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THE GENERALIZED OPEN SETS ON SUPRATOPOLOGY

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Abstract. We introduce the notion of $s\gamma$-sets, and we investigate some properties of $s\gamma$-sets. In particular, we characterize the $s\gamma$-closure by terms of supra-convergence of filters

1. Introduction

Let X be a nonempty set. A subclass $\tau \subset P(X)$ is called a supratopology on X[3] if $X \in \tau$ and τ is closed under arbitrary union. (X, τ) is called a supratopological space. The members of τ are called supraopen sets. The complement of supraopen sets are called supraclosed sets. Let (X, τ) be a supratopological space and $S \subset X$. The supra-closure of S, denoted by $scl(S)$, is the intersection of supraclosed sets including S. And the interior of S, denoted by $sint(S)$, the union of supraopen sets included in S. Let (X, τ) be a topological space. Then τ^* is called a supratopology associated with τ if $\tau \subset \tau^*$. Let (X, τ) and (X, μ) be topological spaces and let τ^* and μ^* be associated supratopologies with τ and μ, respectively. Let (X, τ) be a topological space and $S \subset X$. The closure (resp. interior) of S will be denoted by $cl(S)$ (resp. $int(S)$). A subset S of X is called a semi-open set [2] if $S \subset cl(int(S))$. The complement of a semi-open set is called a semi-closed set. The family of all semi-open sets in X will be denoted by $SO(X)$.

A subset $M(x)$ of a space X is called a supra(resp. semi)-neighborhood of a point x in X if there exists a supra(resp. semi)open set S such that

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$x \in S \subset M(x)$. In [1], R. M. Latif introduced the notion of semi-convergences of filters. And he investigated some characterizations related to semi-continuous functions. Now we recall the concept of semi-convergences of filters. Let $SO(x) = \{A \in SO(X) : x \in A\}$ and let $SO_x = \{A \subset X$: there exists $\mu \subset SO(x)$ such that μ is finite and $\cap\mu \subset A\}$. Then SO_x is called the semi-neighborhood filter at x. For any filter $\mathcal{F}$ on X, we say that $\mathcal{F}$ semi-converges to x if and only if $\mathcal{F}$ is finer than the semi-neighborhood filter SO_x at x. In this paper we introduce the concept of supra-convergences of filters, $s\gamma$-sets, and $s\gamma$ (resp. $s\gamma^*$)-continuity. And we investigate some properties, in particular, a function $f : X \to Y$ is $s\gamma$ (resp. $s\gamma^*$)-continuous if and only if whenever a filter $\mathcal{F}$ supra-converges to x, then $f(\mathcal{F})$ converges(resp. supra-converges) to $f(x)$.

2. sγ-sets

Definition 2.1. Let (X, τ) be a supratopological space. A subset U of X is called an $s\gamma$-set in X if whenever a filter $\mathcal{F}$ on X supra-converges to x and $x \in U$, then $U \in \mathcal{F}$.

The class of all $s\gamma$-sets in X will be denoted by $s\gamma(X)$. In particular, The class of all $s\gamma$-sets induced by the supratopology τ will be denoted by $s\gamma_\tau$.

Remark. From the definition of supra-neighborhood filters and $s\gamma$-sets, easily we can show every supraopen set is an $s\gamma$-set, but the converse is always not true.

Example 2.2. Let X be the real number set and let $S = \{(a, b] : a, b \in R\} \cup \{[c, d) : c, d \in R\}$ be a suprabase for the supratopology τ. For each $x \in X$, since both $(a, x]$ and $[x, b)$ are supraopen sets containing x, $\{x\}$ is an element of $\mathcal{S}_x$. For any filter $\mathcal{F}$ on X, if $\mathcal{F}$ supra-converges to x, then $\mathcal{F}$ includes $\mathcal{S}_x$ and so $\{x\}$ is an $s\gamma$-set. But it is not supra-open.

We recall that; Let (X, τ) be a topological space and τ^* be a supratopology associated with τ. A subset A of X is called an m-set with τ^* if $A \cap T \in \tau^*$ for all $T \in \tau^*$. The class of all m-sets with τ^* will be denoted by τ_m[4].

Obviously, we get the following;

THEOREM 2.3. *If (X, τ) be a topological space and τ^* be a supratopology associated with τ, then $\tau \subset \tau_m \subset \tau^* \subset s\gamma_\tau$.*

THEOREM 2.4. *Let (X, τ) be a supratopological space. The intersection of finitely many supra-open subsets in X is an $s\gamma$-set.*

Proof. Let U_1 and U_2 be supra-open sets in X. For each $x \in U_1 \cap U_2$, it is clearly $U_1 \cap U_2 \in \mathcal{S}_x$, and from the notion of the supra-convergence of filters, whenever every filter $\mathcal{F}$ supra-converges to x, $U_1 \cap U_2 \in \mathcal{F}$. □

DEFINITION 2.5. Let (X, τ) be a supratopological space. The $s\gamma$-interior of a set A in X, denoted by $sI_\gamma(A)$, is the union of all $s\gamma$-sets contained in A.

