[\omega}\Omega_{\mu]} + a_{10}\delta^{\nu}{}_{[\omega}{}^{(3)}k_{\mu]}{}^{\delta}\Omega_{\delta} \\
&+ 2a_{11}{}^{(3)}k^{\nu}{}_{[\omega}{}^{(2)}k_{\mu]}{}^{\delta}\Omega_{\delta} + 2a_{12}{}^{(2)}k^{\nu}{}_{[\omega}{}^{(3)}k_{\mu]}{}^{\delta}\Omega_{\delta} \\
&+ a_{13}{}^{(3)}k_{\omega\mu}{}^{(2)}k^{\nu\delta}\Omega_{\delta}],
\end{aligned} \tag{3.6}
$$

where

$$
\begin{aligned}
a_1 =& \alpha^2\beta(1+4\beta) - 2\alpha\beta(1+\beta+2\beta^2) + \beta(1-13\beta^2) - C, \\
a_2 =& 2\alpha^3\beta - \alpha^2\beta(\alpha-2\beta) + 2\alpha\beta^2(1-2\beta) + \beta^2(3\beta-4) + C, \\
a_3 =& \beta^2(2\alpha^2 - 5\alpha - 9\beta + 7) - C, \\
a_4 =& -2\alpha^3\beta + \alpha^2\beta(1+12\beta) - 9\alpha\beta^2 - \beta(3+5\beta+18\beta^2), \\
a_5 =& 2\alpha^4 - \alpha^3(2\beta+3) - \alpha^2(1+9\beta+4\beta^2) \\
&+ \alpha(2-10\beta-\beta^2+8\beta^3) + \beta(6+13\beta+19\beta^2), \\
a_6 =& -2\alpha^4 + \alpha^3(1+18\beta) + 2\alpha^2\beta(1-8\beta) - \alpha(2+16\beta \\
&+ 59\beta^2 + 8\beta^3) + \beta(27\beta^2 - 58\beta - 10) - 1 + 2C, \\
a_7 =& -\alpha^2\beta(1+4\beta) + 2\alpha\beta(1+\beta) + \beta(13\beta^2 + 4\alpha\beta^2 - 1) + C, \\
a_8 =& 3\alpha^3 + \alpha^2(5\beta + 8\beta^2 - 4) - \alpha(2 + 36\beta + 5\beta^2) \\
&+ 7\beta(2 - 6\beta - 3\beta^2) + 3, \\
a_9 =& \alpha^2(1-8\beta) - 2\alpha(1-6\beta^2) + \beta(8\beta^2 + 35\beta - 12) + 1, \\
a_{10} =& 2\alpha^2\beta(-5+2\beta) + 2\alpha\beta(3 - 6\beta + 4\beta^2) + 4\beta(1 + 2\beta - 2\beta^2), \\
a_{11} =& 2\alpha^4 - \alpha^3(1+3\beta) - 4\alpha^2\beta^2 + \alpha(1 + 7\beta + 4\beta^2) \\
&- \beta(3 - 7\alpha - 4\alpha\beta) - 2, \\
a_{12} =& 2\alpha^4 + \alpha^3(2\beta - 15) + \alpha^2(22 - 19\beta + 4\beta^2) \\
&+ \alpha(-8 + 35\beta - 6\beta^2) - 3\beta + 1,
\end{aligned}
$$

$$a_{13} = -4\alpha^4 - \alpha^3(1-8\beta) + 11\alpha^2\beta - \alpha(8-16\beta+21\beta^2) + \beta(5\beta^2+2\beta-10) - 3,$$

where $\alpha = K_2$, $\beta = K_4$,

$$C = (1+\alpha+\beta)[(1-\alpha+5\beta)^2 - 4\beta(2-\alpha)^2]. \tag{3.7}$$

Proof. In virtue of (2.18) and Agreement (3.1), we have

$$\begin{aligned}(\overline{S}-\overline{B})(1+\overline{K_2}+\overline{K_4}) &\times [(1-\overline{K}_2+5\overline{K}_4)^2 - 4\overline{K}_4(2-\overline{K}_2)^2] \\ =4\underset{1}{\overline{B}}(\overline{K}_4-1) &+ \underset{2}{\overline{B}}(1-\overline{K}_2+5\overline{K}_4) + 2\underset{3}{\overline{B}}(1-2\overline{K}_2+(\overline{K}_2)^2-5\overline{K}_4).\end{aligned} \tag{3.8}$$

The relation (3.6) follows by substituting (3.2), (3.3), (3.4), (3.5), (2.16), (3.7) into (3.8). □ □

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