THEOREM 2.6. *Let (X, τ) be a supratopological space and $A \subset X$.*
(a) $sI_\gamma(A) = \{x \in A : A \in \mathcal{S}_x\}$;
(b) A is $s\gamma$-set if and only if $A = sI_\gamma(A)$.

Proof. (a). For each $x \in sI_\gamma(A)$, there exists an $s\gamma$-set U such that $x \in U$ and $U \subset A$. From the notion of $s\gamma$-sets, the subset U is in the supra-neighborhood filter $\mathcal{S}_x$. And since $\mathcal{S}_x$ is a filter, $A \in \mathcal{S}_x$. Conversely, let $A \in \mathcal{S}_x$, then there exist $U_1, ..., U_n \in S(x)$ such that $U{=}U_1 \cap ... \cap U_n \subset A$. By Theorem 2.4. U is an $s\gamma$-set containing x and since $\mathcal{S}_x$ is a filter, $A \in \mathcal{S}_x$. Thus $x \in sI_\gamma(A)$.
(b). Obvious. □

THEOREM 2.7. *Let (X, τ) be a supratopological space. Then the class $s\gamma(X)$ of all $s\gamma$-subsets in X is a topology on X.*

Proof. Since $\emptyset$ and X are supraopen sets, they are also $s\gamma$-sets in X. Let A and B be non-disjoint $s\gamma$-subsets. For $x \in A \cap B$, if a filter $\mathcal{F}$ on X supra-converges to x, then both A and B are elements of $\mathcal{F}$ and since $\mathcal{F}$ is a filter, the intersection of A and B also an element of $\mathcal{F}$. Thus $A \cap B$ is an $s\gamma$-set. For each $\alpha \in I$, let $A_\alpha \in s\gamma(X)$ and $U = \cup A_\alpha$. For each $x \in U$ and for a filter $\mathcal{F}$ supra-converging to x, by the notion of $s\gamma$-sets, there exists a subset A_α of U such that $x \in A_\alpha$ and $A_\alpha \in \mathcal{F}$, and since $\mathcal{F}$ is a filter, U is an element of the filter $\mathcal{F}$. Thus $U = \cup A_\alpha$ is an $s\gamma$-set. □

Let (X, τ) be a supratopological space. For a subset B of X, we call B an $s\gamma$-closed set if the complement of B is an $s\gamma$-set. From Theorem 2.7, the intersection of any family of $s\gamma$-closed sets is an $s\gamma$-closed set and the union of finitely many $s\gamma$-closed sets is an $s\gamma$-closed set. Obviously we obtain the following, by the definition of $s\gamma$-set.

THEOREM 2.8. *Let (X, τ) be a supratopological space. A set G is $s\gamma$-closed if and only if whenever $\mathcal{F}$ supra-converges to x and $G \in \mathcal{F}$, $x \in G$.*

DEFINITION 2.9. Let (X, τ) be a supratopological space and $A \subset X$,

$$scl_\gamma(A) = \{x \in X : A \cap U \neq \emptyset \text{ for all } U \in \mathcal{S}_x\}.$$

We call $scl_\gamma(A)$ the $s\gamma$-closure of A.

Now we can get the following.

THEOREM 2.10. *Let (X, τ) be a supratopological space. For $A \subset X$,*
(1) $A \subset scl_\gamma(A)$;
(2) A is $s\gamma$-closed if and only if $A = scl_\gamma A$;
(3) $sI_\gamma(A) = X - scl_\gamma(X - A)$;
(4) $scl_\gamma(A) = X - sI_\gamma(X - A)$.

THEOREM 2.11. *Let (X, τ) be a supratopological space. $x \in scl_\gamma(A)$ if and only if there exists a filter $\mathcal{F}$ on X such that $A \in \mathcal{F}$ and $\mathcal{F}$ supra-converges to x.*

Proof. Let $x \in scl(A)$, then by the notion of the $s\gamma$-closure, the collection $\mathcal{B} = \{U \cap A : U \in \mathcal{S}_x\}$ is a filter base. The filter $\mathcal{F}$ generated by filter base $\mathcal{B}$ supra-converges to x and $A \in \mathcal{F}$. Suppose that there is a filter $\mathcal{F}$ supra-converging to x such that $A \in \mathcal{F}$. Since $\mathcal{F}$ contains $\mathcal{S}_x$ and $\mathcal{F}$ is a filter, for all $U \in \mathcal{S}_x$, $U \cap A \neq \emptyset$. Thus $x \in scl_\gamma(A)$. □

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